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I declare this is my own work.

INTERNATIONAL AS
FURTHER MATHEMATICS

(9665/FM02) Unit FPSM1 Pure Mathematics, Statistics and Mechanics

Monday 9 January 2023 07:00 GMT Time allowed: 1 hour 30 minutes

Materials
- For this paper you must have the Oxford International AQA Booklet of Formulae and Statistical Tables (enclosed).
- You may use a graphical calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box around each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

For Examiner's Use	
Question	Mark
1	
2	
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11	
12	
TOTAL	

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ANSWER IN THE SPACES PROVIDED

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Section A

Pure Mathematics

Answer **all** questions in the spaces provided.

1 A curve passes through the point (1, –1) and satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x + \frac{2x^3}{y} \quad \text{for} \quad y \neq 0$$

Use Euler's step-by-step method with a step length of 0.1 to estimate the value of y when $x = 1.2$

Give your answer to four decimal places.

[4 marks]

Answer _____

4

Turn over ►

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2 The variables P and Q are related by the equation

$$Q = aP + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

It is given that $P = \frac{1}{x}$ and $Q = \frac{1}{y}$ where $x \neq 0$ and $y \neq 0$

During an experiment the following pairs of values for x and y were obtained.

x	1	2	3	4
y	1.10	0.88	0.83	0.80

2 (a) Complete the table below giving the values of Q to **two** decimal places.

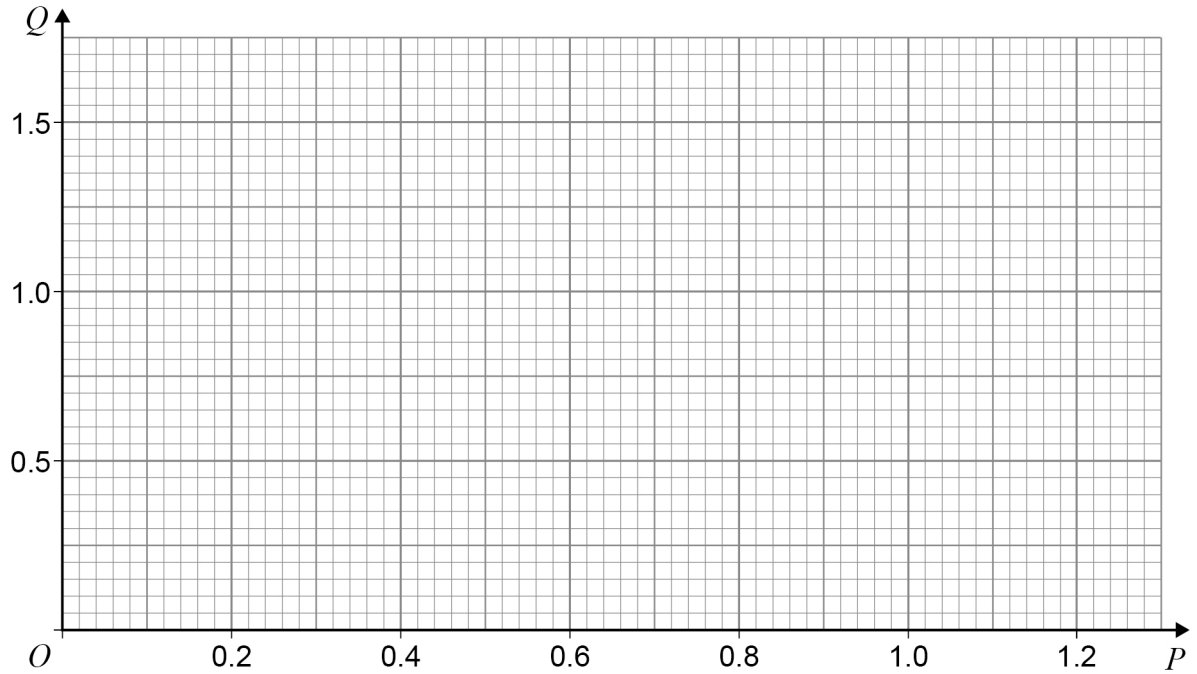
[1 mark]

P	1	0.5	0.33	0.25
Q				

2 (b) On the grid plot Q against P

Draw a line of best fit for the points you have plotted.

[2 marks]



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2 (c) (i) Use your line of best fit to find estimates for a and b **to two significant figures**.

[2 marks]

$a =$ _____ $b =$ _____

2 (c) (ii) Use your values for a and b to write down an equation relating x and y

[1 mark]

Answer _____

2 (d) Use your equation to estimate the value of y when $x = 1.6$

Give your answer to two decimal places.

[1 mark]

Answer _____

7

Turn over ►

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3 The matrix $\mathbf{A}$ is defined by $\mathbf{A} = \begin{bmatrix} 4 & 0 & -3 \\ -2 & 1 & -2 \\ 4 & -2 & 5 \end{bmatrix}$

The matrix $\mathbf{B}$ is defined by $\mathbf{B} = \begin{bmatrix} k & 6k & 3k \\ 0.5 & 8 & 3.5 \\ 0 & 2 & 1 \end{bmatrix}$ where k is a constant.

3 (a) Given that $\mathbf{A} + 4\mathbf{B} = \begin{bmatrix} 20k & 24k & 0 \\ 0 & 33 & 12 \\ 4 & 6 & 9 \end{bmatrix}$ find the value of k

[2 marks]

$k =$ _____

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3 (b) (i) Using your value of k find $\mathbf{AB}$

[2 marks]

Answer _____

3 (b) (ii) Hence find the matrix $\mathbf{C}$ such that $\mathbf{C}\begin{bmatrix} 1 & 6 & 3 \\ 2 & 32 & 14 \\ 0 & 8 & 4 \end{bmatrix} = \mathbf{AB}$

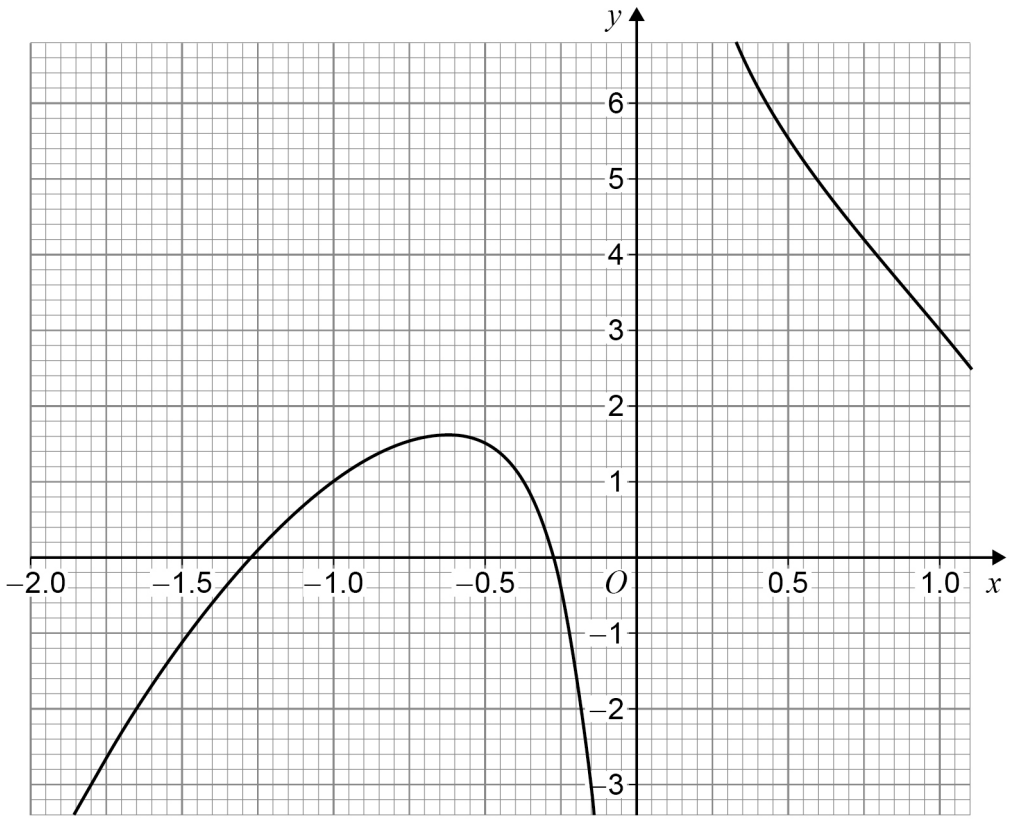
[2 marks]

Answer _____

6

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4 The graph shows part of the curve $y = f(x)$ where $f(x) = \frac{1}{x} - 2x^2 + 4$



The equation $f(x) = 0$ has three roots α, β and γ

The roots α and β are such that $-2 < \alpha < -1$ and $-1 < \beta < 0$

4 (a) Show that $1 < \gamma < 2$

[2 marks]

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4 (b) By drawing a line on the graph opposite, explain why use of the Newton–Raphson method with an initial value of $x_1 = -1$ does not find the root β

[2 marks]

4 (c) Taking $x_1 = -1.3$ as a first approximation to α use the Newton–Raphson method to find a second approximation x_2 to α

Give your answer to four decimal places.

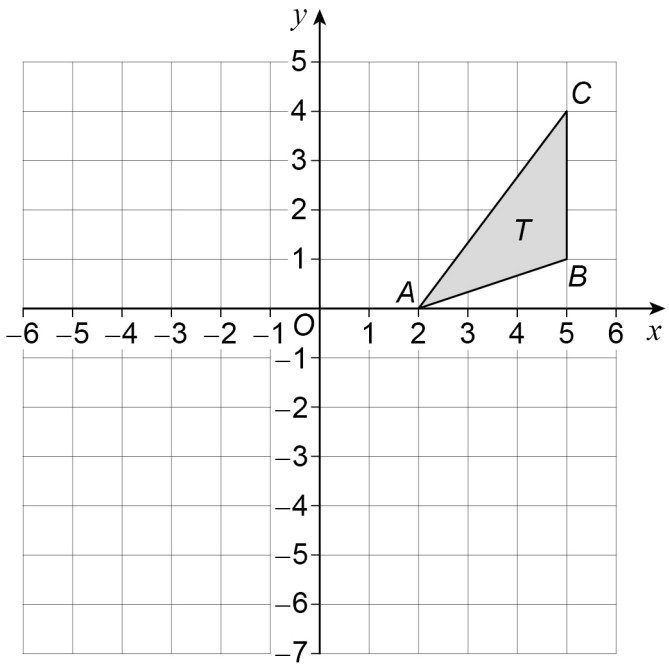
[4 marks]

Answer _____

8

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5 The triangle T with vertices $A(2, 0)$, $B(5, 1)$ and $C(5, 4)$ is shown in the diagram.



The matrix $\mathbf{M} = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$ is used to transform triangle T

The transformation represented by matrix **M** maps ABC onto $A'B'C'$ where A', B' and C' are the vertices of triangle R

The point A' has coordinates $(-1, p)$ where p is a constant.

It is given that p is negative $(p < 0)$

5 (a) (i) Find the exact value of p

[3 marks]

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$p =$ _____

5 (a) (ii) Write down the matrix **M** giving each element as an exact value.

[1 mark]

Answer _____

5 (a) (iii) Describe fully the transformation represented by the matrix **M**

[3 marks]

Question 5 continues on the next page

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5 (b) The matrix **N** is defined by $\mathbf{N} = \begin{bmatrix} c & d \\ d & -c \end{bmatrix}$

The transformation represented by matrix **N** maps triangle R onto triangle S

The point A' is invariant under this transformation.

5 (b) (i) Find the exact values of c and d

[3 marks]

$c =$ _____ $d =$ _____

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5 (b) (ii) Using answers from **parts (a)(ii)** and **(b)(i)**, find the matrix representing the single transformation which maps triangle T onto triangle S

[2 marks]

Answer _____

5 (b) (iii) Describe fully the **single** transformation which maps triangle T onto triangle S

[3 marks]

15

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Section B

Statistics

Answer **all** questions in the spaces provided.

6 The discrete random variable X has probability generating function

$$G_X(t) = (0.2 + 0.8t)^3$$

6 (a) Using differentiation find $\mathrm{E}(X)$

[4 marks]

Answer _____

6 (b) Find $\mathrm{P}(X \geq 2)$

[1 mark]

Answer _____

5

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7 The probability that Imran is late for school on a randomly chosen day is p

The discrete random variable X represents the number of days up to and including the first day Imran is late for school.

The variance of X is 3.75

7 (a) Find the value of p

[3 marks]

$p =$ _____

7 (b) Find the mean of X

[1 mark]

Answer _____

7 (c) Find the probability that the first day Imran is late for school is during the next 5 days.

[2 marks]

Answer _____

6

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8 The discrete random variable X has a uniform distribution and takes values

1, 2, 3, …, 7

The discrete random variable Y has a uniform distribution and takes values

1, 2, 3, …, 10

8 (a) Find $E(4X - 3Y)$

[4 marks]

Answer _____

8 (b) Given that $\text{Var}(X + Y) = 10$ find the value of the correlation between X and Y, giving your answer to three decimal places.

[5 marks]

Answer _____

9

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Section C

Mechanics

Answer **all** questions in the spaces provided.

9 A student is creating a model for the magnitude of the air resistance force, F newtons, acting on a sphere as it moves at a speed of v m s^{-1}

The student assumes that

$$F = kv^n$$

where k and n are constants.

Find the dimensions of k in terms of n

[3 marks]

Answer _____

3

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10 A disc of mass 0.3 kg is sliding at a speed of 6 m s^{-1} on a smooth horizontal surface when it collides with a vertical wall that is perpendicular to its path.

The disc rebounds at a speed of 4 m s^{-1} after the collision with the wall.

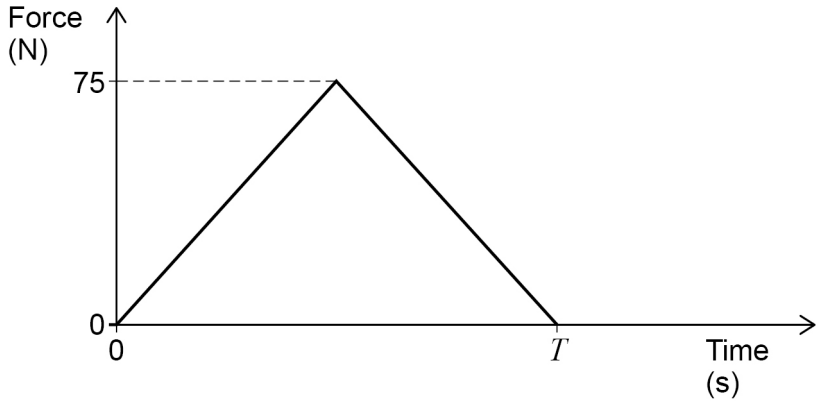
10 (a) Calculate the magnitude of the impulse exerted by the wall on the disc.

[2 marks]

Answer _____

10 (b) The graph shows how the magnitude of the force exerted by the wall on the disc varies with time.

The disc is in contact with the wall for T seconds.



Find the value of T

[2 marks]

$T =$ _____

4

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11 Hannah and Rachel start jogging at the same time from different positions and move with constant velocities on a horizontal field.

The table shows their initial position vectors and constant velocities.

	Initial Position Vector (m)	**Constant Velocity (m s^{-1})**
Hannah	$-20\mathbf{i}+450\mathbf{j}$	$2\mathbf{i}-0.5\mathbf{j}$
Rachel	$80\mathbf{i}+150\mathbf{j}$	$\mathbf{i}+U\mathbf{j}$

U is a constant.

Hannah and Rachel meet at a position on the field and then stop.

Find the value of U

[4 marks]

$U =$ _____

4

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12 Two spheres, P and Q, of the same radius are at rest on a smooth horizontal table.

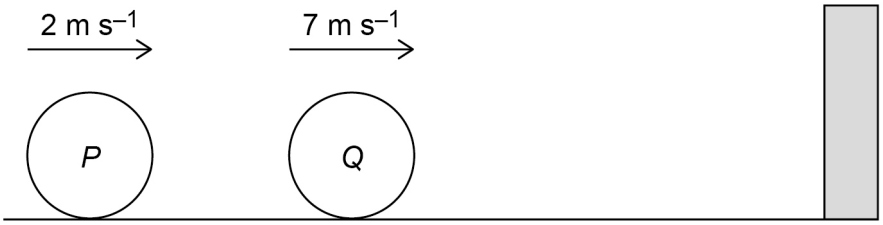
A vertical wall is perpendicular to the straight line through the centres of P and Q

Sphere Q is closer to the wall than sphere P

The spheres are set into motion so that they both move directly towards the wall.

Sphere P has mass 0.3 kg and initially moves with speed 2 m s^{-1}

Sphere Q has mass 0.5 kg and initially moves with speed 7 m s^{-1}



The coefficient of restitution between each sphere and the wall is 0.8

The coefficient of restitution between the two spheres is 0.5

12 (a) Find the magnitude of the impulse on P when the spheres collide, giving your answer to **three significant figures**.

[7 marks]

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Answer _____

12 (b) When the spheres collide, they are in contact for 0.075 seconds.

Find the magnitude of the average force exerted by one sphere on the other during the collision.

[2 marks]

Answer _____

9

END OF QUESTIONS

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