OXFORD

INTERNATIONAL AQA EXAMINATIONS

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# INTERNATIONAL AS
# FURTHER MATHEMATICS

(9665/FM02) Pure Maths, Statistics and Mechanics Unit FPSM1

Thursday 24 January 2019 07:00 GMT Time allowed: 1 hour 30 minutes

**Materials**
- For this paper you must have the Oxford International AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer the questions in the spaces provided. Do not write outside the box on each page or on blank pages.
- If you need extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do all rough work in this book. Cross through any work you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- There are three sections to this paper.
- The maximum mark for this paper is 80. There are 40 marks for **Section A**, 20 marks for **Section B** and 20 marks for **Section C**.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- Show all necessary working; otherwise marks may be lost.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| **TOTAL** | |

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## Section A

Answer **all** questions in the spaces provided.

**1** A curve passes through the point (1.2, 0.8) and satisfies the differential equation

$$\frac{dy}{dx} = \frac{1}{x^2+y^2}$$

Use Euler's step-by-step method with a step length of 0.1 to estimate the value of $y$ when $x = 1.4$

Give your answer to three decimal places.

**[5 marks]**

Answer _____

5

*Do not write outside the box*

**2** The equation

$$x^3 - 8x - 12 = 0$$

has a real root, $\alpha$, in the interval $3 < x < 4$

Use linear interpolation to show that $\alpha$ is in the interval $\frac{96}{29} < x < 4$

**[5 marks]**

5

**Turn over for the next question**

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**3** The variables $x$ and $y$ are thought to be related by an equation of the form

$$y = ax^n$$

where $a$ and $n$ are constants.

Let $Y = \log_{10} y$ and $X = \log_{10} x$

**3 (a)** Show that there is a linear relationship between $X$ and $Y$.

**[3 marks]**

**3 (b)** An experiment gives the following values of $x$ and $y$.

| $x$ | 2 | 4 | 6 | 8 |
|---|---|---|---|---|
| $y$ | 19 | 92 | 232 | 458 |

Complete the table below.

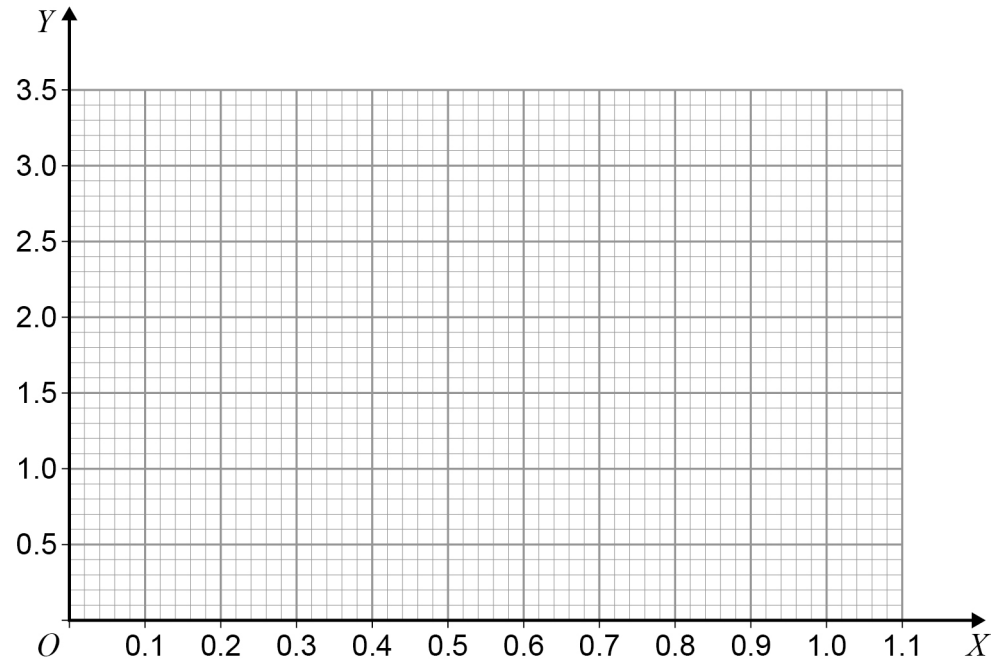
| $X$ | 0.301 | | | |
|---|---|---|---|---|
| $Y$ | 1.28 | | | |

**[2 marks]**

*Do not write outside the box*

**3 (c)** On the grid below, draw a linear graph relating $X$ and $Y$.

**[2 marks]**




**3 (d)** Use your graph to find estimates for $a$ and $n$.

**[4 marks]**

$a =$ _____

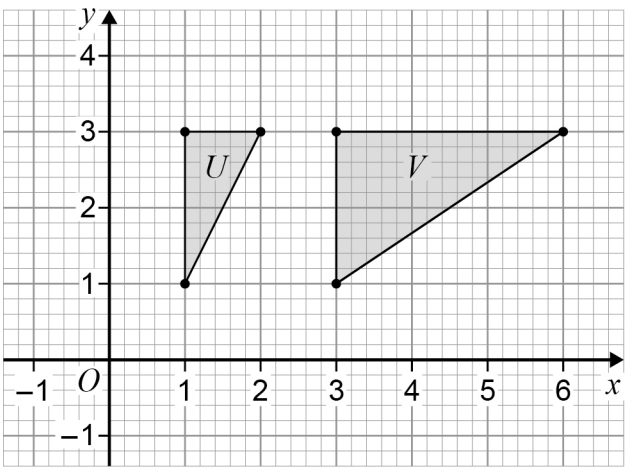
$n =$ _____

11

**Turn over ►**

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**4 (a)** The matrix $\mathbf{P}$ represents the transformation which maps triangle $U$ to triangle $V$ in the diagram below.




Write down the matrix $\mathbf{P}$.

**[1 mark]**

Answer _____

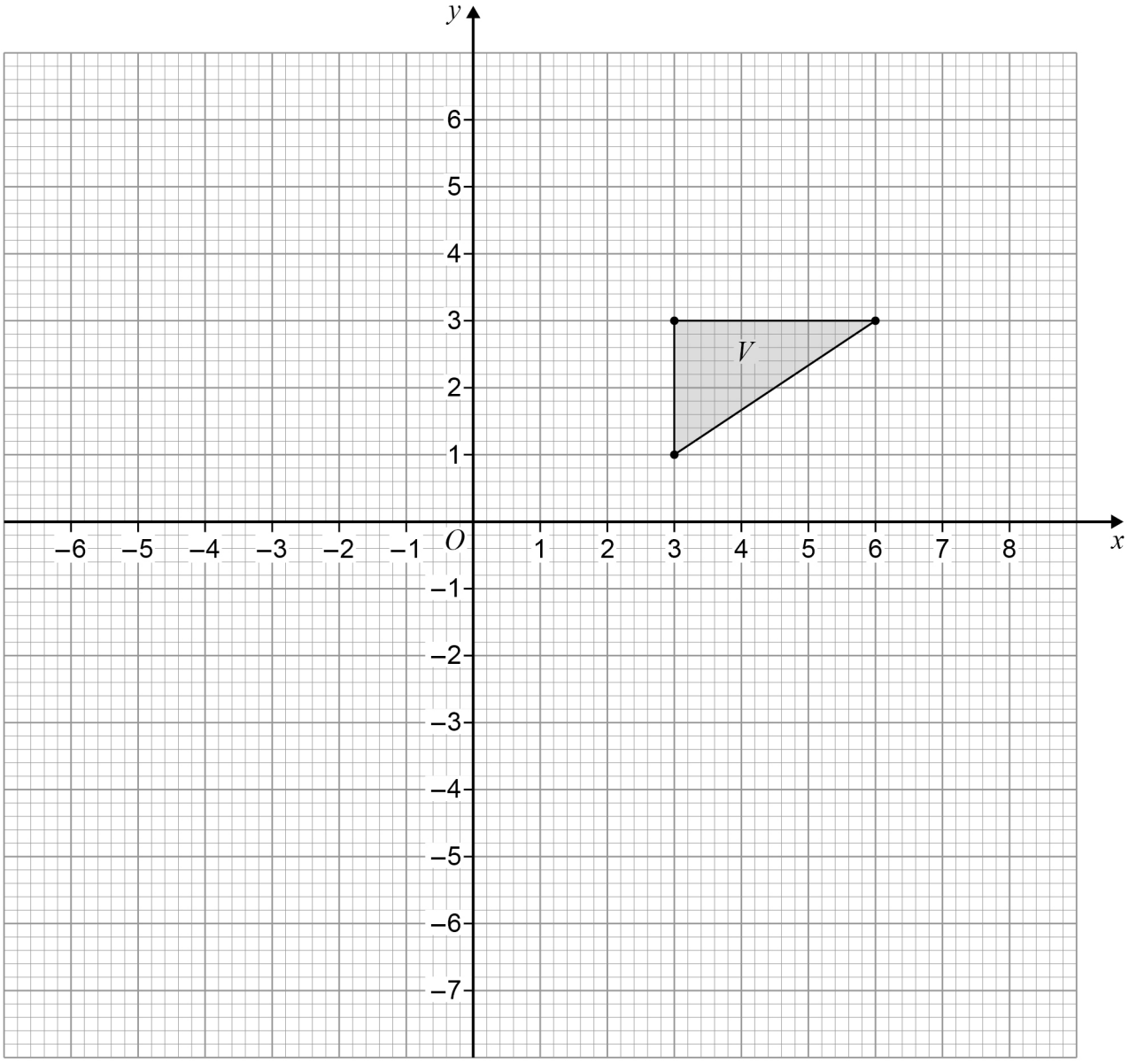
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**4 (b)** The transformation R is a reflection in the line $y = -x$

R maps triangle $V$ to triangle $W$.

Draw triangle $W$ on the diagram below.

**[2 marks]**




**Question 4 continues on the next page**

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**4 (c)** The matrix $\mathbf{Q}$ represents the transformation R.

Find the determinant of $\mathbf{QP}$.

**[3 marks]**

Answer _____

6

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**ANSWER IN THE SPACES PROVIDED**

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**5** The matrix $\mathbf{B}$ is defined by $\mathbf{B} = \begin{bmatrix} -1 & 1 \\ -1 & -1 \end{bmatrix}$

The matrix $\mathbf{C}$ is defined by $\mathbf{C} = \begin{bmatrix} 2 & 4 \\ 2 & 6 \end{bmatrix}$

The matrix $\mathbf{A}$ is such that $\mathbf{AB} = \mathbf{C}$.

**5 (a)** Find $\mathbf{A}$.

**[4 marks]**

Answer _____

*Do not write outside the box*

**5 (b)** Show that $\mathbf{B}^4 = k\mathbf{I}$ for some integer $k$, where $\mathbf{I}$ is the 2 × 2 identity matrix.

**[4 marks]**

**5 (c)** The matrix $\mathbf{B}$ represents a combination of an enlargement and a rotation.

**5 (c) (i)** Find the scale factor of the enlargement.

**[1 mark]**

Answer _____

**5 (c) (ii)** Find the angle of the rotation.

**[1 mark]**

Answer _____

*Do not write outside the box*

**5 (d)** Use your answer to part **(b)** to find $\mathbf{B}^{21}$

**[3 marks]**

Answer _____

**13**

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## Section B

Answer **all** questions in the spaces provided.

**6** $X$ is a discrete uniform distribution where $x$ can take the value 1, 2 or 3

**6 (a)** Find P($X$ > 1)

**[1 mark]**

Answer _____

**6 (b)** Derive the probability generating function, $G_X(t)$, of $X$.

**[2 marks]**

Answer _____

*Do not write outside the box*

**6 (c)** $Y$ is a discrete random variable with probability generating function $G_Y(t)$ where

$$G_Y(t) = 0.6 + 0.4t$$

Given that $X$ and $Y$ are independent, find $P(X + Y = 1)$

**[3 marks]**

Answer _____

6

**Turn over ►**

*Do not write outside the box*

**7** To prepare for a tennis tournament, Holly plays practice tennis matches until she wins a match.

The results of different tennis matches are independent.

The probability that Holly wins a practice tennis match is 0.6

Let $H$ represent the event that Holly plays more than 3 practice tennis matches.

**7 (a)** Find P($H$).

**[1 mark]**

Answer _____

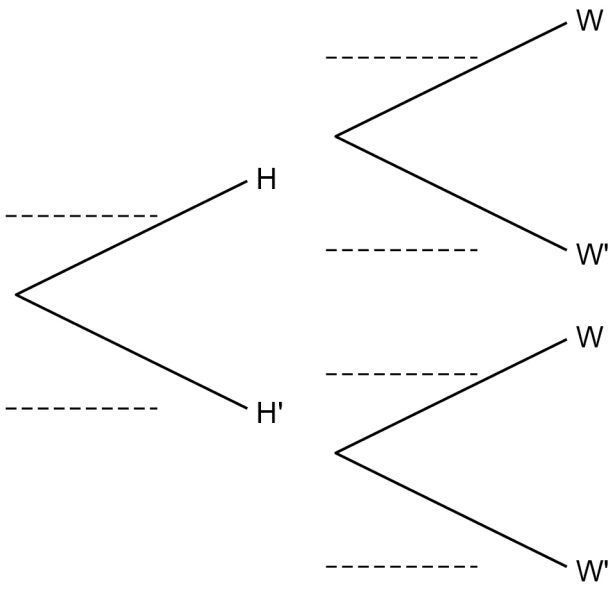
**7 (b)** Let $W$ represent the event that Holly wins the tennis tournament.

Given that P($W$) = 0.7 and P($H \mid W$) = 0.01, complete the tree diagram opposite by finding the probability for each branch.

**[5 marks]**

**6**

*Do not write outside the box*




**Turn over for the next question**

*Do not write outside the box*

**8** The discrete random variable $P$ has:

$E(P) = 2$

$E(P^2) = 5$

$E(P^3) = 14.6$

The discrete random variable $Q$ is such that $Q = 2P^2 - 5$

**8 (a)** Find $\mathrm{Cov}(P, Q)$.

**[5 marks]**

Answer _____

*Do not write outside the box*

**8 (b)** The variance of $Q$ is 8

Find $\mathrm{Var}(P + Q)$.

**[3 marks]**

Answer _____

8

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## Section C

Answer **all** questions in the spaces provided.

**9** A model for the resistance force, $R$ newtons, acting on a sphere moving with speed $v$ m s$^{-1}$ through a fluid is given by

$$R = kv^{\frac{3}{2}}$$

where $k$ is a constant.

Find the dimensions of $k$ in terms of $\mathrm{M}$, $\mathrm{L}$ and $\mathrm{T}$.

**[2 marks]**

Answer _____

2

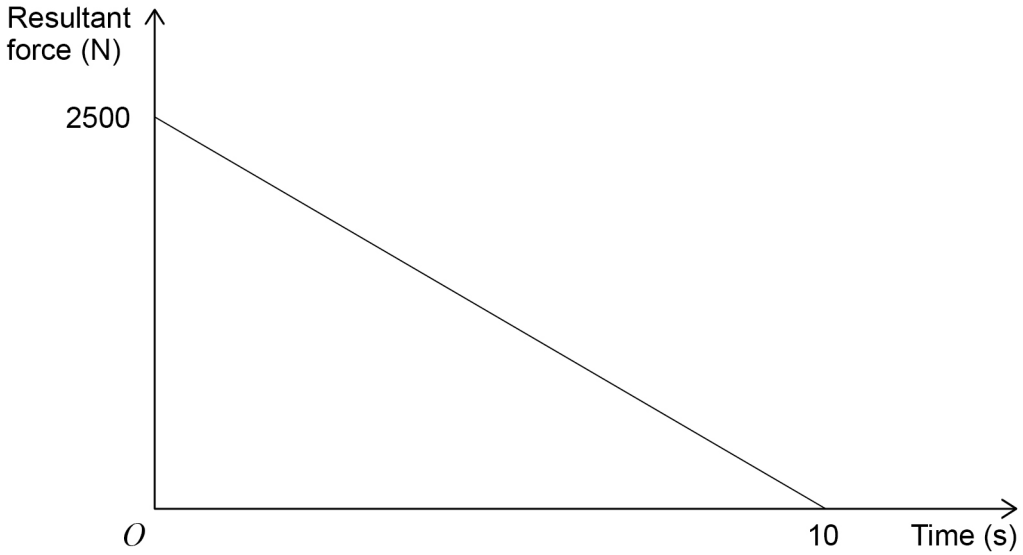
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**10** During a 10-second interval, the speed of a van decreases from 20 m $s^{-1}$ to $U$ m $s^{-1}$ as it moves along a straight road.

The mass of the van is 2000 kg.

The graph shows how the magnitude of the resultant force acting on the van varies during the 10 seconds.




**10 (a)** Calculate the magnitude of the impulse on the van over the 10-second interval.

**[1 mark]**

Answer _____

*Do not write outside the box*

**10 (b)** Find $U$.

**[3 marks]**

Answer _____

4

**Turn over ►**

*Do not write outside the box*

**11** Two discs, *A* and *B*, are the same size.

Disc *A* has mass $m$ and disc *B* has mass $4m$.

The discs are moving in the same direction along a straight line on a smooth horizontal surface when they collide.

Just before the collision, disc *A* has speed $5U$ and disc *B* has speed $U$.

The coefficient of restitution between the two discs is $e$.

**11 (a)** Find the speed of *B* after the collision, giving your answer in terms of $e$ and $U$.

**[5 marks]**

Answer _____

*Do not write outside the box*

**11 (b)** After the collision, $A$ moves in the opposite direction.

Find an inequality that $e$ must satisfy.

**[3 marks]**

Answer _____

8

**Turn over ►**

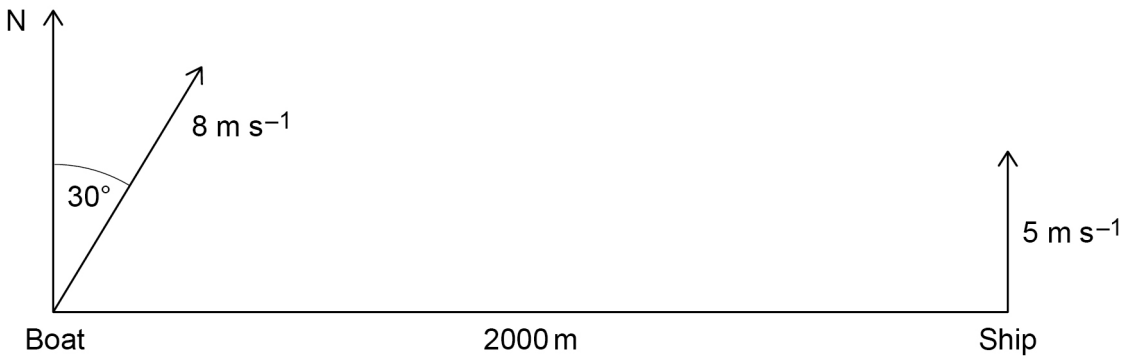
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**12** A boat is moving with a constant velocity of 8 m $s^{-1}$ on a bearing of 030°

A ship is moving north with a constant velocity of 5 m $s^{-1}$

Initially the ship is 2000 metres east of the boat.

The diagram shows the initial positions and the velocities of the boat and the ship.




Find the minimum distance between the boat and the ship giving your answer to 3 significant figures.

**[6 marks]**

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Answer _____

6

**END OF QUESTIONS**

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