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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Friday 7 June 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 4 2 8 5 R A 0 1 3 2

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The quadratic equation

$$kx^2 - (2k + 6)x + 16 = 0$$

has equal roots.

Find the two possible values of k

(4)

(Total for Question 1 is 4 marks)

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2 (a) Expand $\frac{2}{\sqrt{1+3x}}$ in ascending powers of x up to and including the term in x^3

Express each coefficient as a fraction in its simplest terms where appropriate.

(4)

(b) State the range of values of x for which the expansion is valid.

(1)

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Question 2 continued

(Total for Question 2 is 5 marks)

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3 O, A and B are fixed points such that

$$\left|\overrightarrow{OA}\right| = 3\sqrt{5} \qquad \overrightarrow{AB} = \mathbf{i} + 3a\mathbf{j} \qquad \overrightarrow{OB} = 7\mathbf{i} + 2a\mathbf{j}$$

Given that $a > 0$

(a) find the value of a

(4)

(b) Hence find a unit vector parallel to $\overrightarrow{OA}$

(2)

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Question 3 continued

(Total for Question 3 is 6 marks)

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4 $f(x) = px^3 + qx^2 - 37x - 12q$ where p and q are constants.

When $f'(x)$ is divided by $(x + 2)$ the remainder is -33

Given that $(x + 5)$ is a factor of $f(x)$

(a) (i) show that $p = 2$

(ii) find the value of q

(6)

(b) Hence, use algebra to factorise $f(x)$ completely.

(3)

(c) Hence solve the equation $f(x) = 0$

(2)

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Question 4 continued

(Total for Question 4 is 11 marks)

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5 The force F newtons between two magnetic poles is given by the formula

$$F = \frac{3}{20r^2}$$

where r is the distance, in centimetres, between the poles.

The distance between the two poles is increasing at a constant rate of 0.7 cm/s

Find the rate of change of F, in newtons/s to 3 significant figures, when the distance between the poles is 2.8 cm

(6)

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Question 5 continued

(Total for Question 5 is 6 marks)

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6

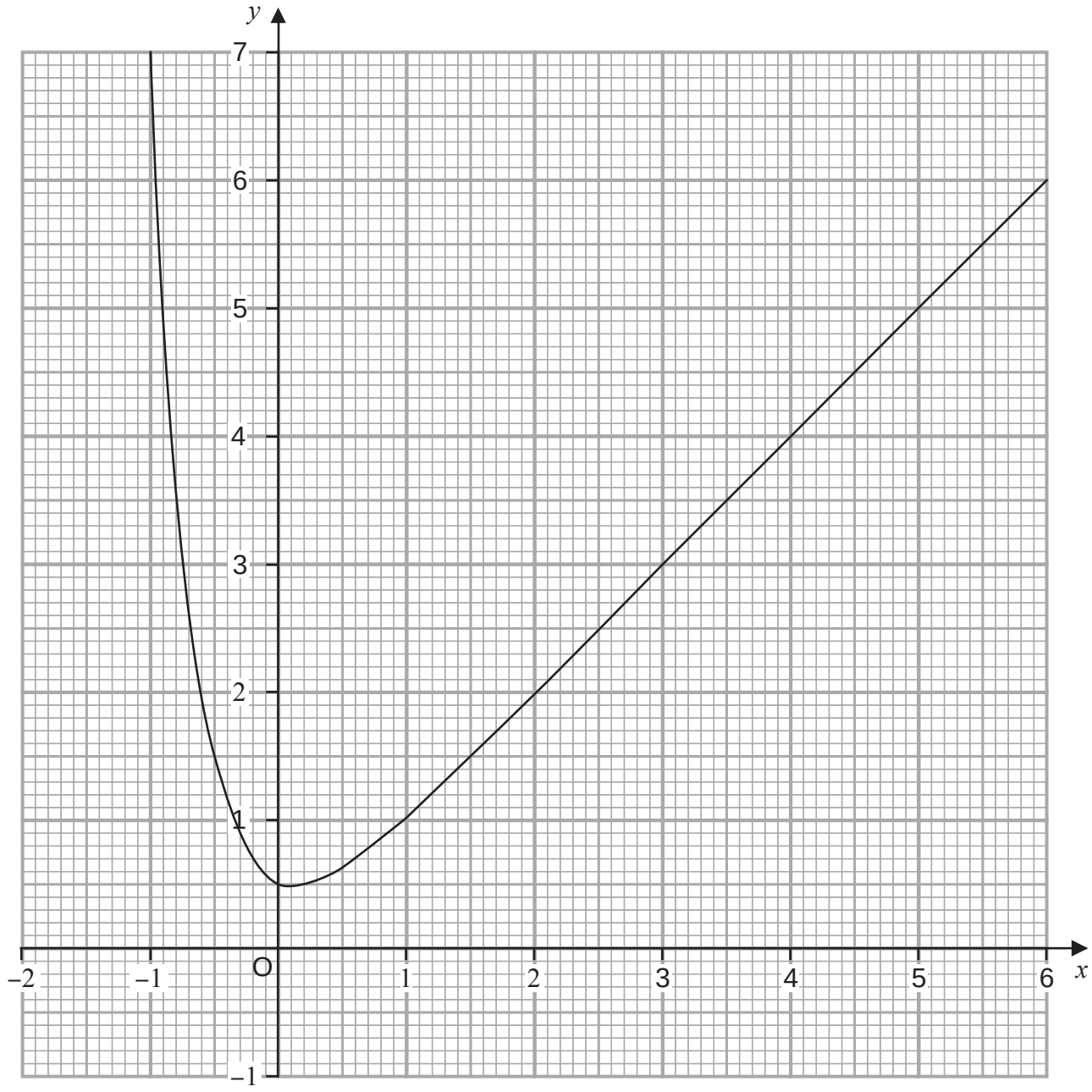


Figure 1

Figure 1 shows part of the graph of the curve with equation $y = x + 2^{-(4x+1)}$

By drawing a suitable straight line on the graph, obtain an estimate, to one decimal place, of the roots of the equation $\log_2(8 - 3x) + 4x = 0$ in the interval $-2 \leqslant x \leqslant 6$

(7)

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Question 6 continued

(Total for Question 6 is 7 marks)

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7

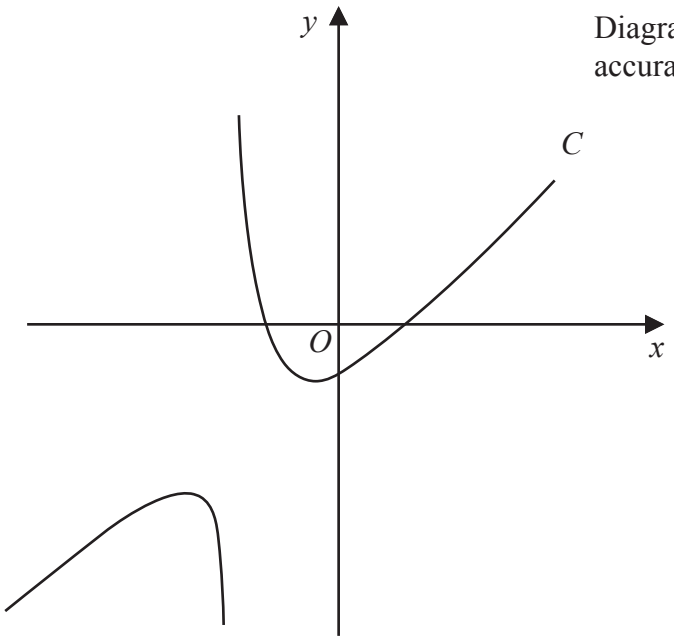


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{x^2 - 1}{4x + 5} \quad \text{where} \quad x \neq -\frac{5}{4}$$

(a) Write down the equation of the asymptote to C that is parallel to the y-axis.

(1)

The line l is the normal to C at the point where $x = -1$

(b) Find an equation of l

(7)

The line l meets C again at the point D

(c) Find the coordinates of D

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 14 marks)

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8 The sum of the first 2 terms of a geometric series G is 360
The sum of the 2nd and 3rd terms of G is 288

The nth term of G is U_n

(a) Show that $U_n = A\left(\frac{4}{5}\right)^{n-1}$ where A is an integer to be found.

(7)

(b) Explain why G is convergent.

(1)

(c) Hence find the sum to infinity of G

(2)

(d) Find the least number of terms for which the sum is greater than 978

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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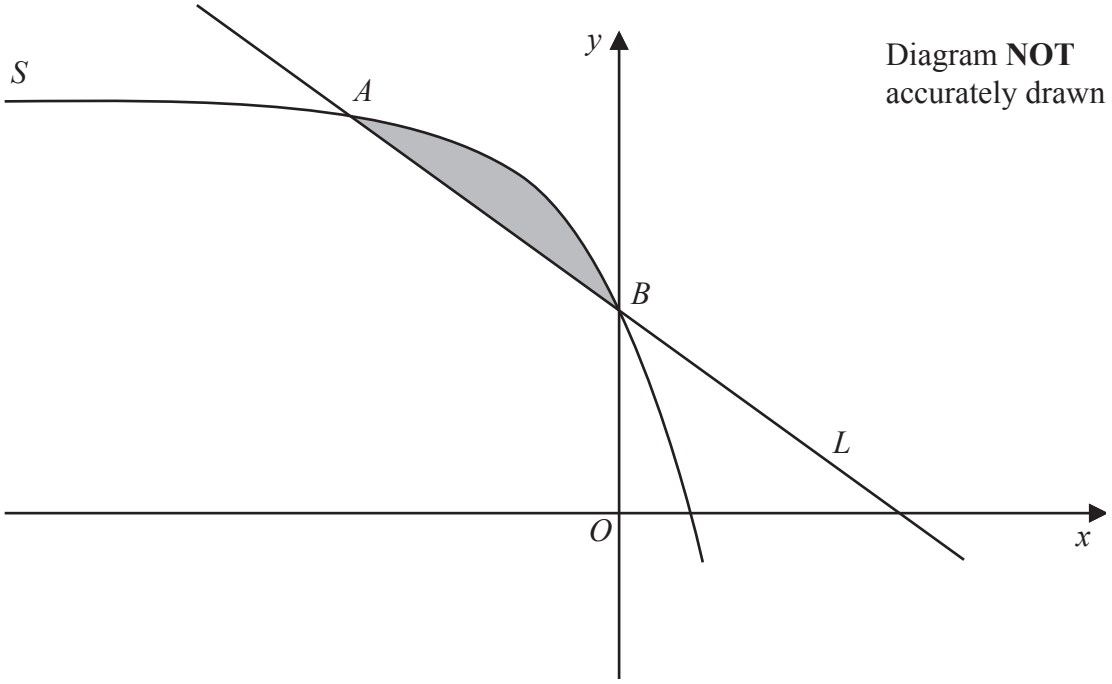


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a sketch of part of the curve S with equation $y = -2e^{3x} + 4$ and the line L

The curve S has intersections with the line L at the points A and B with x coordinates $x = -1$ and $x = 0$ respectively.

The finite region bounded by S and L is shown shaded in Figure 3

Use calculus to find the exact area of this region.

Give your answer in the form $\dfrac{a + be^{-c}}{c}$ where a, b and c are integers to be found.

(8)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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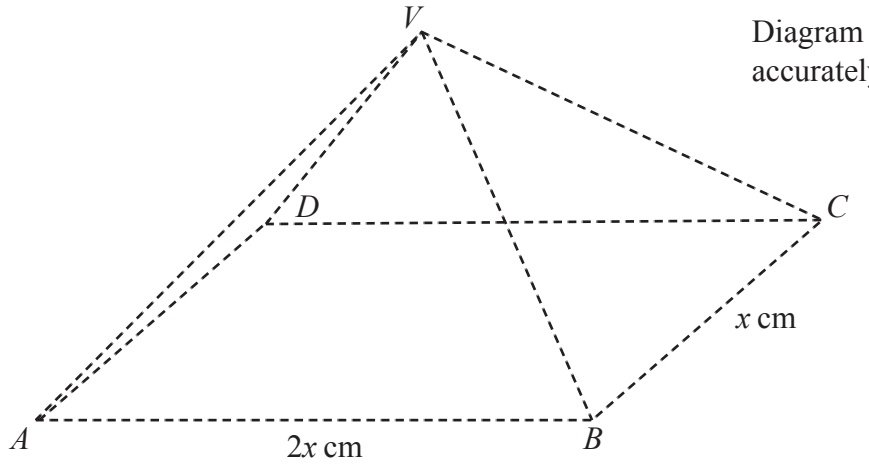


Figure 4

Figure 4 shows a right pyramid $ABCDV$

The base of the pyramid is a rectangle where,

$$AB = DC = 2x \text{ cm} \quad AD = BC = x \text{ cm}$$

The edges VA, VB, VC and VD are all of equal length.

The angle between VA and $ABCD$ is 45°

(a) Show that $VA = \frac{\sqrt{10}}{2} x$ cm

(3)

(b) Find in cm, the exact height of the pyramid in terms of x

(2)

Find, in degrees to one decimal place,

(c) the size of angle VBA

(2)

(d) the size of the obtuse angle between the plane AVC and the plane BVD

(4)

Given that the volume of the pyramid is $9\sqrt{5}$ cm^3

(e) find the value of x

(2)

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 13 marks)

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11 (a) Using a formula on page 2 show that $\cos 2A = 2\cos^2 A - 1$

(2)

(b) Hence show that $\left(2\cos^2 A - 1\right)^2 = \dfrac{\cos 4A + 1}{2}$

(3)

The curve with equation $y = \dfrac{\sin 2x}{2} + \dfrac{\left(2\cos^2 x - 1\right)^2}{2} + \dfrac{1}{8}$ has a stationary point P in the range $0 \leqslant x \leqslant \dfrac{\pi}{6}$

(c) Find the exact coordinates of P

(7)

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

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