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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

## Wednesday 18 October 2023

Morning (Time: 1 hour 30 minutes)

Paper reference **WMA13/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, $\alpha$, in the interval $[1, 2]$

**(2)**

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root $\alpha$.

(b) (i) Find the value of $x_2$ to 4 decimal places.

(ii) Find, by repeated iteration, the value of $\alpha$, giving your answer to 4 decimal places.

**(3)**

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**Question 1 continued**

**(Total for Question 1 is 5 marks)**

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**2.** The function f is defined by

$$f(x) = \frac{x+3}{x-4} \qquad x \in \mathbb{R}, x \neq 4$$

(a) Find ff(6)

**(2)**

(b) Find $f^{-1}$

**(3)**

The function g is defined by

$$g(x) = x^2 + 5 \qquad x \in \mathbb{R}, x > 0$$

(c) Find the exact value of $a$ for which

$$gf(a) = 7$$

**(3)**

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**Question 2 continued**

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**Question 2 continued**

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**Question 2 continued**

**(Total for Question 2 is 8 marks)**

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**3.** (a) Using the identity for $\cos(A + B)$, prove that

$$\cos 2A \equiv 2\cos^2 A - 1$$

**(2)**

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

**(4)**

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**Question 3 continued**

**(Total for Question 3 is 6 marks)**

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**4.** A new mobile phone is released for sale.

The total sales $N$ of this phone, in **thousands**, is modelled by the equation

$$N = 125 - Ae^{-0.109t} \qquad t \geqslant 0$$

where $A$ is a constant and $t$ is the time in months after the phone was released for sale.

Given that when $t = 0$, $N = 32$

(a) state the value of $A$.

**(1)**

Given that when $t = T$ the total sales of the phone was 100 000

(b) find, according to the model, the value of $T$. Give your answer to 2 decimal places.

**(3)**

(c) Find, according to the model, the rate of increase in total sales when $t = 7$, giving your answer to 3 significant figures.

(*Solutions relying entirely on calculator technology are not acceptable*.)

**(2)**

The total sales of the mobile phone is expected to reach 150 000

Using this information,

(d) give a reason why the given equation is not suitable for modelling the total sales of the phone.

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 7 marks)**

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**5.** The curve $C$ has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where $k$ is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where $A$ and $B$ are constants to be found.

**(3)**

Given that $C$ has exactly three turning points,

(b) find the $x$ coordinate of each of these points. Give your answer in terms of $k$ where appropriate.

**(3)**

(c) find the upper limit to the value for $k$.

**(1)**

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**Question 5 continued**

**(Total for Question 5 is 7 marks)**

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**6.** An area of sea floor is being monitored.

The area of the sea floor, $S\,\text{km}^2$, covered by coral reefs is modelled by the equation

$$S = pq^t$$

where $p$ and $q$ are constants and $t$ is the number of years after monitoring began.

Given that

$$\log_{10} S = 4.5 - 0.006t$$

(a) find, according to the model, the area of sea floor covered by coral reefs when $t = 2$

**(2)**

(b) find a complete equation for the model in the form

$$S = pq^t$$

giving the value of $p$ and the value of $q$ each to 3 significant figures.

**(3)**

(c) With reference to the model, interpret the value of the constant $q$

**(1)**

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**Question 6 continued**

**(Total for Question 6 is 6 marks)**

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**7.**

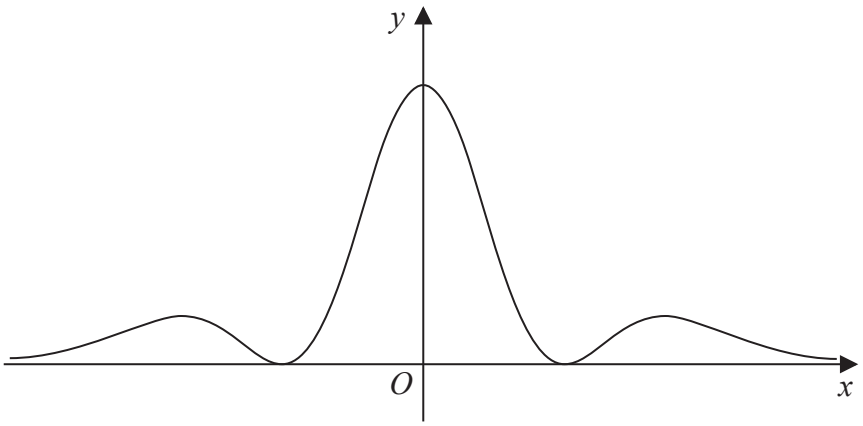


**Figure 1**

Figure 1 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}\left(2x^2 - 3\right)^2$$

(a) Find the range of f

**(2)**

(b) Show that

$$\mathrm{f}'(x) = 2x\left(2x^2 - 3\right)\mathrm{e}^{-x^2}\left(A - Bx^2\right)$$

where $A$ and $B$ are constants to be found.

**(4)**

Given that the line $y = k$, where $k$ is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of $k$

**(4)**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 10 marks)**

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**8.** (a) Prove that

$$2\,\text{cosec}^2 2\theta(1 - \cos 2\theta) \equiv 1 + \tan^2\theta$$

**(4)**

(b) Hence solve for $0 < x < 360°$, where $x \neq (90n)°$, $n \in \mathbb{N}$, the equation

$$2\,\text{cosec}^2 2x(1 - \cos 2x) = 4 + 3\sec x$$

giving your answers to one decimal place.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(4)**

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**Question 8 continued**

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**Question 8 continued**

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**Question 8 continued**

**(Total for Question 8 is 8 marks)**

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**9.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

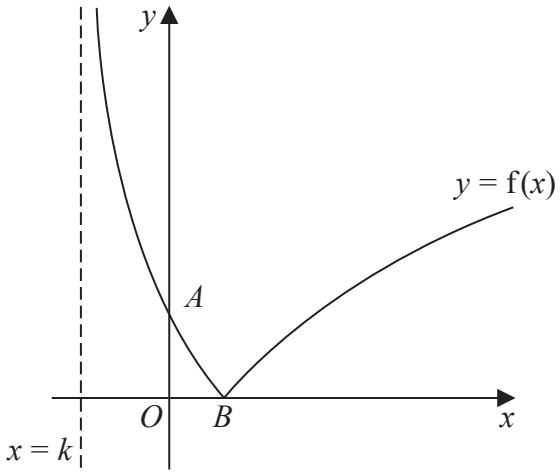


**Figure 2**

Figure 2 shows a sketch of the curve with equation

$$y = |2 - 4\ln(x + 1)| \qquad x > k$$

where $k$ is a constant.

Given that the curve

- has an asymptote at $x = k$
- cuts the $y$-axis at point $A$
- meets the $x$-axis at point $B$

as shown in Figure 2,

(a) state the value of $k$

**(1)**

(b) (i) find the $y$ coordinate of $A$

(ii) find the exact $x$ coordinate of $B$

**(3)**

(c) Using algebra and showing your working, find the set of values of $x$ such that

$$|2 - 4\ln(x + 1)| > 3$$

**(5)**

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**Question 9 continued**

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**Question 9 continued**

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**Question 9 continued**

**(Total for Question 9 is 9 marks)**

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**10.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve $C$ has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point $P$ with $x$ coordinate $\frac{1}{4}$ lies on $C$

(a) Find the exact $y$ coordinate of $P$

**(2)**

(b) Find $\frac{dx}{dy}$

**(2)**

(c) Hence show that $\frac{dy}{dx}$ can be written in the form

$$\frac{dy}{dx} = \frac{1}{\sqrt{q + r(x + s)^2}}$$

where $q$, $r$ and $s$ are constants to be found.

**(3)**

Using the answer to part (c),

(d) (i) state the $x$ coordinate of the point where the value of $\frac{dy}{dx}$ is a minimum,

(ii) state the value of $\frac{dy}{dx}$ at this point.

**(2)**

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**Question 10 continued**

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**Question 10 continued**

**(Total for Question 10 is 9 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

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