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Please check the examination details below before entering your candidate infor on

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Find the set of values of x for which

(a) $2(3x - 1) < 4 - 3x$

(2)

(b) $3x^2 - 8x - 3 < 0$

(4)

(c) **both** $2(3x - 1) < 4 - 3x$ **and** $3x^2 - 8x - 3 < 0$

(1)

(Total for Question 1 is 7 marks)

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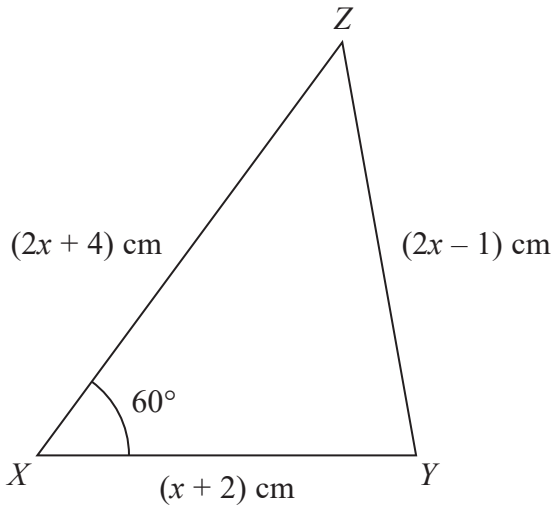
2



Diagram
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Figure 1

Figure 1 shows triangle XYZ in which

$XY = (x + 2)$ cm $XZ = (2x + 4)$ cm $YZ = (2x - 1)$ cm and $\angle YXZ = 60°$

Find the value of x

Give your answer in the form $p + q\sqrt{3}$ where p and q are integers to be found.

(4)

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Question 2 continued

(Total for Question 2 is 4 marks)

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3

$$f(x) = 8x^2 + 10x - 3$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A, B and C are constants,

(a) find the value of A, the value of B and the value of C.

(3)

(b) Hence, or otherwise, find,

(i) the value of x for which $f(x)$ has a minimum,

(ii) the minimum value of $f(x)$.

(2)

The curve C has equation $y = f(x)$.

(c) Find the x coordinate of each of the points where C crosses the x-axis.

(2)

The straight line l has equation $y = 2x + 13$

(d) Use algebra to find the coordinates of the two points of intersection of C and l.

(4)

Using the same axes and the results of parts (b), (c) and (d),

(e) sketch the curve C and the straight line l.

(2)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 13 marks)

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4 The equation of a curve is $y = x^3 \sin x$

Find an equation of the tangent to the curve at the point on the curve where $x = \frac{1}{2}\pi$

Give your answer in the form $y = mx + c$

(7)

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Question 4 continued

(Total for Question 4 is 7 marks)

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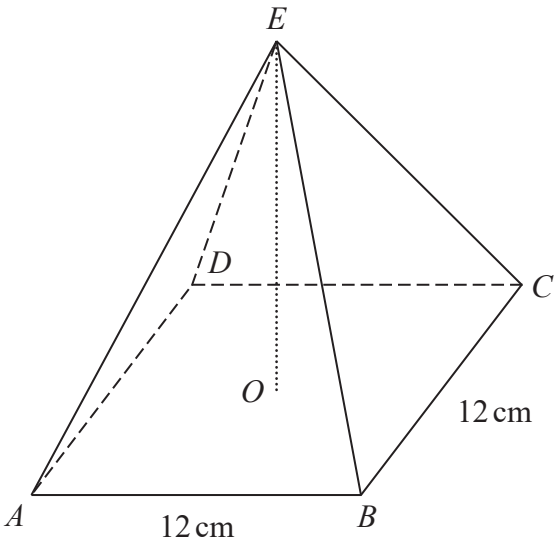

Diagram
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Figure 2

Figure 2 shows a right pyramid with a square base $ABCD$ and vertex E.

The base of the pyramid is horizontal with $AB = BC = 12$ cm.
The diagonals of the base intersect at the point O.

The vertex E of the pyramid is vertically above O and the angle between EA and the plane $ABCD$ is 30°

The height of the pyramid is h cm.

(a) Find the exact value of h

(3)

The point F lies on AD such that $AF : FD = 1 : 4$

(b) Calculate, to the nearest degree, the size of the angle EFO.

(4)

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Question 5 continued

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 7 marks)

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6 The nth term of a geometric series G is U_n

The first three terms of G are given by

$$U_1 = q(4p + 1) \qquad U_2 = q(2p + 3) \qquad U_3 = q(2p - 3)$$

(a) Find the possible values of p

(5)

Given that G is convergent with sum to infinity 250

(b) find the value of q

(3)

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Question 6 continued

(Total for Question 6 is 8 marks)

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7 $y = e^{2x} \cos 2x$

(a) Show that

$$\frac{dy}{dx} = 2y - 2e^{2x} \sin 2x$$

(4)

(b) Hence show that

$$\frac{d^2y}{dx^2} = 4\frac{dy}{dx} - 8y$$

(5)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

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8 The quadratic equation

$$x^2 - 4k\sqrt{2}x + 2k^4 - 1 = 0$$

where k is a positive constant, has roots α and β

Given that $\alpha^2 + \beta^2 = 66$ and that $\alpha^3 + \beta^3 = p\sqrt{2}$ where p is an integer,

find the value of p

(11)

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 11 marks)

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9 A cube has edges of length x cm.

The total surface area, A cm^2, of the cube is increasing at a constant rate of 0.45 cm^2/s

Find the rate of increase, in cm^3/s, of the volume of the cube at the instant when the total surface area of the cube is 384 cm^2

(7)

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Question 9 continued

(Total for Question 9 is 7 marks)

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10 Using formulae given on page 2

(a) show that

(i) $\sin 2\theta = 2\sin\theta\cos\theta$

(ii) $\cos 2\theta = 2\cos^2\theta - 1$

(5)

Given that $\theta \neq (90° + 180°n)$ where $n \in \mathbb{Z}$

(b) use the results from part (a) to show that $\sin 2\theta - \tan\theta$ can be written as $\tan\theta\cos 2\theta$

(4)

(c) Solve for $0 < x < 360$

$$\sin 2x° - \tan x° = 0$$

(4)

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Question 10 continued

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Question 10 continued

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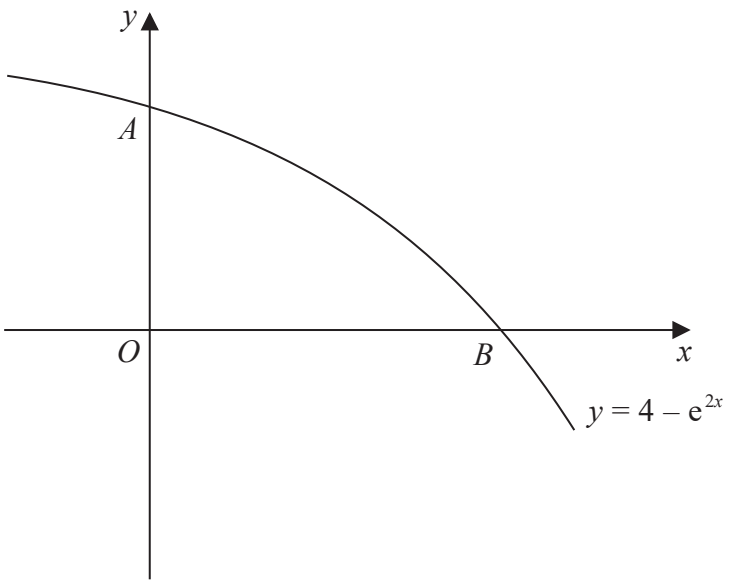
11



Diagram
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Figure 3

Figure 3 shows part of the curve C with equation $y = 4 - e^{2x}$
The curve C crosses the y-axis at the point A and the x-axis at the point B.

(a) (i) Write down the y coordinate of point A.

(ii) Show that the x coordinate of B is $x = \ln 2$

(3)

The line l is the normal to C at the point B.

(b) Find an equation for l, giving your answer in the form $y = mx + c$

(4)

The finite region R is bounded by C, l and the y-axis.

(c) Using calculus, find the area of R.
Give your answer to one decimal place.

(7)

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Question 11 continued

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