www.dynamicpapers.com

Please check the examination details below before entering your candidate infor on

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01R**

Further Pure Mathematics
PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P71818A

©2023 Pearson Education Ltd.

J:1/1/1/

P 7 1 8 1 8 A 0 1 2 8

Pearson

www.dynamicpapers.com

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that $\dfrac{a-\sqrt{48}}{\sqrt{3}+1}$ can be written in the form $b\sqrt{3}-9$ where a and b are integers,

find the value of a and the value of b
Show your working clearly.

(4)

(Total for Question 1 is 4 marks)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

2 In ΔABC,

$$\angle BAC = 50° \qquad AB = 10\,\text{cm} \qquad BC = 9\,\text{cm}$$

Given that $\angle BCA = x°$

find the two possible values, to one decimal place, of x

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 3 marks)

www.dynamicpapers.com

3 An arithmetic series has first term 16 and common difference –5

The sum to n terms of this series is S_n

Given that $S_n < -450$

find the least value of n

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 4 marks)

www.dynamicpapers.com

4 O, A and B are fixed points such that

$$\overrightarrow{OA} = p\mathbf{i} + 2p\mathbf{j} \qquad \overrightarrow{OB} = 5\mathbf{i} + 9p\mathbf{j}$$

Given that $\overrightarrow{AB}$ is parallel to $(\mathbf{i} - 2\mathbf{j})$

(a) find the value of p

(6)

(b) Hence find $\overrightarrow{AB}$ as a simplified expression in terms of **i** and **j**

(2)

(c) Find a unit vector parallel to $\overrightarrow{OA}$

Give your answer in the form $\frac{\sqrt{a}}{5}(b\mathbf{i} + c\mathbf{j})$ where a, b and c are integers to be found.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 12 marks)

www.dynamicpapers.com

5 $f(x) = 2ax^3 + x^2 - bx + 3a$ where a and b are integers.

Given that $(x + 2)$ and $(x - 1)$ are both factors of $f(x)$

(a) show that $a = 2$ and find the value of b

(5)

(b) Hence factorise $f(x)$ completely.

(2)

Hence, given that $h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6$

(c) solve the equation $h(y) = 0$

Where appropriate give your answers to 3 decimal places.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 5 continued

(Total for Question 5 is 12 marks)

www.dynamicpapers.com

6 The curve C has equation $y = \dfrac{e^{(x^2+1)}}{x^2+1}$

(a) Show that $\dfrac{dy}{dx} = \dfrac{Kx^3e^{(x^2+1)}}{(x^2+1)^2}$ where K is a constant whose value is to be found.

(5)

(b) Find an equation of the tangent to C at the point on C where $x = -1$
Simplify your answer.

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 10 marks)

www.dynamicpapers.com

7 A particle P is moving along the x-axis. At time t seconds $(t \geqslant 0)$ the velocity of P is v m/s where

$$v = t^2 - 10t + 28$$

(a) Find the velocity of P when $t = 1$

(1)

Given that the distance of P from the origin is 24 m when $t = 3$

(b) find the distance of P from the origin when $t = 5$

(5)

(c) Find the acceleration of P when $t = 9$

(2)

(d) (i) Show that there are no values of t for which P is instantaneously at rest.

(ii) Find the least magnitude of the velocity of P

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

www.dynamicpapers.com

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 11 marks)

www.dynamicpapers.com

8

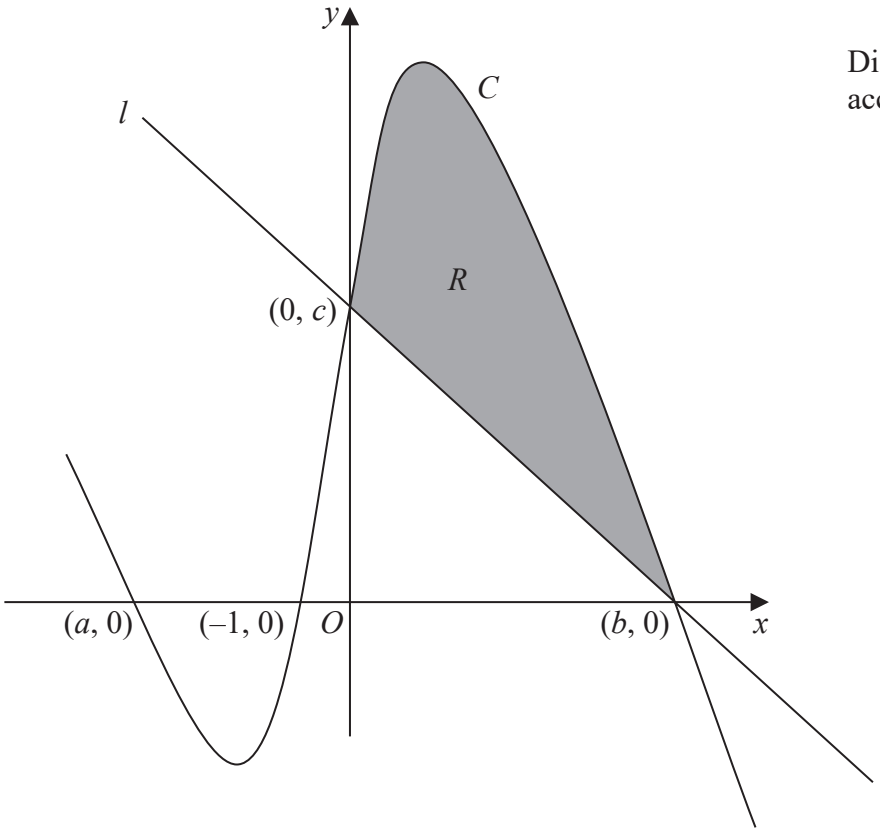

Diagram
accurately dr

Figure 1

Figure 1 shows part of the curve C with equation $y = f(x)$

The curve C passes through the points with coordinates

$$(a, 0), (-1, 0), (b, 0) \text{ and } (0, c)$$

Given that $f'(x) = 17 + 2x - 3x^2$

(a) show that the equation of C is $y = 15 + 17x + x^2 - x^3$

(4)

(b) Find the value of a, the value of b and the value of c

(6)

The straight line l intersects C at the points with coordinates $(b, 0)$ and $(0, c)$

The region R, shown shaded in Figure 1, is bounded by l and C

(c) Use algebraic integration to find the exact area of region R

(5)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 15 marks)

www.dynamicpapers.com

9

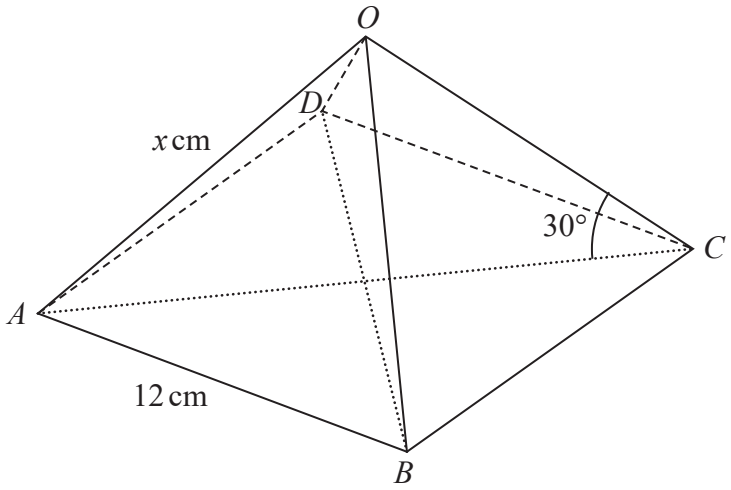

Diagram **N**
accurately dra

Figure 2

Figure 2 shows the right pyramid $OABCD$ with a square base $ABCD$ of side 12 cm.

$$OA = OB = OC = OD = x \text{ cm} \quad \text{and} \quad \angle OAC = \angle ODB = \angle OCA = \angle OBD = 30°$$

(a) Find the exact length of AC

(2)

(b) Show that $x = 4\sqrt{6}$

(2)

(c) Find the total surface area, to the nearest cm^2, of the pyramid.

(5)

(d) Find the size of the obtuse angle, to the nearest degree, between the plane OAB and the plane OBC

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

www.dynamicpapers.com

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 13 marks)

www.dynamicpapers.com

10 Using formulae from page 2

(a) show that $\cos(A - B) - \cos(A + B) = 2 \sin A \sin B$

(2

(b) Hence show that $\cos 5\theta - \cos 9\theta = 2 \sin 7\theta \sin 2\theta$

(1)

(c) Solve the equation

$$\cos 5\theta - \cos 9\theta = \sqrt{3} \sin 7\theta \quad \text{for} \quad 0 < \theta \leqslant \frac{1}{3}\pi$$

Give your solutions in terms of π

(7)

(d) Using calculus and showing your working, evaluate

$$\int_0^{\frac{\pi}{7}} 8 \sin 7x \cos 2x \tan 2x \, \mathrm{d}x$$

Give your answer to 3 decimal places.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 10 continued

www.dynamicpapers.com

Question 10 continued

(Total for Question 10 is 16 marks)

TOTAL FOR PAPER IS 100 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA