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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Time 2 hours

Paper reference **4PM1/02**

Further Pure Mathematics

PAPER 2



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

P66027RRA

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Pearson

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$ $\quad$ $\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$ $\quad$ $\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Use the formula for $\cos(A + B)$ to show that $\cos 2A = 1 - 2\sin^2 A$

(2)

Given that $\sin A = \frac{x + 1}{2}$ and $\cos 2A = \frac{4 - y}{3}$

(b) show that $y = \frac{1}{2}(3x^2 + 6x + 5)$

(3)

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Question 1 continued

(Total for Question 1 is 5 marks)

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2 The finite region enclosed by the curve with equation $y = 4 - x^2$ and the line with equation $y = x + 2$ is rotated through 360° about the x-axis.

Use algebraic integration to find the exact volume of the solid formed.

(6)

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Question 2 continued

(Total for Question 2 is 6 marks)

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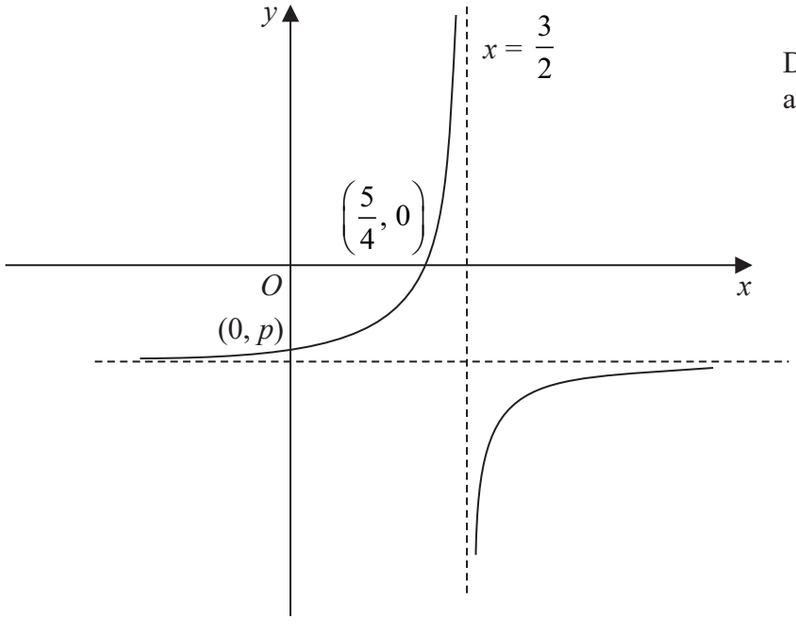

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sketch of the curve with equation

$$y = \frac{a - bx}{cx - d} \qquad x \neq \frac{d}{c}$$

where a, c and d are prime numbers and b is an integer.

The asymptote to the curve that is parallel to the y-axis has equation $x = \frac{3}{2}$

(a) Write down the value of c and the value of d

(2)

The curve crosses the x-axis at the point $\left(\frac{5}{4}, 0\right)$

(b) Find the value of a and the value of b

(2)

The curve crosses the y-axis at the point $(0, p)$ where p is a rational number.

(c) Find the value of p

(2)

(d) Find an equation of the asymptote to the curve that is parallel to the x-axis.

(1)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

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4 The curve C has equation $y = 2x^2 + px + q$ where p and q are integers.

The curve C has a stationary point at $(3, -5)$

(a) Show that $p = -12$ and find the value of q

(4)

(b) State, giving a reason, the nature of the stationary point.

(1)

(c) Find an equation of the normal to C at the point on C where $x = 1$

Give your answer in the form $ax + by + c = 0$

(6)

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Question 4 continued

(Total for Question 4 is 11 marks)

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5 y and x vary so that $y = xe^{-2x}$

Given that the value of x increases by 3%, use calculus to find, in terms of x, an estimate for the percentage change in y
Give your answer in the form $a(b - cx)$ where a, b and c are integers.

(6)

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Question 5 continued

(Total for Question 5 is 6 marks)

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6 A particle P is moving along the x-axis. At time t seconds ($t \geqslant 0$) the displacement, s metres, of P from the origin O, is given by $s = t^3 - 4t^2 - 16t - 8$

(a) Find the distance of P from O when $t = 0$

(1)

(b) Find the value of t for which P is instantaneously at rest.

(4)

(c) Find the value of t for which P is accelerating at $10\,\text{m/s}^2$ in the positive x direction.

(3)

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Question 6 continued

(Total for Question 6 is 8 marks)

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7 A geometric series G has first term a and common ratio r

The sum of the first three terms of G is $\frac{61}{6}$

The sum to infinity of G is $\frac{125}{6}$

(a) (i) Show that $r = \frac{4}{5}$

(ii) Find the value of a

(6)

The sum of the first n terms of G is S_n

Given that $S_n > 19.8$

(b) show that $n \lg\left(\frac{4}{5}\right) < \lg\left(\frac{31}{625}\right)$

(2)

(c) Hence find the least value of n

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 10 marks)

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8 (a) Complete the table of values for $y = 2x + \frac{3}{x^2} - 3$ giving your answers to 2 decimal places where appropriate.

x	0.5	0.75	1	1.5	2	3	4	5
y	10		2				5.19	7.12

(2)

(b) On the grid opposite, draw the graph of $y = 2x + \frac{3}{x^2} - 3$ for $0.5 \leqslant x \leqslant 5$

(2)

(c) By drawing a suitable straight line on the grid, obtain estimates, to one decimal place, of the roots of the equation $4x^3 - 10x^2 + 3 = 0$ for $0.5 \leqslant x \leqslant 5$

(5)

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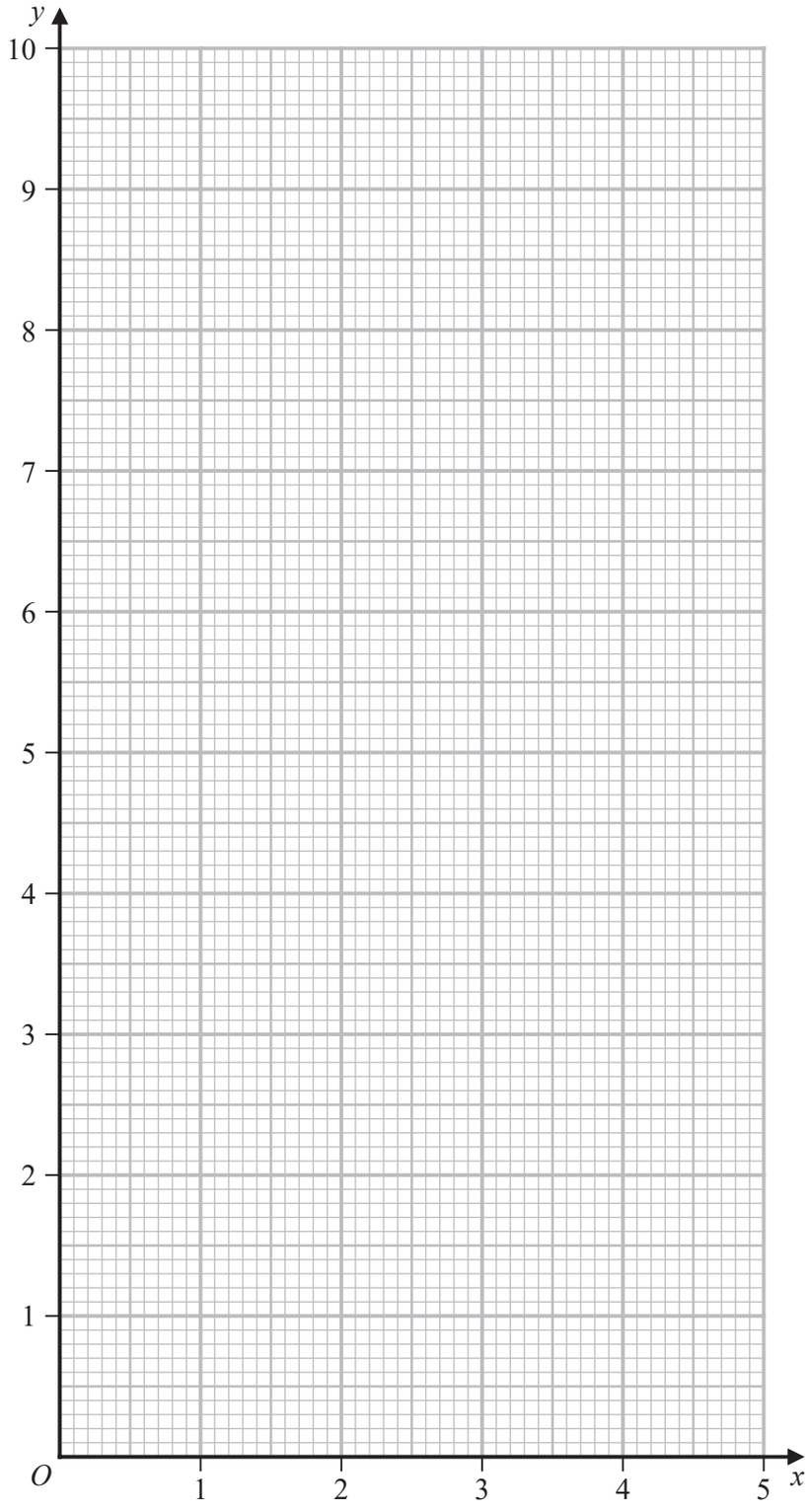
Question 8 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 8 continued

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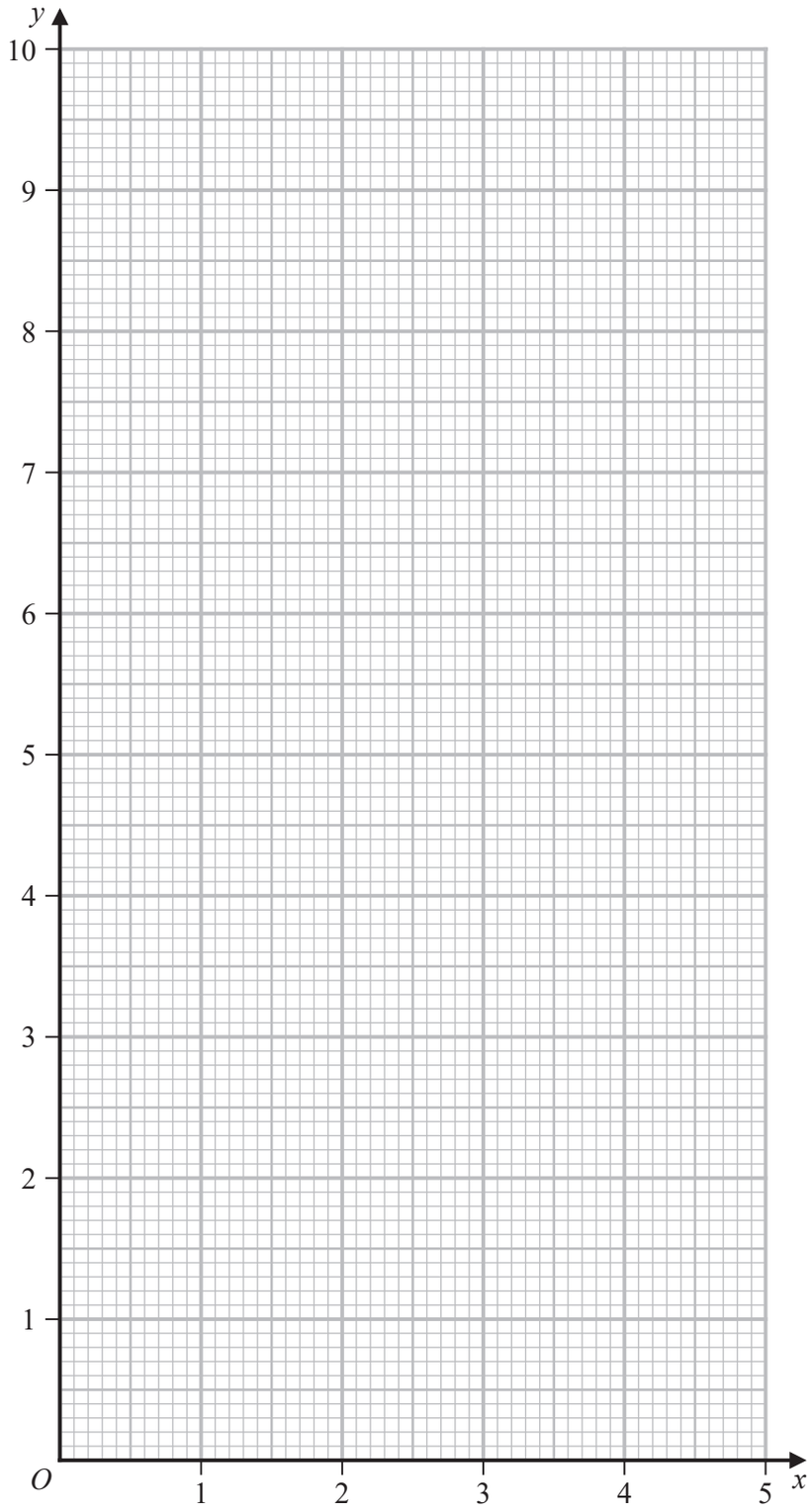
Only use this grid if you need to redraw your graph.



(Total for Question 8 is 9 marks)

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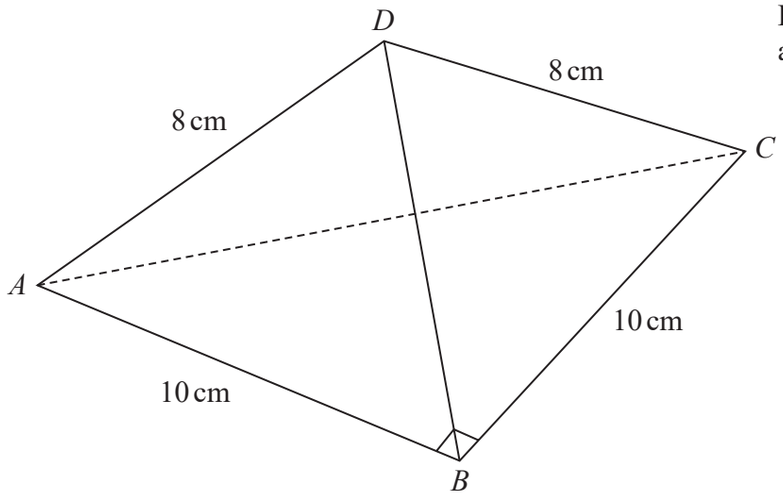

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows a triangular pyramid $ABCD$ with base ABC

$AB = BC = 10$ cm $\quad AD = CD = 8$ cm $\quad \angle ABC = 90°$

(a) Find the exact length of AC

Give your answer in the form $p\sqrt{q}$ cm where p is an integer and q is a prime number.

(2)

The point M is the midpoint of AC

(b) Find the exact length of BM

Give your answer in the form $m\sqrt{n}$ cm where both m and n are prime numbers.

(2)

Given that $BD = 6$ cm,

(c) find, in degrees to one decimal place, the size of the acute angle between the plane ACD and the plane ABC

(4)

The base ABC of the pyramid is placed on a horizontal plane.

(d) Find, in cm to 3 significant figures, the vertical height of D above the base.

(2)

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Question 9 continued

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Question 9 continued

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Question 9 continued

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10 (a) Show that $\dfrac{3}{\sqrt{9-3x}} = \left(1-\dfrac{x}{3}\right)^{-\frac{1}{2}}$

(2)

(b) Hence expand $\dfrac{3}{\sqrt{9-3x}}$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(3)

$$f(x) = \frac{1+2x}{\sqrt{9-3x}}$$

(c) Find the expansion of $3f(x)$ in ascending powers of x up to and including the term in x^3 expressing each coefficient as an exact fraction in its lowest terms.

(4)

(d) Hence, using algebraic integration, obtain an approximation to 6 significant figures for

$$\int_{0.1}^{0.2} \frac{1+2x}{\sqrt{9-3x}}\,dx$$

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 13 marks)

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11 The points A and B have coordinates $(-3, -5)$ and $(7, 5)$ respectively.

(a) Find an equation for the line AB

(2)

The point C has coordinates $(p, 1)$ where $p < 0$

Given that AC and BC are perpendicular,

(b) prove that $p = -5$

(7)

The point D, where BCD is a straight line, is such that C divides BD in the ratio $4 : 3$

(c) Find the coordinates of D

(2)

(d) (i) Find the exact length of AC

(ii) Hence, or otherwise, find the area of triangle ABD

(4)

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Question 11 continued

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Question 11 continued

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