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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Time 2 hours

Paper reference **4PM1/01**

Further Pure Mathematics

PAPER 1



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

P66026RA

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\qquad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The roots of the equation $4x^2 - 3x - 8 = 0$ are α and β

Without solving this equation, form a quadratic equation, with integer coefficients, which has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

(7)

(Total for Question 1 is 7 marks)

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2 $f(x) = 2x^2 + (p - 1)x - 2p$ where p is a constant.

Find the set of values of p for which the equation $f(x) = 0$ has two distinct real roots.

(5)

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Question 2 continued

(Total for Question 2 is 5 marks)

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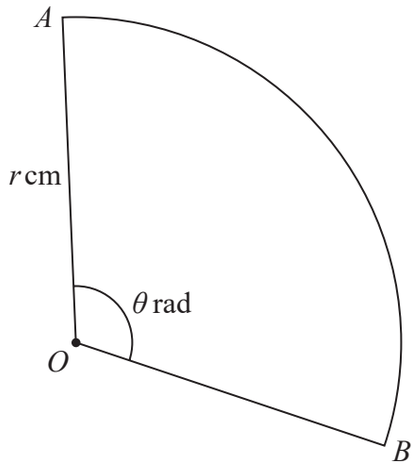

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the sector AOB of a circle with centre O and radius r cm, where r is an integer. The size of angle AOB is θ radians.

The sector has an area of 16.8 cm^2 and a perimeter of 16.4 cm.

Calculate

(i) the value of r

(ii) the value of θ

(8)

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Question 3 continued

(Total for Question 3 is 8 marks)

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4 $y = \dfrac{\sin 2x}{\sqrt{x^2 - 9}} \qquad |x| > 3$

Show that $\dfrac{dy}{dx} = \dfrac{2(x^2 - 9)\cos 2x - x\sin 2x}{\sqrt{(x^2 - 9)^3}}$

(5)

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Question 4 continued

(Total for Question 4 is 5 marks)

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5 Solve the equation

$$\log_3 \sqrt{x-5} + \log_9 (x+3) - 1 = 0$$

Show clear algebraic working.

(7)

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Question 5 continued

(Total for Question 5 is 7 marks)

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6 The volume of a sphere with radius r cm is increasing at a constant rate of $3\,\text{cm}^3/\text{s}$.

Find the rate, in cm^2/s, at which the surface area of the sphere is increasing when $r = 10$

(6)

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Question 6 continued

(Total for Question 6 is 6 marks)

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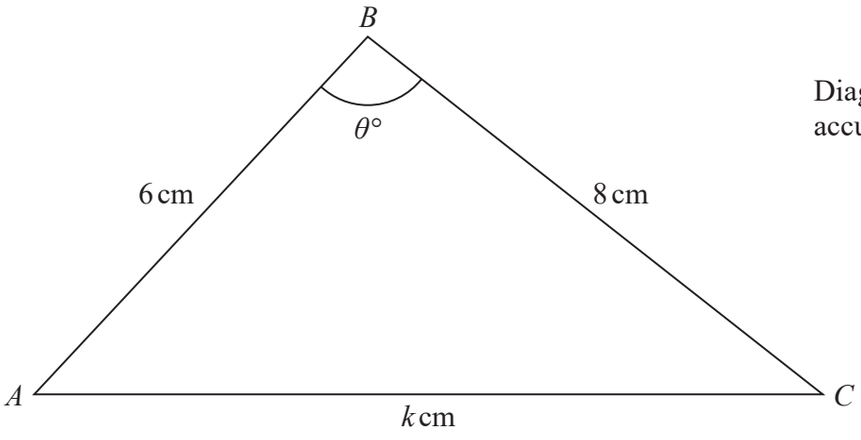

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows triangle ABC

$$AB = 6\,\text{cm} \quad BC = 8\,\text{cm} \quad AC = k\,\text{cm} \quad \angle ABC = \theta°$$

(a) Show that $\cos\theta° = \dfrac{100 - k^2}{96}$

(2)

The area of triangle ABC is $\sqrt{455}$ cm^2

(b) Find the two possible values of k

(7)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 9 marks)

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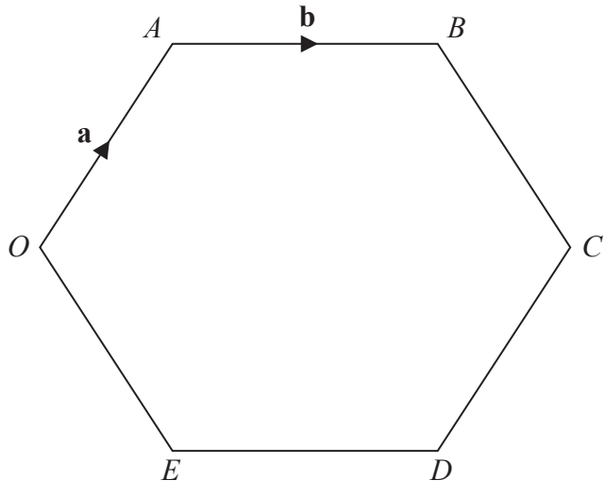

Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows the regular hexagon $OABCDE$ with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{AB} = \mathbf{b}$

(a) Find $\overrightarrow{OB}$ in terms of **a** and **b**

(1)

(b) Find $\overrightarrow{BC}$ as a simplified expression in terms of **a** and **b**

(3)

The point M divides BC in the ratio $2:1$

(c) Find $\overrightarrow{OM}$ as a simplified expression in terms of **a** and **b**

(2)

The point Y is such that OMY and ABY are straight lines.

(d) Use a vector method to find $AB:BY$

(5)

The area of hexagon $OABCDE$ is $60\,\text{cm}^2$

(e) Find the area of triangle OAY

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

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9 (a) Show that $\sum_{r=1}^{n}(5r-1) = \frac{n}{2}(3+5n)$

(3)

(b) Hence, or otherwise, evaluate $\sum_{r=10}^{20}(5r-1)$

(3)

The sum of the first n terms of an arithmetic series is S_n where $S_n = \sum_{r=1}^{n}(5r-1)$

The rth term of this series is u_r

Given that $S_n = 12u_{n+1} + 52$

(c) find the value of n

(5)

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Question 9 continued

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 11 marks)

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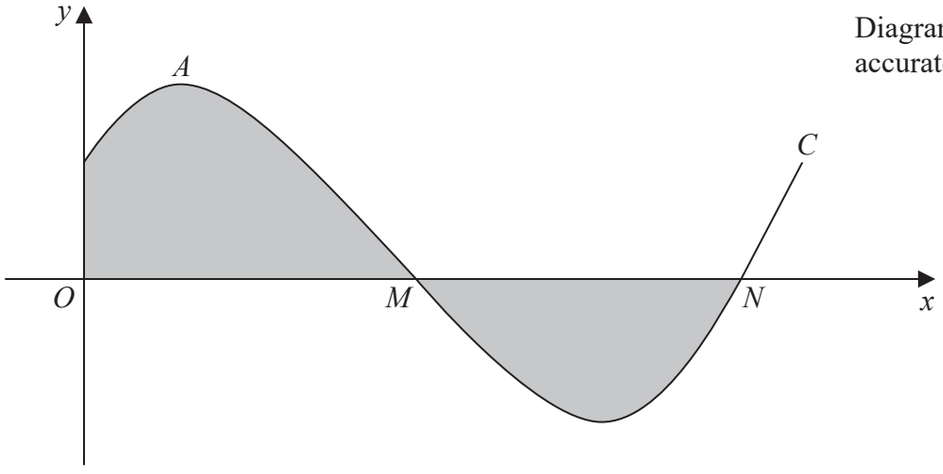

Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows the curve C with equation $y = \frac{1}{2} + \sin 3x$ where $0 \leqslant x < \frac{2\pi}{3}$

The curve C crosses the x-axis at the points M and N

(a) Show that the coordinates of M are $\left(\frac{7\pi}{18}, 0\right)$ and find the coordinates of N

(3)

The curve C has a maximum at the point A

(b) Find the coordinates of A

(4)

(c) Find an equation of the tangent to C at M

Give your answer in the form $ay + b\sqrt{3}\,x - c\sqrt{3}\,\pi = 0$ where a, b and c are integers to be found.

(4)

The finite region, shown shaded in Figure 4, is bounded by the curve C, the y-axis and the part of the x-axis from O to N

(d) Use algebraic integration to find, to 3 significant figures, the total area of the shaded region.

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 15 marks)

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11 $f'(x) = ax^2 - 14x - 10$ where $a \in \mathbb{Z}$

Given that $(x - 4)$ is a factor of $f(x)$ and that when $f(x)$ is divided by $(x + 1)$ the remainder is 25

(a) show that $a = 6$

(6)

(b) Hence use algebra to solve the equation $f(x) = 0$

(6)

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

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