www.dynamicpapers.com

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/02R**

Further Pure Mathematics
PAPER 2R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

P71642A

©2022 Pearson Education Ltd.

Q:1/1/1/1/

P 7 1 6 4 2 A 0 1 3 6

Pearson

www.dynamicpapers.com

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\qquad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\qquad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\qquad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

There are no questions on this page.

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The position vector of the point A is $(3\mathbf{i} - 2\mathbf{j})$, referred to a fixed origin O.

The point B is such that $\overrightarrow{AB} = (6\mathbf{i} + 8\mathbf{j})$

(a) Find the position vector of B as a simplified expression in terms of **i** and **j**

(2)

(b) Find the magnitude of vector $\overrightarrow{AB}$

(1)

(c) Find a unit vector, in terms of **i** and **j**, that is parallel to $\overrightarrow{AB}$

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 1 continued

(Total for Question 1 is 5 marks)

www.dynamicpapers.com

2 When poured from a pipe, concrete is formed into the shape of a cuboid with a square base of side x and with a height of $3x$

The volume of the cuboid increases at a constant rate of $8\,m^3/s$

Find the rate of increase, in m/s, of x when $x = 2$ metres.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 2 continued

(Total for Question 2 is 6 marks)

www.dynamicpapers.com

3 A geometric series has first term a and common ratio r, where $r > 0$

Given that the 3rd term of the series is 5 and that the 5th term of the series is $\frac{5}{2}$

(a) find

(i) the exact value of r

(ii) the value of a

(4)

(b) Find the sum to infinity of this series.
Give your answer in the form $p + q\sqrt{2}$ where p and q are integers.

(2)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

(Total for Question 3 is 6 marks)

www.dynamicpapers.com

4 $f(x) = x^3 + px^2 + qx + 7$ where p and q are integers.

$(x + 1)$ is a factor of $f(x)$
The remainder when $f(x)$ is divided by $(x + 2)$ is -5

(a) Find the value of p and the value of q

(5)

(b) Hence, show that $f(x) = 0$ has only one real root.

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 4 continued

(Total for Question 4 is 8 marks)

www.dynamicpapers.com

5 (a) Complete the table of values for $y = e^{3x-2}$ giving your answers to 2 decimal places.

x	0	0.25	0.5	0.75	1
y	0.14				2.72

(2)

(b) On the grid opposite, draw the graph of $y = e^{3x-2}$ for $0 \leqslant x \leqslant 1$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $3x = 2 + \ln(3 - x)$

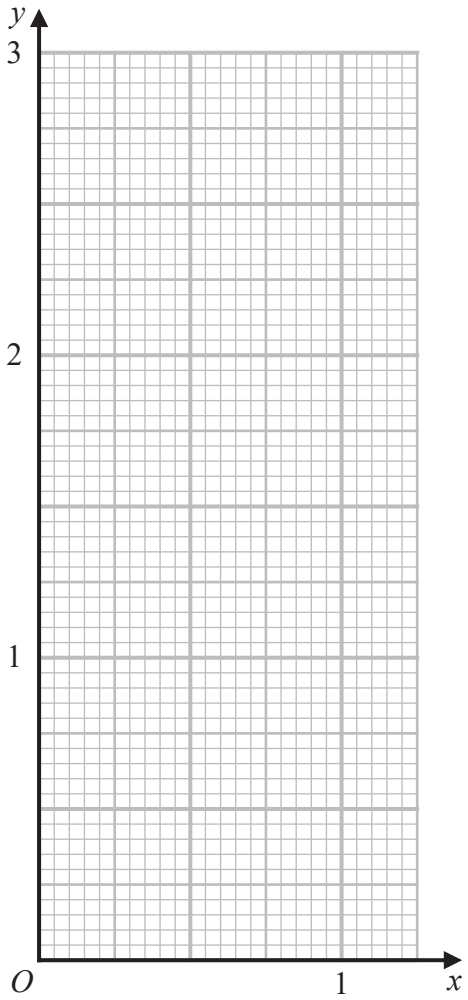
(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued



Turn over for a spare grid if you need to redraw your graph.

www.dynamicpapers.com

Question 5 continued

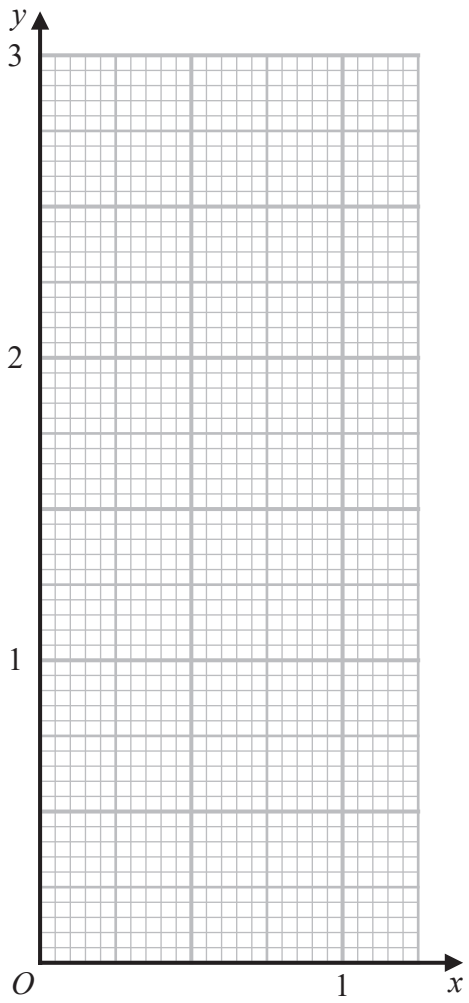
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 5 continued

Only use this grid if you need to redraw your graph.



(Total for Question 5 is 7 marks)

www.dynamicpapers.com

6

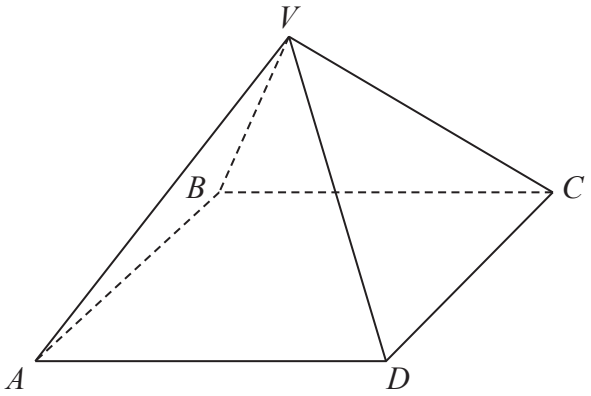


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a right pyramid *VABCD* with vertex *V* and square base *ABCD*.

Each of the edges of the pyramid has the same length.

Find the size, in degrees to one decimal place, of the angle between the plane *CVD* and the base *ABCD*.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 6 continued

(Total for Question 6 is 6 marks)

www.dynamicpapers.com

7 (a) Solve the equation

$$\cos(3x - 15)° = \frac{\sqrt{3}}{2} \quad \text{for} \quad 0 \leqslant x < 180$$

(4)

(b) Solve, giving your solutions to one decimal place where appropriate,

$$3\tan y° + 4\sin y° = 0 \quad \text{for} \quad -180 \leqslant y < 180$$

(4)

(c) Solve, giving your solutions to one decimal place where appropriate,

$$\cos\theta° = 3\sin^2\theta° - 1 \quad \text{for} \quad -180 \leqslant \theta < 180$$

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 7 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 7 continued

(Total for Question 7 is 12 marks)

www.dynamicpapers.com

8

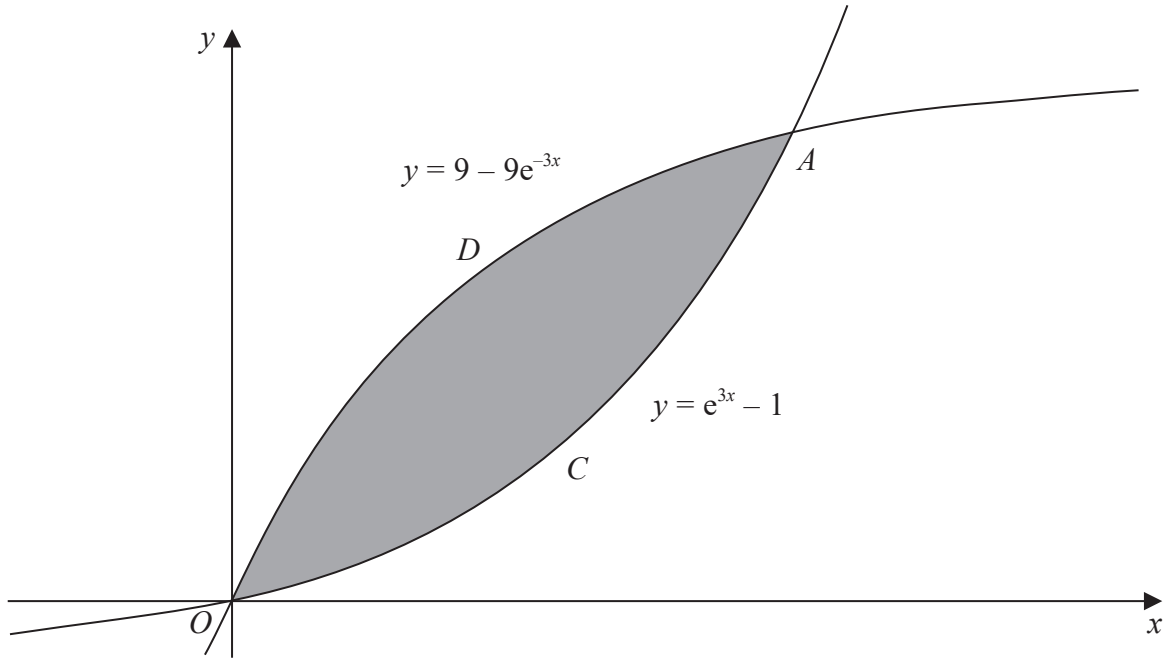


Figure 2

Figure 2 shows part of the curve C with equation $y = e^{3x} - 1$ and part of the curve D with equation $y = 9 - 9e^{-3x}$

The curves intersect at the origin O and the point A.

(a) (i) Show that the x coordinate of the point A satisfies the equation

$$(e^{3x})^2 - 10e^{3x} + 9 = 0$$

(ii) Hence, show that the x coordinate of the point A is $\frac{1}{3}\ln 9$

(5)

The finite region bounded by C and by D is shown shaded in Figure 2.

(b) Use calculus to find the exact area of this region.

(6)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 8 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 8 continued

(Total for Question 8 is 11 marks)

www.dynamicpapers.com

9 (a) Write $\frac{3}{(3-x)^3}$ in the form $a(1-bx)^{-3}$

where a and b are fractions in their lowest terms.

(2)

(b) Expand $\frac{3}{(3-x)^3}$ in ascending powers of x up to and including the term in x^3

Express each coefficient as a fraction in its lowest terms.

(3)

(c) (i) Use a suitable value of x with your expansion in part (b), to obtain an approximation for $\frac{24}{125}$ to 5 decimal places.

(ii) Find the percentage error, to 2 decimal places, of your approximation from the actual value.

(4)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

Question 9 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 9 continued

(Total for Question 9 is 9 marks)

www.dynamicpapers.com

10 A curve C has equation

$$y = \frac{7x - 2}{2x - 3} \qquad x \neq \frac{3}{2}$$

(a) Write down an equation of the asymptote to C that is

(i) parallel to the y-axis,

(ii) parallel to the x-axis.

(2)

(b) Find the coordinates of the points of intersection of C with the coordinate axes.

(2)

(c) Using calculus, show that at every point on the curve, the gradient of C is negative.

(4)

(d) Using the axes on the opposite page, sketch C.
Show clearly and label with their equation any asymptotes and the coordinates of the points of intersection of C with the coordinate axes.

(3)

The straight line l is the normal to C at the point A.
The x coordinate of A is positive and the gradient of l is 17
The line l also intersects C at the point B.

(e) Find the exact coordinates of B.

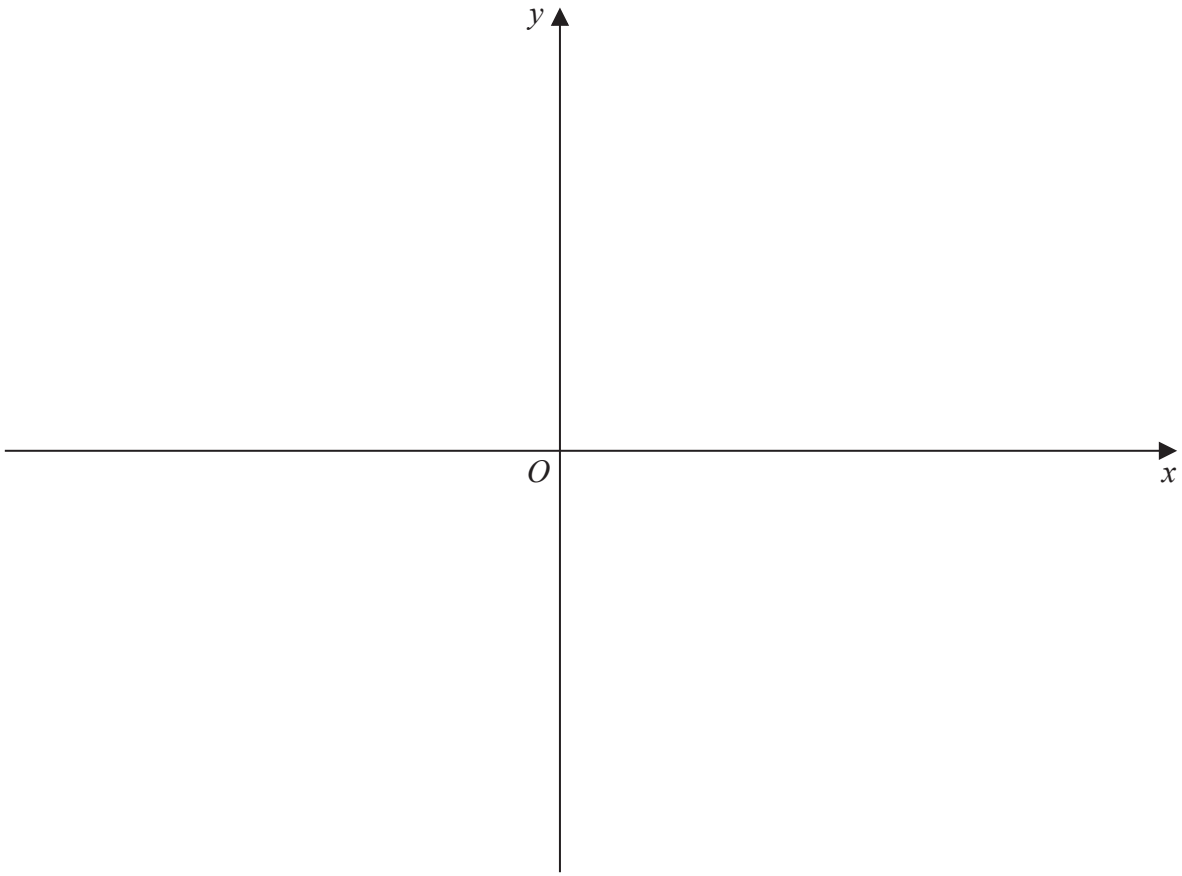
(7)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 10 continued



www.dynamicpapers.com

Question 10 continued

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 10 continued

(Total for Question 10 is 18 marks)

www.dynamicpapers.com

11

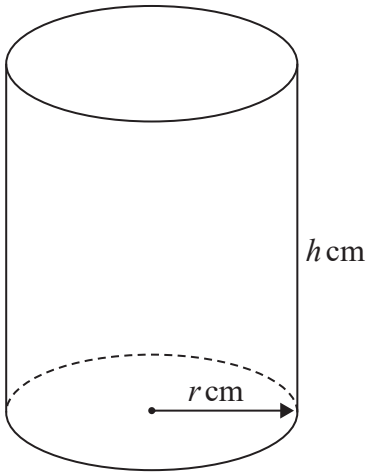


Diagram **NOT** accurately drawn

Figure 3

Figure 3 shows a solid metal right circular cylinder of radius r cm and height h cm.

The total surface area of the cylinder is $600\,\text{cm}^2$

The volume of the cylinder is $V\,\text{cm}^3$

(a) Show that $V = 300r - \pi r^3$

(4)

Given that r can vary,

(b) (i) use calculus to show that the exact value of r for which V is a maximum is

$$r = \sqrt{\frac{100}{\pi}}$$

(ii) justify that this value of r gives a maximum value of V

(5)

The cylinder is melted down and reformed into a sphere of radius p cm.

(c) Find, to one decimal place, the greatest possible value of p

(3)

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

www.dynamicpapers.com

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA

Question 11 continued

www.dynamicpapers.com

Question 11 continued

(Total for Question 11 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS

DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA
DO NOT WRITE IN THIS AREA