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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel International GCSE

Centre Number | Candidate Number

Thursday 18 June 2020

Morning (Time: 2 hours) | Paper Reference **4PM1/02R**

Further Pure Mathematics

Paper 2R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

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Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The nth term of an arithmetic series A is a_n

The nth term of a geometric series G is t_n

For these two series

$$a_1 = t_1 \qquad a_{10} = t_3 = 48 \qquad a_{10} = 4t_2$$

Find

(i) the common ratio of G,

(ii) the common difference of A.

(6)

(Total for Question 1 is 6 marks)

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2 $f(x) = x^3 + px + q$ where p and q are constants.

The remainder when $f(x)$ is divided by $(x - 1)$ is -12

The remainder when $f(x)$ is divided by $(x - 4)$ is 30

(a) Find the value of p and the value of q.

(6)

Using your values of p and q

(b) show that $f(3) = 0$

(1)

(c) Express $f(x)$ as a product of linear factors.

(3)

(d) Hence solve the equation $f(x) = 0$

(1)

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Question 2 continued

(Total for Question 2 is 11 marks)

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3

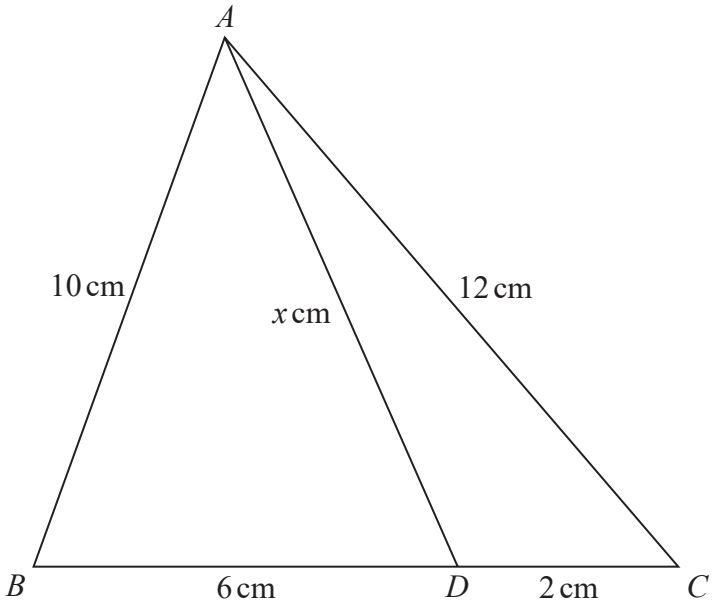

Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows the triangle ABC in which $AB = 10\,\text{cm}$ and $AC = 12\,\text{cm}$.
The point D lies on BC such that $BD = 6\,\text{cm}$, $DC = 2\,\text{cm}$ and $AD = x\,\text{cm}$.

(a) Show that $x = 11$

(4)

(b) Find the area, in cm^2 to 3 significant figures, of triangle ADB.

(4)

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Question 3 continued

(Total for Question 3 is 8 marks)

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4 (a) Complete the table of values for $y = 2x + 1 + \frac{2}{x^2}$

Give your answers to 2 decimal places where appropriate.

x	0.5	1	1.5	2	2.5	3	3.5
y		5			6.32		8.16

(2)

(b) On the grid opposite, draw the graph of $y = 2x + 1 + \frac{2}{x^2}$ for $0.5 \leqslant x \leqslant 3.5$

(2)

(c) Use your graph to obtain estimates, to 1 decimal place, of the roots of the equation

$$2x + \frac{2}{x^2} = 7 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

(2)

(d) By drawing a suitable straight line on the grid, obtain estimates, to 1 decimal place, of the roots of the equation

$$\frac{3x}{2} + \frac{2}{x^2} = 5 \quad \text{in the interval } 0.5 \leqslant x \leqslant 3.5$$

(5)

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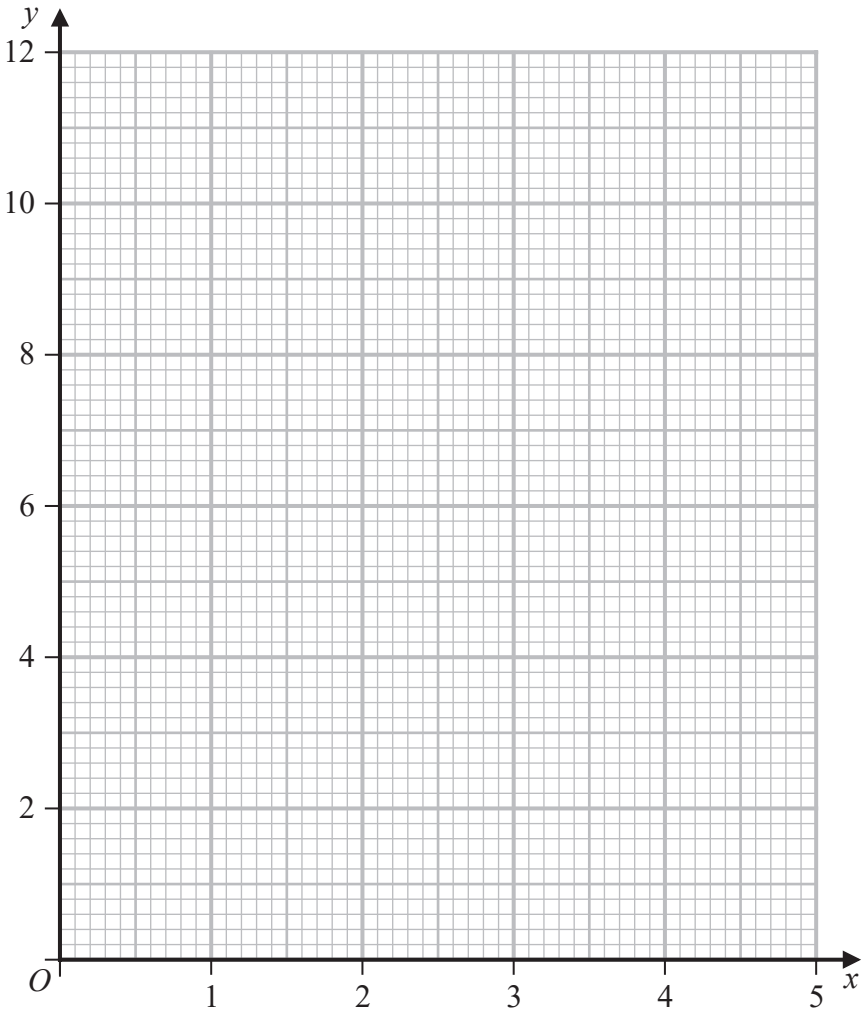
Question 4 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 4 continued

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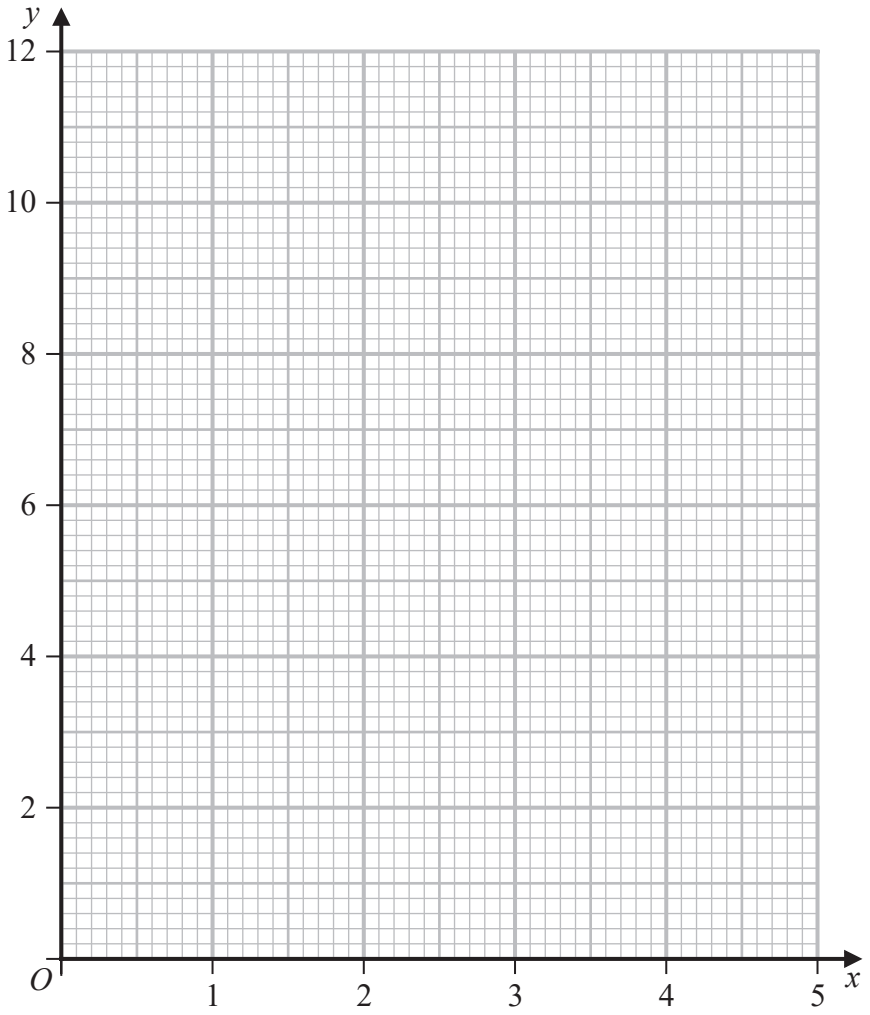
Question 4 continued

Only use this grid if you need to redraw your graph.



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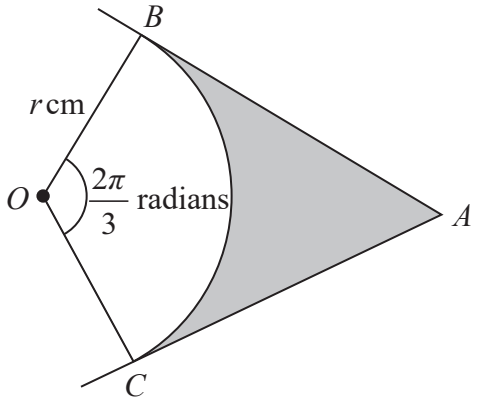

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Figure 2

In Figure 2, AB and AC are tangents to a circle with centre O and radius r cm.

The points B and C lie on the circle so that OBC is a sector of this circle and $\angle BOC = \frac{2\pi}{3}$ radians.

Given that the area of the shaded region is $10\,\text{cm}^2$,

find, to 3 significant figures, the value of r.

(8)

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Question 5 continued

(Total for Question 5 is 8 marks)

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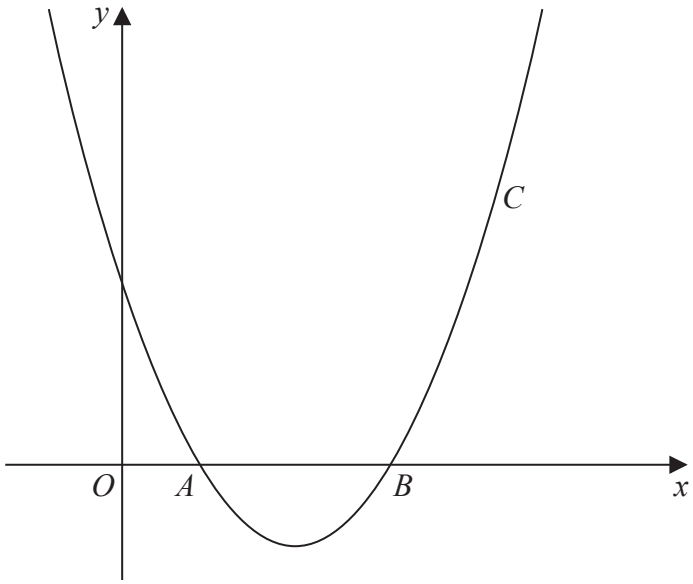

Diagram **NOT** accurately drawn

Figure 3

The curve C with equation $y = x^2 - 5x + 4$ crosses the x-axis at the points A and B, as shown in Figure 3

(a) Find the coordinates of A and the coordinates of B.

(3)

The tangent to C at A meets the tangent to C at B at the point T.

(b) Find the coordinates of T.

(6)

The normal to C at A meets the normal to C at B at the point N.

(c) Find the coordinates of N.

(3)

(d) Find the area of the quadrilateral $ATBN$.

(3)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 15 marks)

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7 (a) Find the set of values of k for which the equation $kx^2 - 4x + 2k = 7$ has real roots

(4)

Given that the roots of the equation $kx^2 - 4x + 2k = 7$ are α and β,

(b) form a quadratic equation with roots $\frac{\alpha + 1}{\alpha}$ and $\frac{\beta + 1}{\beta}$

Give each coefficient in terms of k.

(8)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 12 marks)

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8 Solve the equation $\log_3 x - 2\log_x 3 = 1$

(7)

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Question 8 continued

(Total for Question 8 is 7 marks)

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9 Given that

$$x = e^{-t} \sin 2t$$

show that

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = 0$$

(8)

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Question 9 continued

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 8 marks)

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10 $f(x) = 32x^3 - 33x + 1$

(a) Show that $f(1) = 0$

(1)

(b) Hence using an algebraic method solve $f(x) = 0$

(4)

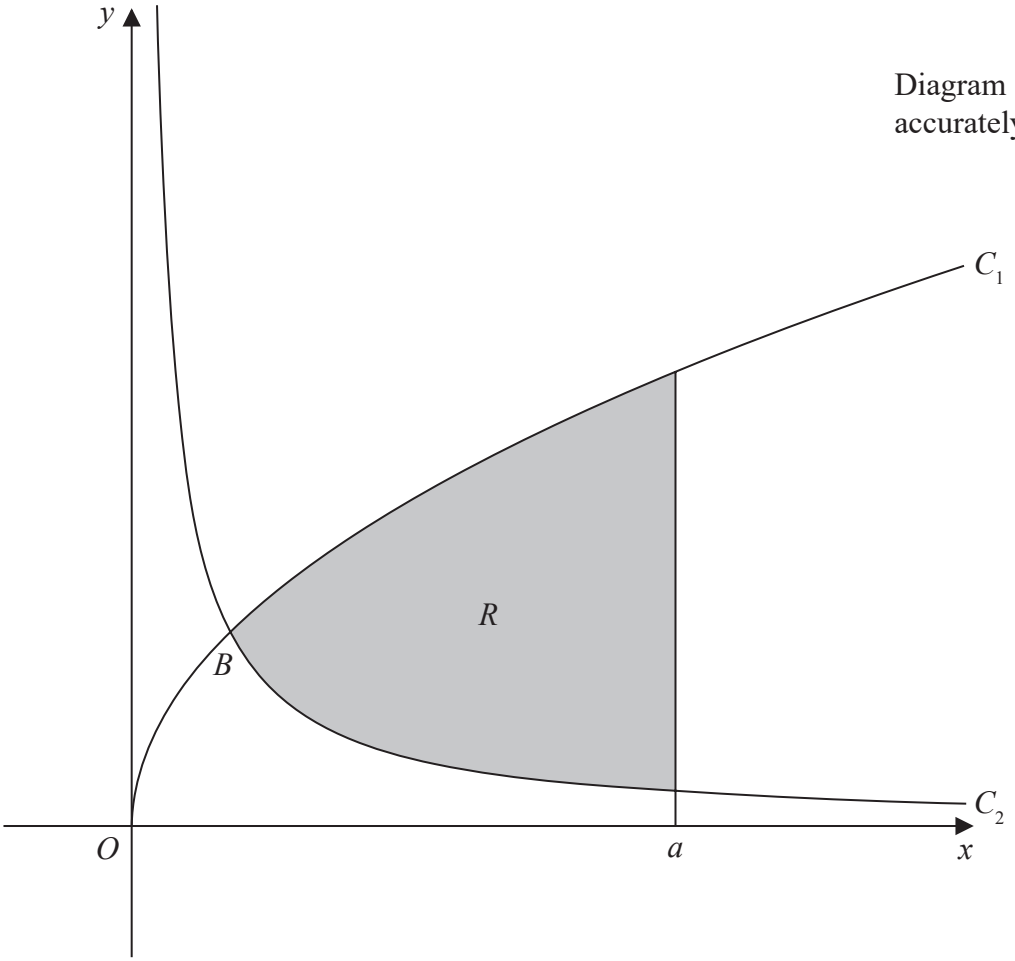

Diagram **NOT** accurately drawn

Figure 4

The region R, shown shaded in Figure 4, is bounded by the curve C_1 with equation $y = \sqrt{x}$, by the curve C_2 with equation $y = \frac{1}{8x}$ and by the line with equation $x = a$

The curves C_1 and C_2 intersect at the point B, with x coordinate p, where $p < a$

(c) Find the value of p.

(2)

The region R is rotated through 360° about the x-axis to generate a solid with volume $\frac{27\pi}{64}$

(d) Use algebraic integration to find the value of a.

(7)

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS

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