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# Mark Scheme (Results)

## January 2022

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 1

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**General Marking Guidance**

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
  - M marks: method marks
  - A marks: accuracy marks
  - B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
  - cao – correct answer only
  - ft – follow through
  - isw – ignore subsequent working
  - SC - special case
  - oe – or equivalent (and appropriate)
  - dep – dependent
  - indep – independent
  - awrt – answer which rounds to
  - eeoo – each error or omission

- **No working**
  If no working is shown then correct answers normally score full marks
  If no working is shown then incorrect (even though nearly correct) answers score no marks.

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- **With working**
  You must always check the working in the body of the script (and on any diagram irrespective of whether the final answer is correct or incorrect and award any marks appropriate from the mark scheme.
  If it is clear from the working that the "correct" answer has been obtained from incorrect working, award 0 marks.
  If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
  If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
  If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
  It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
  It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
  Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
  Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

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## General Principles for Further Pure Mathematics Marking

*(but note that specific mark schemes may sometimes override these general principles)*

**Method mark for solving a 3 term quadratic equation:**

1. Factorisation:

$\left(x^2+bx+c\right)=\left(x+p\right)\left(x+q\right)$, where $|pq|=|c|$ leading to $x=....$

$\left(ax^2+bx+c\right)=\left(mx+p\right)\left(nx+q\right)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=....$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=....$

3. Completing the square:

$x^2+bx+c=0:\ \left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ leading to $x=....$

4. Use of calculators

Unless the question specifically states 'show' or 'prove' accept correct answers from no working. If an incorrect solution is given without any working do not award the Method mark.

**Method marks for differentiation and integration:**

1. Differentiation

Power of at least one term decreased by 1. $\left(x^n\rightarrow x^{n-1}\right)$

2. Integration:

Power of at least one term increased by 1. $\left(x^n\rightarrow x^{n+1}\right)$

**Use of a formula:**

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

**or**, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

**Answers without working:**

The rubric states "Without sufficient working, correct answers may be awarded no marks".

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General policy is that if it could be done "in your head" detailed working would not be requ .
(Mark schemes may override this eg in a case of "prove or show....")

**Exact answers:**
When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

**Rounding answers (where accuracy is specified in the question)**
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

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# January 2022
# 4PM1 Paper 1
# Mark Scheme

| Question | Scheme | Marks |
|---|---|---|
| 1 | $\int \cos 4\theta \ \mathrm{d}\theta = \left[\frac{\sin 4\theta}{4}\right]$<br>For an attempt to evaluate **their** integral using the given values and reach a value<br>$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = \left[\frac{\sin 4\theta}{4}\right]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \frac{\sin\left(4\times\frac{\pi}{3}\right)}{4} - \frac{\sin\left(4\times\frac{\pi}{4}\right)}{4} = \ldots$<br>$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = -\frac{\sqrt{3}}{8}$ | M1A1<br><br><br>M1<br><br>A1<br>[4] |
| | | **Total 4 marks** |

| Mark | Notes |
|---|---|
| **M1** | For an attempt to integrate $\cos 4\theta$ obtaining:<br>$\pm\frac{\sin 4\theta}{4}$<br>For this mark ignore incorrect / absent limits |
| **A1** | For the correct integrated expression:<br>$\frac{\sin 4\theta}{4}$ |
| **M1** | For an attempt to evaluate **their** integral using the given values and reach a value.<br>Must be substituting into $k \sin 4\theta$<br>Condone candidates who convert to working in degrees to evaluate. |
| **A1** | For the correct value $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos 4\theta \ \mathrm{d}\theta = -\frac{\sqrt{3}}{8}$<br>Note: question requires answer to be given in the form $-\frac{\sqrt{a}}{b}$ and therefore equivalent answers are not acceptable. |

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| Question number | Scheme | Marks |
|---|---|---|
| 2 (a) | $\mathrm{f}(x)=2\left(x^2-6x\right)+5$ OR $\mathrm{f}(x)=2\left(x^2-6x+\frac{5}{2}\right)$ | M1 |
| | $\mathrm{f}(x)=2\left(\left[x-3\right]^2-k\right)+5$ OR $\mathrm{f}(x)=2\left(\left[x-3\right]^2-k+\frac{5}{2}\right)$ | M1 |
| | $\mathrm{f}(x)=2\left[x-3\right]^2-13$ | A1 |
| | $a=2,\ b=-3,\ c=-13$ Allow values embedded in f ($x$) | [3] |
| **ALT – Equating coefficients** | | |
| | $ax^2+2abx+\left(ab^2+c\right)\equiv 2x^2-12x+5\Rightarrow a=2$ | [M1 |
| | $2ab=-12\Rightarrow 2\times 2\times b=-12\Rightarrow b=-3$ | M1 |
| | $ab^2+c=5\Rightarrow 2\times(-3)^2+c=5\Rightarrow c=5-18=-13$ | A1] |
| (b) | $2\left[x-'3'\right]^2-'13'>37\Rightarrow 2\left[x-'3'\right]^2=50\Rightarrow\left[x-3\right]^2=25\Rightarrow x='3'\pm\ldots$ | M1 |
| | Critical values $x=-2,8$ | A1 |
| | $\{x<-2\}\cup\{x>8\}$ | M1A1 [4] |
| **ALT - solving inequality without use of completed square form** | | |
| | $2x^2-12x-32>0\ \Rightarrow\ x^2-6x-16>0\ \Rightarrow\ (x+2)(x-8)>0$ | [M1 |
| | Critical values $x=-2,8$ | A1 |
| | $\{x<-2\}\cup\{x>8\}$ | M1A1] |
| | | **Total 7 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **M1** | Starts process to complete the square by taking out 2 as a common factor<br>$\mathrm{f}(x)=2(x^2-6x)+5$ OR $\mathrm{f}(x)=2(x^2-6x+\frac{5}{2})$ |
| | **M1** | Attempts to complete the square:<br>$\mathrm{f}(x)=2\left(\left[x-3\right]^2-k\right)+5$ OR $\mathrm{f}(x)=2\left(\left[x-3\right]^2-k+\frac{5}{2}\right)$ |
| | **A1** | Correctly completes the square to obtain:<br>$a=2,\ b=-3,\ c=-13$<br>Allow embedded answers i.e. $\mathrm{f}(x)=2\left[x-3\right]^2-13$ |
| **ALT – Equating coefficients** | | |
| | **M1** | Expands the given form and equates to f($x$).<br>Begins process of comparing coefficients, establishing that $a=2$.<br>$ax^2+2abx+\left(ab^2+c\right)\equiv 2x^2-12x+5\Rightarrow a=2$ |
| | **M1** | Equates coefficient of $x$ and solves for $b$.<br>$2ab=-12\Rightarrow 2\times 2\times b=-12\Rightarrow b=-3$<br>Equates constant terms and attempts to solve for $c$. |

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| | | |
|---|---|---|
| | | $ab^2 + c = 5 \Rightarrow 2\times(-3)^2 + c = 5 \Rightarrow c = 5 - 18 = -13$ |
| | **A1** | For correct values of *a*, *b* and *c*.<br>$a = 2,\ b = -3,\ c = -13$<br>Allow embedded answers i.e. $\mathrm{f}(x) = 2[x-3]^2 - 13$ |
| **(b)** | **M1** | Sets f($x$) > 37 and uses their result from part (a)<br>[provided it is in the form $\mathrm{f}(x) = 2(x \pm P)^2 \pm Q$ ]<br>and attempts to find **two** critical values<br>$2[x-'3']^2 - '13' > 37 \Rightarrow 2[x-'3']^2 = 50 \Rightarrow [x-3]^2 = 25 \Rightarrow x = '3' \pm ...$<br>Condone use of = rather than > |
| | **A1** | For both critical values of $x = -2,\ 8$ |
| | **M1** | For choosing the outside region for their cv's [provided there are two values]<br>$\{x < '-2'\} \cup \{x > '8'\}$ or any other correct notation.<br>Condone 'AND' for this mark |
| | **A1** | For the correct region with correct values<br>$\{x < -2\} \cup \{x > 8\}$ or any other correct notation.<br>Must **not** use 'AND' for this mark. |
| **Alt – solving inequality without use of completed square form** | | |
| | **M1** | Sets f($x$) > 37 and attempts to find **two** critical values.<br>See general guidance on what constitutes an attempt to solve a quadratic.<br>Condone use of = rather than > |
| | **A1** | For both critical values of $x = -2,\ 8$ |
| | **M1** | For choosing the outside region for their cv's [provided there are two values]<br>$\{x < '-2'\} \cup \{x > '8'\}$ or any other correct notation.<br>Condone 'AND' for this mark |
| | **A1** | For the correct region with correct values<br>$\{x < -2\} \cup \{x > 8\}$ or any other correct notation.<br>Must **not** use 'AND' for this mark. |

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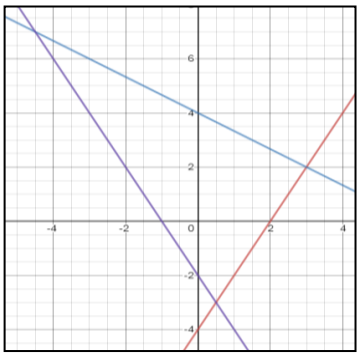
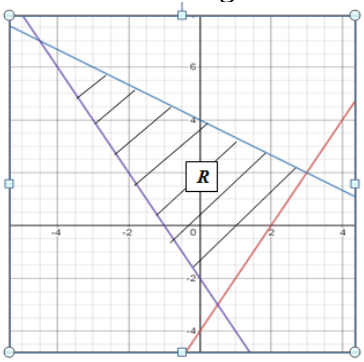
| Question number | Scheme | Marks |
|---|---|---|
| 3(a)(i) | $\frac{ar^4}{ar} = \frac{\frac{135}{1024}}{\frac{5}{16}}$ | M1 |
| | $r = \sqrt[3]{\frac{\frac{135}{1024}}{\frac{5}{16}}} = \left(\sqrt[3]{\frac{135\times16}{5\times1024}}\right) = ...$ | M1 |
| | $r = \frac{3}{4}$ oe | A1 |
| (a)(ii) | $ar = \frac{5}{16} \Rightarrow a = \frac{\frac{5}{16}}{\frac{3}{4}} = \left(\frac{5}{12}\right)$ | M1 |
| | $a = \frac{5}{12}$ | A1 |
| | | [5] |
| (b) | $S = \frac{\frac{5}{12}}{1-\frac{3}{4}} = ...$ | M1 |
| | $S = \frac{5}{3}$ | A1 |
| | | [2] |
| | | **Total 7 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)(i)** | **M1** | For $\frac{ar^4}{ar} = \frac{\frac{135}{1024}}{\frac{5}{16}}$ or $\frac{ar}{ar^4} = \frac{\frac{5}{16}}{\frac{135}{1024}}$ |
| | **M1** | For rearranging to find a value for $r$<br>$r = \sqrt[3]{\frac{\frac{135}{1024}}{\frac{5}{16}}} = \left(\sqrt[3]{\frac{135\times16}{5\times1024}}\right) = ...$ |
| | **A1** | For the correct value of $r = \frac{3}{4}$ oe |
| **(a)(ii)** | **M1** | For attempting to find a value for $a$ using their $r$<br>$ar = \frac{5}{16} \Rightarrow a = \frac{\frac{5}{16}}{\frac{3}{4}} = \left(\frac{5}{12}\right)$ |
| | **A1** | For the correct value of $a = \frac{5}{12}$ |

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| | | |
|---|---|---|
| **(b)** | **M1** | For using the correct formula for the sum to infinity using their $a$ and $r$ provided $\|r\| <$<br>$S = \frac{'\frac{5}{12}'}{1-'\frac{3}{4}'} = ...$<br>$\frac{'5'}{12} \neq \frac{5}{16}$ , $\frac{'5'}{12} \neq \frac{135}{1024}$ |
| | **A1** | For the correct value of $S = \frac{5}{3}$<br>**Note:** Must be the exact value. |

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<table>
<tr><th>Question</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4 (a)</td><td>$y = 2x - 4$ Intersections with axes at $(0, -4)$ $(2, 0)$<br>$2x + 3y = 12$ Intersections with $y$ axes at $(0, 4)$ and. $(6, 0)$<br>$y + 2x + 2 = 0$ Intersections with $y$ axes at $(0, -2)$ and. $(-1, 0)$<br><br></td><td>B1<br>B1<br>B1<br>[3]</td></tr>
<tr><td>(b)</td><td>For the correct region shaded in or out<br></td><td>B1ft<br>[1]</td></tr>
<tr><td>(c)</td><td>Points of intersection are:<br><table><tr><td>$(0.5, -3)$</td><td>$(3,\ 2)$</td><td>$(-4.5,\ 7)$</td></tr></table><br><table><tr><td>Vertex</td><td>$(0.5, -3)$</td><td>$(3,\ 2)$</td><td>$(-4.5,\ 7)$</td></tr><tr><td>$P = x - 2y$</td><td>6.5</td><td>$-1$</td><td>$-18.5$</td></tr><tr><td></td><td></td><td></td><td>Least</td></tr></table><br>For $P = -18.5$</td><td>M1<br>A1<br><br>dM1<br><br><br>A1<br>[4]</td></tr>
<tr><td></td><td colspan="2">ALT- objective line approach</td></tr>
</table>

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| | | |
|---|---|---|
| | Slope of objective line is $\frac{1}{2}$<br>$\left(-\frac{9}{2},\ 7\right)$<br>$P = -\frac{9}{2} - 2(7)$<br>For $P = -18.5$ | [M1<br>A1<br>M1<br>A1] |
| | | **Total 8 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **B1B1B1** | B1 for each line drawn correctly<br>$y = 2x - 4$ Intersections with axes at $(0, -4)$ $(2, 0)$<br>$2x + 3y = 12$ Intersections with $y$ axes at (0, 4) and. (6, 0)<br>$y + 2x + 2 = 0$ Intersections with $y$ axes at $(0, -2)$ and. $(-1, 0)$<br>Minimum length of line is 4 units horizontally or 4 units vertically. |
| **(b)** | **B1ft** | For the correct region marked - allow shaded in or out<br>Ft for shading the closed region from their lines. Must be a closed region. |
| **(c)** | **M1** | For attempting to find the correct coordinates of at least one intersection either by reading values from their graphs or by solving simultaneous equations. If they are solving simultaneous equations, they must find a value for $x$ and a corresponding value for $y$ |
| | **A1** | For at least one correct point of intersection<br>$(0.5, -3)$ $(3,\ 2)$ $(-4.5,\ 7)$ |
| | **dM1** | For substituting one point of intersection into the given $P$ ft their coordinates |
| | **A1** | For identifying $P = -18.5$ |
| **ALT – objective line approach** | | |
| | **M1** | For attempt to use objective line approach.<br>Identifies that the slope of objective line is $\frac{1}{2}$<br>Identifies the intersection of $2x + 3y = 12$ and $y + 2x + 2 = 0$ as the point where $P$ is least. |
| | **A1** | For finding the correct coordinates $\left(-\frac{9}{2},\ 7\right)$ |
| | **M1** | For substituting their $\left(-\frac{9}{2},\ 7\right)$ into $P$. |
| | **A1** | For $P = -18.5$ |

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| Question number | Scheme | Marks |
|---|---|---|
| 5(a) | $f(-2) = 0, \quad f(-3) = 21$<br>$a(-2)^3 + 5b(-2)^2 + 8a(-2) - 4b = 0$<br>$a(-3)^3 + 5b(-3)^2 + 8a(-3) - 4b = 21$<br>$2b = 3a$<br>$41b = 51a + 21$<br>$a = 2$*, $b = 3$ | M1<br>A1<br>M1<br>A1cso<br>A1<br>[5] |
| (b) | $x+2\overline{)2x^3+15x^2+16x-12}$ $\quad 2x^2+11x-6$<br>**ALT**<br>$2x^3+15x^2+16x-12=(x+2)(Ax^2+Bx+C)\Rightarrow(x+2)(2x^2+11x-6)$<br>$2x^2+11x-6=(2x-1)(x+6)$<br>$(2x-1)(x+6)=0 \Rightarrow 2x-1=0, x+6=0$<br>$x=-6,\ -2,\ \frac{1}{2}$ | M1<br>M1<br>M1<br>A1<br>[4] |
| | | **Total 9 marks** |

| Question | Marks | Scheme |
|---|---|---|
| **(a)** | **M1** | For attempting either $f(-2)=0$ or $f(-3)=21$<br>Allow $f(\pm 2)=0$ or $f(\pm 3)=21$ for this mark.<br>Allow for $f(\pm 3) = 21$ with $a = 2$ assumed. |
| | **A1** | For both correct equations in terms of $a$ and $b$<br>$a(-2)^3 + 5b(-2)^2 + 8a(-2) - 4b = 0$<br>$a(-3)^3 + 5b(-3)^2 + 8a(-3) - 4b = 21$<br>Evaluation not required for this mark, just the correct substitution. |
| | **M1** | Attempts to solve their two linear simultaneous equation in $a$ and $b$<br>$2b = 3a$<br>$41b = 51a + 21$<br>Condone one slip provided consistent addition or subtraction if using elimination. |
| | **A1 cso** | For $a = 2$* |
| | **A1** | For $b = 3$ |
| **(b)** | **M1** | For attempting division of $f(x)=2x^3+15x^2+16x-12$ by $(x+2)$ getting as far as $2x^2 + \cdots$<br>$x+2\overline{)2x^3+15x^2+16x-12}$ $\quad 2x^2+11x-6$<br>**ALT** |

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| | | |
|---|---|---|
| | | Equates coefficients to find the 3TQ factor<br>$2x^3+15x^2+16x-12=(x+2)(Ax^2+Bx+C)\Rightarrow(x+2)(2x^2+11x-6)$<br>Must get as far as $(x+2)(2x^2+\cdots)$ for the mark. |
| | **M1** | For attempting to factorise **their** 3TQ, but it must be a 3TQ<br>$2x^2+11x-6=(2x-1)(x+6)$<br>Refer to general guidance for what constitutes an attempt to factorise. |
| | **M1** | An attempt to solve f ($x$) = 0 |
| | **A1** | For $x=-6,\ -2,\ \frac{1}{2}$<br>**Note:** Correct answers with no working scores M0M0M0A0 |

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| Question number | Scheme | Marks |
|---|---|---|
| **6(a)(i)** | $\frac{\sin 30^o}{x} = \frac{\sin ACB}{x+3}$ | M1 |
| | $\sin\theta^o = \frac{x+3}{2x}$ * | A1 cso |
| **(ii)** | $\cos^2\theta^o = 1 - \left(\frac{x+3}{2x}\right)^2$ | M1 |
| | $\cos^2\theta^o = \frac{(2x)^2 - (x+3)^2}{(2x)^2}$ | M1 |
| | $\cos\theta^o = \frac{\sqrt{3x^2-6x-9}}{2x}$ * | A1 |
| | | [5] |
| **Alt – use of right-angled triangle with Pythagoras' theorem** | | |
| | Adjacent $= \sqrt{(2x)^2 - (x+3)^2}$ | [M1 |
| | $\cos\theta = \frac{\sqrt{(2x)^2-(x+3)^2}}{2x}$ | M1 |
| | $\cos\theta^o = \frac{\sqrt{3x^2-6x-9}}{2x}$ * | A1] |
| **(b)** | $\frac{\angle BAC}{30^o} = \frac{7}{2} \Rightarrow \angle BAC = 105^o$ | B1 |
| | $\theta = 180 - 30 - 105 = 45$ | B1 |
| | $\cos 45^o = \frac{\sqrt{2}}{2} = \frac{\sqrt{3x^2-6x-9}}{2x} \Rightarrow 2x^2 = 3x^2 - 6x - 9 \Rightarrow x^2 - 6x - 9 = 0$ | M1 |
| | $x^2 - 6x - 9 = 0 \Rightarrow (x-3)^2 - 18 = 0 \Rightarrow x = ...$ | M1 |
| | $x = 3 + 3\sqrt{2}$ | A1 |
| | | [5] |
| **Alt – last three marks** | | |
| | $\sin\theta^o = \frac{x+3}{2x} = \frac{\sqrt{2}}{2} \Rightarrow x + 3 = \sqrt{2}x$ | [M1 |
| | $x + 3 = \sqrt{2}x \Rightarrow x(\sqrt{2} - 1) = 3 \Rightarrow x = \frac{3}{\sqrt{2}-1}$ | M1 |
| | $x = 3 + 3\sqrt{2}$ | A1] |
| | | **Total 10 marks** |

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| Part | Marks | Scheme |
|---|---|---|
| **(a) (i)** | **M1** | For using a correct sine rule to give, $\frac{\sin 30^o}{x} = \frac{\sin ACB}{x+3}$ |
| | **A1 cso** | For correctly obtaining the expression for $\sin\theta$ $\sin\theta^o = \frac{x+3}{2x}$ * |
| **(ii)** | **M1** | For using the Pythagorean identity $\cos^2\theta^o = 1-\left(\frac{x+3}{2x}\right)^2$ |
| | **M1** | For simplifying to form a single fraction $\cos^2\theta^o = \frac{(2x)^2-(x+3)^2}{(2x)^2}$ |
| | **A1 cso** | For simplifying to achieve the given expression, $\cos\theta^o = \frac{\sqrt{3x^2-6x-9}}{2x}$ * Note this is a show question |
| **Alt – use of right-angled triangle with Pythagoras' theorem** | | |
| | **M1** | For use of a right-angled triangle with Pythagoras' theorem to determine the adjacent Adjacent $= \sqrt{(2x)^2-(x+3)^2}$ |
| | **M1** | For use of cosine ratio $\cos\theta = \frac{\sqrt{(2x)^2-(x+3)^2}}{2x}$ |
| | **A1 cso** | For simplifying to achieve the given expression, $\cos\theta^o = \frac{\sqrt{3x^2-6x-9}}{2x}$ * Note this is a show question |
| **(b)** | **B1** | For finding the size of $\angle BAC$ $\frac{\angle BAC}{30^o} = \frac{7}{2} \Rightarrow \angle BAC = 105^o$ |
| | **B1** | For finding the value of $\theta = 180-30-105 = 45$ |
| | **M1** | For substituting the value of $\angle ABC$ into the given expression for $\cos\theta$ and forming a 3TQ, condone arithmetic errors in rearrangement. $\cos 45^o = \frac{\sqrt{2}}{2} = \frac{\sqrt{3x^2-6x-9}}{2x} \Rightarrow 2x^2 = 3x^2-6x-9 \Rightarrow x^2-6x-9=0$ Allow for use of their 45˚ but this must come from an attempt at working with the ratio. Do not allow if their 45˚ is 30˚. |
| | **M1** | For an attempt to solve their 3TQ by any valid method (see general guidance) $x^2-6x-9=0 \Rightarrow (x-3)^2-18=0 \Rightarrow x=...$ |
| | **A1 cao** | For the correct value of $x$ in the correct form $x = 3+3\sqrt{2}$ Allow $a=3, b=2$ |
| **Alternative method** | | |
| | **M1** | For substituting the value of $\angle ABC$ into the given expression for $\sin\theta$ and forming a linear equation $\sin\theta^o = \frac{x+3}{2x} = \frac{\sqrt{2}}{2} \Rightarrow x+3 = \sqrt{2}x$ |

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| | | |
|---|---|---|
| | **M1** | For an attempt to solve their linear equation<br>$x+3=\sqrt{2}x \Rightarrow x\left(\sqrt{2}-1\right)=3 \Rightarrow x=\dfrac{3}{\sqrt{2}-1}$ |
| | **A1 cao** | For the correct value of $x$ in the correct form $x=3+3\sqrt{2}$<br>Allow $a=3$, $b=2$ |

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| Question | Scheme | Marks |
|---|---|---|
| **7(a)** | 4 | B1<br>[1] |
| **(b)** | **Working in $\log_2$**<br>$2\log_4 x = \frac{2\log_2 x}{\log_2 4} = \frac{2\log_2 x}{2} = [\log_2 x]$<br>$\log_2 16 + \frac{2\log_2 x}{2} = \log_2 y$<br>$\log_2 16x = \log_2 y$ OR $\log_2\left(\frac{x}{y}\right) = -4 \Rightarrow \frac{x}{y} = 2^{-4}$<br>$y = 16x^*$ | M1<br>M1<br>M1<br>A1 cso<br>[4] |
| | **ALT** | |
| | **Working in $\log_4$**<br>$\log_2 y = \frac{\log_4 y}{\log_4 2} = \frac{\log_4 y}{\frac{1}{2}} = 2\log_4 y = [\log_4 y^2]$<br>$\log_4 256 + \log_4 x^2 = \log_4 y^2$<br>$\log_4\left(256x^2\right) = \log_4 y^2$ OR $2\log_4\left(\frac{y}{x}\right) = 4 \Rightarrow \frac{y}{x} = 4^2$<br>$256x^2 = y^2 \Rightarrow y = 16x^*$ | [M1<br>M1<br>M1<br>A1] |
| **(c)** | $16x = 4x + 5$<br>$16x = 4x + 5 \Rightarrow 12x = 5 \Rightarrow x = ...$<br>$x = \frac{5}{12}$ | B1<br>M1<br>A1<br>[3] |
| | | **Total 8 marks** |

| Part | Marks | Scheme |
|---|---|---|
| **(a)** | **B1** | States 4 only |
| **(b)** | **M1** | For an attempt to change the base of $\log_4 x$ to base 2 using $\log_a x = \frac{\log_b x}{\log_b a}$<br>$\log_4 x = \frac{\log_2 x}{\log_2 4}\left[= \frac{\log_2 x}{2}\right]$ |
| | **M1** | An attempt to rewrite the equation in terms of $\log_2$<br>$\log_2 16 + \frac{2\log_2 x}{'2'} = \log_2 y$<br>F.t. their '2' from attempted change of base. |
| | **M1** | Uses $\log A + \log B = \log AB$ to correctly combine the logs<br>$\log_2 16x = \log_2 y$<br>OR<br>Uses $\log A - \log B = \log\frac{A}{B}$ to correctly combine the logs and removes logs<br>$\log_2\left(\frac{x}{y}\right) = -4$ **and** $\frac{x}{y} = 2^{-4}$ (this approach will score the second and third M marks at this stage) |
| | **A1** | For correctly obtaining $y = 16x^*$ |

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| | | |
|---|---|---|
| **Alt – working in $\log_4$** | | |
| | **M1** | For an attempt to change the base of $\log_2 y$ to base 4 using $\log_a y = \frac{\log_b y}{\log_b a}$<br>$\log_2 y = \frac{\log_4 y}{\log_4 2}\left[= \frac{\log_4 y}{\frac{1}{2}} = 2\log_4 y\right]$ |
| | **M1** | For dealing with the indices **and** writing $4 = \log_4 256$<br>$\log_4 256 + \log_4 x^2 = \log_4 y^2$ |
| | **M1** | Uses $\log A + \log B = \log AB$ to correctly combine the logs<br>$\log_4\left(256x^2\right) = \log_4 y^2$<br>OR<br>Uses $\log A - \log B = \log\frac{A}{B}$ to correctly combine the logs and removes logs<br>$2\log_4\left(\frac{y}{x}\right) = 4$ **and** $\frac{y}{x} = 4^2$ (this approach will score the second and third M marks at this stage) |
| | **A1** | For correctly obtaining $y = 16x$* |
| **(c)** | **B1** | For writing down $16x = 4x + 5$ |
| | **M1** | For an attempt to solve the equation<br>$16x = 4x + 5 \Rightarrow 12x = 5 \Rightarrow x = ...$ |
| | **A1** | For $x = \frac{5}{12}$ |
| | **Note:** This is a 'hence' question. Condone candidates working without using given results, but the first mark is not awarded until the candidate reaches $16x = 4x + 5$ | |

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| Question | Scheme | Mark |
|---|---|---|
| **8 (a)** | $90\pi = \pi r^2 h \Rightarrow h = \dfrac{90}{r^2}$<br>$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \dfrac{90}{r^2}$<br>$S = 2\pi r^2 + \dfrac{2\times 90\pi}{r} = 2\pi r^2 + \dfrac{180\pi}{r}$ * | B1<br>M1<br>A1cso<br>[3] |
| **(b)** | $\dfrac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \dfrac{180\pi}{r^2}$<br>$\dfrac{\mathrm{d}S}{\mathrm{d}r} = 0 \Rightarrow 4\pi r - \dfrac{180\pi}{r^2} = 0 \Rightarrow 4\pi r = \dfrac{180\pi}{r^2} \Rightarrow r^3 = 45 \Rightarrow r = ...$<br>$r = 3.55689... \Rightarrow r \approx 3.56$<br>$\dfrac{\mathrm{d}^2S}{\mathrm{d}r^2} = 4\pi + \dfrac{360\pi}{r^3}$<br>$\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{360\pi}{r^3} \Rightarrow \left(\dfrac{d^2S}{dr^2} = 37.699\ldots\right)$<br>$37.699 > 0 \Rightarrow$ hence minimum | M1<br>M1<br>A1<br>M1<br>A1ft<br>[5] |
| **(c)** | $S = 2\pi \times 3.556...^2 + \dfrac{2\times 90\pi}{3.556..} = ...$<br>$S = 238.4769... \Rightarrow S = 238\ \left(\mathrm{cm}^2\right)$ | M1<br>A1<br>[2] |
| | | **Total 10 marks** |

| Part | Marks | Scheme |
|---|---|---|
| **(a)** | **B1** | For finding an expression for *h* in terms of *r*<br>$90\pi = \pi r^2 h \Rightarrow h = \dfrac{90}{r^2}$<br>Award for finding an expression for *hr* in terms of *r*<br>$90\pi = \pi r^2 h \Rightarrow hr = \frac{90}{r}$ |
| | **M1** | For substituting their expression for *h* into a **correct** formula for the closed surface area of a cylinder<br>$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi r \times \frac{'90'}{r^2}$<br>Or for substitution of their expression for *hr* into a **correct** formula for the closed surface area of a cylinder<br>$S = 2\pi r^2 + 2\pi rh \Rightarrow S = 2\pi r^2 + 2\pi \times \frac{'90'}{r}$ |
| | **A1 cso** | For the correct expression for the area as shown |

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| | | |
|---|---|---|
| | | $S = 2\pi r^2 + \frac{2\times 90\pi}{r} = 2\pi r^2 + \frac{180\pi}{r}$<br>Must have the $S =$ for this mark. |
| **(b)** | **M1** | For attempting to differentiate the given expression for *S* at least one power to decrease and neither power to increase.<br>$\frac{\mathrm{d}S}{\mathrm{d}r} = 4\pi r - \frac{180\pi}{r^2}$ |
| | **M1** | Sets their $\frac{\mathrm{d}S}{\mathrm{d}r} = 0$ and attempts to solve for *r*<br>$4\pi r - \frac{180\pi}{r^2} = 0 \Rightarrow 4\pi r = \frac{180\pi}{r^2} \Rightarrow r^3 = 45 \Rightarrow r = ...$ |
| | **A1** | For the correct value of $r = 3.55689... \Rightarrow r \approx 3.56$<br>Accept awrt 3.56 |
| | **M1** | For attempting to differentiate their expression for $\frac{\mathrm{d}S}{\mathrm{d}r}$ at least one power to decrease and neither power to increase.<br>$\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 4\pi + \frac{360\pi}{r^3}$ |
| | **A1ft** | For correct work throughout $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 4\pi + \frac{180\pi}{r^3} \Rightarrow \left(\frac{\mathrm{d}^2S}{\mathrm{d}r^2} = 37.699...\right)$<br>$37.699 > 0 \Rightarrow$ hence minimum<br>Evaluation not required as both terms positive so $\frac{\mathrm{d}^2S}{\mathrm{d}r^2} > 0$ hence minimum<br>Indication of positive or >0 required.<br>If $\frac{\mathrm{d}^2S}{\mathrm{d}r^2}$ evaluated incorrectly then do not award. If evaluated then accept awrt 38 |
| **(c)** | **M1** | For substituting their value of *r* into the **given** expression for *S*<br>$S = 2\pi \times '3.556'...^2 + \frac{2\times 90\pi}{'3.556'..} = ...$<br>Their value of *r* >0 |
| | **A1** | $S = 238.4769... \Rightarrow S = 238\ (\mathrm{cm}^2)$<br>Accept awrt 238 |

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| Question | Scheme | Ma |
|---|---|---|
| **9 (a)** | $(1-2x)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\times -2x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!}+\ldots$ | M1 |
| | $(1-2x)^{-\frac{1}{2}} = 1+x+\frac{3}{2}x^2+\frac{5}{2}x^3+\ldots$ | A1A1 |
| | . | [3] |
| **(b)** | $\frac{1}{\sqrt{0.96}} = \frac{1}{\sqrt{\frac{96}{100}}} = \frac{10}{4\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}} = \ldots$ | M1 |
| | $\frac{10\sqrt{6}}{4\times 6} = \frac{5\sqrt{6}}{12}$ * | A1 cso [2] |
| **ALT – confirming given result** | | |
| | $\frac{1}{\sqrt{0.96}} = \frac{5\sqrt{6}}{12} \Rightarrow 12 = \sqrt{0.96}\times 5\sqrt{6}$ | [M1 |
| | $12^2 = 0.96\left(5\sqrt{6}\right)^2 = 0.96\times 5^2\times 6$ * | A1cso] |
| **(c)** | $\frac{1}{\left(5\sqrt{6}-12\right)}\times\frac{\left(5\sqrt{6}+12\right)}{\left(5\sqrt{6}+12\right)}$ | M1 |
| | $= \frac{5\sqrt{6}+12}{150-12^2} = \frac{5\sqrt{6}+12}{6} = \frac{5\sqrt{6}}{6}+2$ | A1 [2] |
| **(d)** | $1-2x = 0.96 \Rightarrow 2x = 0.04 \Rightarrow x = 0.02$ | B1 |
| | $\frac{9}{5\sqrt{6}-12} = 9\left(2\times\left[\frac{5\sqrt{6}}{12}\right]+2\right) =: 9\times\left[2\left(1+0.02+\frac{3}{2}\times 0.02^2+\frac{5}{2}\times 0.02^3\right)+2\right] = \ldots$ | M1:M1 |
| | 36.37116 | A1 [4] |
| | | **Total 11 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **M1** | For an attempt to use the Binomial Expansion<br>The minimally acceptable attempt is as follows;<br>• The power of $x$ must be correct in each term. $\left[x,\ x^2 \text{ and } x^3\right]$<br>• The first term is 1<br>• The denominators are correct<br>• $-2x$ correct in each term<br>$(1-2x)^{-\frac{1}{2}} = 1+\left(-\frac{1}{2}\times -2x\right)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)(-2x)^2}{2!}+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)(-2x)^3}{3!}+\ldots$ |
| | **A1** | The first term and one algebraic term correct and simplified |

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|---|---|---|
| | | $(1-2x)^{-\frac{1}{2}} = 1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3 + \ldots$ |
| | **A1** | Fully correct simplified expansion as shown above. |
| **(b)** | **M1** | For changing 0.96 to $\frac{96}{100}$ or equivalent fraction and attempting to multiply numerator and denominator by either $\sqrt{6}$ $\left(\text{or } \sqrt{96}\right)$<br>$\frac{1}{\sqrt{0.96}} = \frac{1}{\sqrt{\frac{96}{100}}} = \frac{10}{4\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \ldots$ |
| | **A1 cso** | For the correct answer as shown with no errors $\frac{10\sqrt{6}}{4\times 6} = \frac{5\sqrt{6}}{12}*$ |
| **ALT – confirming given result** | | |
| | **M1** | For rearranging and squaring OR for squaring on both sides |
| | **A1 cso** | For showing that the two sides of the result are equal<br>$12^2 = 0.96\left(5\sqrt{6}\right)^2 = 0.96 \times 5^2 \times 6 = 144$ |
| **(c)** | **M1** | For multiplying numerator and denominator by $5\sqrt{6}+12$<br>$\frac{1}{\left(5\sqrt{6}-12\right)} \times \frac{\left(5\sqrt{6}+12\right)}{\left(5\sqrt{6}+12\right)}$ [Can be implied by $\frac{\left(5\sqrt{6}+12\right)}{\left(150-12^2\right)}$ seen] |
| | **A1** | For a **correct** expansion of brackets throughout.<br>$\frac{1}{(5\sqrt{6}-12)} \times \frac{(5\sqrt{6}+12)}{(5\sqrt{6}+12)}\left[= \frac{5\sqrt{6}+12}{150-12^2} =\right] \frac{5\sqrt{6}+12}{6} = \frac{5\sqrt{6}}{6} + 2$ |
| **(d)** | **B1** | For finding the required value of $x$<br>$1-2x = 0.96 \Rightarrow 2x = 0.04 \Rightarrow x = 0.02$ |
| | **M1** | For substituting their value of $x$ **provided** it is $-\frac{1}{2} < x < \frac{1}{2}$ into the expansion as follows:<br>$\frac{9}{5\sqrt{6}-12} = 9\left(2 \times \left[\frac{5\sqrt{6}}{12}\right] + 2\right) =$<br>**Note:** Must show substitution if $x$ is incorrect. |
| | **M1** | For substituting their expansion for $\frac{5\sqrt{6}}{12}$<br>$9 \times \left[2\left(1 + 0.02 + \frac{3}{2} \times 0.02^2 + \frac{5}{2} \times 0.02^3\right) + 2\right] = \cdots$ |
| | **A1** | For the value of 36.37116 [The calculator value is 36.37117] |

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| Question | Scheme | M |
|---|---|---|
| **10 (a)** | For the correct value of $a = 10$ | B1<br>[1] |
| **(b)** | Gradient of line $L_2$ $\quad m = -\frac{1}{2}$<br>$y - '10' = '-\frac{1}{2}'(x-2)$<br>$y - 10 = -\frac{1}{2}(x-2)$ oe<br>$x + 2y - 22 = 0$ * | B1<br>M1<br>A1<br>A1 cso<br>[4] |
| **(c)** | Coordinates of point $A$ are $(-3,\ 0)$<br>Coordinates of point $B$ are $(22,\ 0)$<br>Length of $AC$<br>$\left(5\sqrt{2}\right)^2 = (m - '-3')^2 + n^2 \quad [\Rightarrow 50 = m^2 + 6m + 9 + n^2]$<br>Gradient of $BC$<br>$\frac{1}{4} = \frac{n}{m-'22'} \Rightarrow n = \frac{m-'22'}{4} \Rightarrow n^2 = \frac{m^2-44m+484}{16}$<br>$50 = m^2 + 6m + 9 + \frac{m^2 - 44m + 484}{16}$<br>[or $50 = (22+4n)^2 + 6(22+4n) + 9 + n^2$]<br>$17m^2 + 52m - 172 = 0$ OR $17n^2 + 200n + 575 = 0$<br>e.g. $m = \frac{-52 \pm \sqrt{52^2 - 4\times 17 \times -172}}{2\times 17} \Rightarrow m = 2,\ (-5.058...)$<br>$m = 2$ and $n = -5$ | B1<br>B1<br>M1<br>M1<br>ddM1<br>A1<br>M1<br>A1A1<br>[9] |
| **(d)** | Area$_{APB} = \frac{1}{2}('10') \times ('22' - -'3') = (125)$<br>Area$_{ABC} = \frac{1}{2}('5') \times ('22' - '-3') = \left(\frac{125}{2}\right)$<br>Area of quadrilateral $ACBP$ $= \frac{375}{2}$ (units)$^2$ | M1<br>A1<br>A1<br>[3] |
| | **ALT** | |
| | Area$_{ACBP} = \frac{1}{2}('25') \times ('15') = \ldots$<br>Area of quadrilateral $ACBP$ $= \frac{375}{2}$ (units)$^2$ | [M1A1<br>A1] |
| | **ALT** | |

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| | | |
|---|---|---|
| | $A=\frac{1}{2}\begin{bmatrix}'-3' & 2 & '22' & '2' & '-3'\\ 0 & '10' & 0 & '-5' & '0'\end{bmatrix}$ | [M1 |
| | $A=\frac{1}{2}\left(\left[(-3)\times10+2\times0+22\times(-5)+2\times0\right]-\left[2\times0+22\times10+2\times0+(-3)\times-5\right]\right)=...$ | A1 |
| | Area of quadrilateral $APBC$ $=\frac{375}{2}$ (units)$^2$ | A1] |
| | | **Total 17 marks** |

| Part | Mark | Notes |
|---|---|---|
| **(a)** | **B1** | For the correct value of $a = 10$.<br>Accept embedded i.e. $P = (2,10)$ |
| **(b)** | **B1** | For the correct gradient of line $L_2$ $\quad m=-\frac{1}{2}$ |
| | **M1** | For a correct attempt at the equation of line $L_2$ using their gradient and their value for $a$<br>$y-'10'='-\frac{1}{2}'(x-2)$ |
| | **A1** | For the correct equation in any form<br>$y-10=-\frac{1}{2}(x-4)$ oe |
| | **A1 cso** | For the correct equation in the required form $\quad x+2y-22=0$ *<br>[Accept for example $22-x-2y=0$ provided all terms on one side] |
| **(c)** | **B1** | Coordinates of point $A$ are $(-3,\ 0)$ |
| | **B1** | Coordinates of point $B$ are $(22,\ 0)$ |
| | **M1** | Length of $AC$<br>$\left(5\sqrt{2}\right)^2=(m-'-3')^2+n^2$ $\quad[\Rightarrow 50=m^2+6m+9+n^2]$<br>Allow use of '$-3$' provided this is an $x$-intercept i.e. ('$-3$', 0) |
| | **M1** | Gradient of $BC$<br>$\frac{1}{4}=\frac{n}{m-'22'}\left[\Rightarrow n=\frac{m-'22'}{4}\Rightarrow n^2=\frac{m^2-44m+484}{16}\right]$<br>Allow use of '22' provided this is an $x$-intercept i.e. ('22', 0) |
| | **ddM1** | For attempting to form an equation in $m$<br>e.g.<br>$50=m^2+6m+9+\frac{m^2-44m+484}{16}$<br>OR, For attempting to form an equation in $n$<br>e.g.<br>$50=\left(22+4n\right)^2+6\left(22+4n\right)+9+n^2$ |
| | **A1** | For the correct 3TQ in either $m$ or $n$<br>$17m^2+52m-172=0$ $\quad$ OR $\quad 17n^2+200n+575=0$ |
| | **M1** | For attempting to solve their 3TQ to find a value for $m$ or $n$ by any valid method<br>e.g. $m=\frac{-52\pm\sqrt{52^2-4\times17\times-172}}{2\times17}\Rightarrow m=2,\ (-5.058...)$<br>If a calculator is used with the incorrect 3TQ award only with a full method seen. |
| | **A1** | For the value of $m$ **or** $n$<br>$m=2$ or $n=-5$<br>If a second value for $m$ or $n$ is seen then condone for this mark. |
| | **A1** | For the value of $m$ **and** $n$<br>$m=2$ and $n=-5$ |

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| | | |
|---|---|---|
| | | Any other values of *m* and *n* must be rejected. |
| **(d)** | **M1** | For either area of triangle *APB* or *ABC*<br>$\text{Area}_{APB} = \frac{1}{2}(\text{'10'}) \times (\text{'22'} - -\text{'3'}) = (125)$ or $\text{Area}_{ABC} = \frac{1}{2}(\text{'5'}) \times (\text{'22'} - \text{'}-3\text{'}) = \left(\frac{125}{2}\right)$ |
| | **A1** | Either area of triangle *APB* or *ABC* correct |
| | **A1** | Area of quadrilateral *ACBP* = $\frac{375}{2}$ (units)$^2$ |
| **ALT** | | |
| | **M1A1** | $\text{Area}_{ACBP} = \frac{1}{2}(\text{'25'}) \times (\text{'15'}) = \ldots$ |
| | **A1** | Area of quadrilateral *ACBP* = $\frac{375}{2}$ (units)$^2$ |
| **ALT – determinant method** | | |
| | **M1** | For using the 'determinant method'<br>e.g. $A = \frac{1}{2}\begin{bmatrix} \text{'}-3\text{'} & 2 & \text{'22'} & \text{'2'} & \text{'}-3\text{'} \\ 0 & \text{'10'} & 0 & \text{'}-5\text{'} & \text{'0'} \end{bmatrix}$ |
| | **A1** | For a correct evaluation of their determinants using **their** values<br>e.g. $A = \frac{1}{2}\left(\left[(-3)\times 10 + 2\times 0 + 22\times(-5) + 2\times 0\right] - \left[2\times 0 + 22\times 10 + 2\times 0 + (-3)\times -5\right]\right) = \ldots$ |
| | **A1** | Area of quadrilateral *APBC* = $\frac{375}{2}$ (units)$^2$ |

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| Question | Scheme | Marks |
|---|---|---|
| **11 (a)** | $\frac{e^{4x}}{32}$ **and** $8x^2-4x+1$ | M1 |
| | $\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{e^{4x}}{8}(8x^2-4x+1)$ | A1A1 |
| | $\frac{dy}{dx}=\frac{16xe^{4x}}{32}-\frac{4e^{4x}}{32}+\frac{8x^2e^{4x}}{8}-\frac{4xe^{4x}}{8}+\frac{e^{4x}}{8}$ | dM1 |
| | $\frac{dy}{dx}=x^2e^{4x}*$ | A1 cso [5] |
| **(b)** | Volume $=\pi\int_{-2}^{0}(3xe^{2x})^2\,dx$ | M1 |
| | $V=\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4x}}{32}(8x^2-4x+1)\right]_{-2}^{0}$ | M1 |
| | $\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4\times0}}{32}(8\times0^2-4\times0+1)\right]-\pi\left[\frac{9e^{4\times-2}}{32}(8\times-2^2-4\times-2+1)\right]$ | dM1 |
| | $V=0.87142.....\approx0.87$ (2sf) | A1 [4] |
| | **SC - attempts integration by parts.** | |
| | Volume $=\pi\int_{-2}^{0}(3xe^{2x})^2\,dx$ | M1 |
| | $\int x^2e^{4x}\,dx=\frac{x^2e^{4x}}{'4'}-\int 2x\frac{e^{4x}}{'4'}\,dx$ | |
| | $\int x^2e^{4x}\,dx=\frac{x^2e^{4x}}{'4'}-\left[\frac{2xe^{4x}}{'16'}-\int\frac{2e^{4x}}{'16'}\right]=\frac{x^2e^{4x}}{'4'}\pm\frac{2xe^{4x}}{'16'}\pm\frac{e^{4x}}{'32'}$ | M1 |
| | $\pi\int_{-2}^{0}9x^2e^{4x}\,dx=9\pi\left[\frac{x^2e^{4x}}{'4'}-\frac{2xe^{4x}}{'16'}+\frac{e^{4x}}{'32'}\right]_{-2}^{0}$ | |
| | $=9\pi\left[\left(0-0+\frac{1}{32}\right)-\left(\frac{(-2)^2e^{4(-2)}}{'4'}-\frac{2(-2)e^{4(-2)}}{'16'}+\frac{e^{4(-2)}}{'32'}\right)\right]$ | dM1 |
| | For the correct volume of $0.87142.....\approx0.87$ rounded correctly to 2sf | A1 [4] |
| | | **Total 9 marks** |

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| Part | Mark | Notes |
|---|---|---|
| **(a)** | **M1** | For using product rule correctly with an attempt to differentiate both<br>$\frac{e^{4x}}{32}$ **and** $8x^2-4x+1$<br>Correct application of $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$ is required<br>For attempt to differentiate:<br>$8x^2-4x+1\rightarrow 16x-4$<br>$\frac{e^{4x}}{32}\rightarrow ke^{4x}$ where $k\neq 1$ or $\frac{1}{32}$ or $\frac{1}{128}$ |
| | **A1** | For either $\frac{e^{4x}}{32}(16x-4)$ OR $\frac{e^{4x}}{8}(8x^2-4x+1)$ o.e. in both cases |
| | **A1** | For $\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{e^{4x}}{8}(8x^2-4x+1)$ fully correct<br>Accept $\frac{dy}{dx}=\frac{e^{4x}}{32}(16x-4)+\frac{4e^{4x}}{32}(8x^2-4x+1)$ |
| | **dM1** | For multiplying out both sets of brackets, or factorising, their $\frac{dy}{dx}$ provided the earlier M mark has been achieved<br>e.g., $\frac{dy}{dx}=\frac{16xe^{4x}}{32}-\frac{4e^{4x}}{32}+\frac{8x^2e^{4x}}{8}-\frac{4xe^{4x}}{8}+\frac{e^{4x}}{8}$<br>Some working to collect terms between product rule and final expression. |
| | **A1 cso** | For the correct expression only $\frac{dy}{dx}=x^2e^{4x}*$ |
| **(b)** | **M1** | This is a strategy mark for using the correct expression with the correct limits for the volume of rotation.<br>Volume = $\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\,dx$ |
| | **M1** | For using the **given** result from part (a) to integrate<br>$V=\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4x}}{32}(8x^2-4x+1)\right]_{-2}^{0}$<br>Ignore $\pi$ and limits for this mark even if they are missing or incorrect. |
| | **dM1** | For applying the correct limits in an attempt to evaluate the integral<br>$\pi\int_{-2}^{0}9x^2e^{4x}\,dx=\pi\left[\frac{9e^{4\times0}}{32}(8\times0^2-4\times0+1)\right]-\pi\left[\frac{9e^{4\times-2}}{32}(8\times-2^2-4\times-2+1)\right]$<br>Dependent on previous method mark.<br>Condone omission of $\pi$ for this mark. |
| | **A1** | For the correct volume of $0.87142.....\approx 0.87$ rounded correctly to 2sf |
| **SC -attempts integration by parts.** | | |
| | **M1** | This is a strategy mark for using the correct expression with the correct limits for the volume of rotation. |

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|---|---|---|
| | | Volume = $\pi\int_{-2}^{0}\left(3xe^{2x}\right)^2\ dx$ |
| | **M1** | For an attempt to integrate by parts.<br>• They must use the correct formula<br>• $\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$ **TWICE** in the correct direction<br><br>First integration – ignore $\pi$ and $3^2$ for this mark<br><br>$\int x^2e^{4x}\ dx = \frac{x^2e^{4x}}{'4'} - \int 2x\frac{e^{4x}}{'4'}\,dx$<br><br>Second integration<br><br>$\int x^2e^{4x}\ dx = \frac{x^2e^{4x}}{'4'} - \left[\frac{2xe^{4x}}{'16'} - \int\frac{2e^{4x}}{'16'}\right] = \frac{x^2e^{4x}}{'4'} \pm \frac{2xe^{4x}}{'16'} \pm \frac{e^{4x}}{'32'}$ |
| | **dM1** | For applying the correct limits in an attempt to evaluate the integral<br><br>$\pi\int_{-2}^{0}9x^2e^{4x}\ dx = 9\pi\left[\frac{x^2e^{4x}}{'4'} - \frac{2xe^{4x}}{'16'} + \frac{e^{4x}}{'32'}\right]_{-2}^{0}$<br>$= 9\pi\left[\left(0-0+\frac{1}{32}\right) - \left(\frac{(-2)^2e^{4(-2)}}{'4'} - \frac{2(-2)e^{4(-2)}}{'16'} + \frac{e^{4(-2)}}{'32'}\right)\right]$<br>Dependent on previous method mark.<br>Condone omission of $\pi$ for this mark. |
| | **A1** | For the correct volume of $0.87142..... \approx 0.87$ rounded correctly to 2sf |

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