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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel International GCSE

Time 2 hours

Paper reference **4PM1/01R**

Further Pure Mathematics

PAPER 1R



Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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International GCSE in Further Pure Mathematics Formulae shee

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times$ slant height

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$ $\quad$ $\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$ $\quad$ $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ $\quad$ $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \frac{\log_b x}{\log_b a}$

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Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) On the grid below, draw the graph of the line with equation

(i) $4x + 5y = 20$ (ii) $3y - 4x = -12$

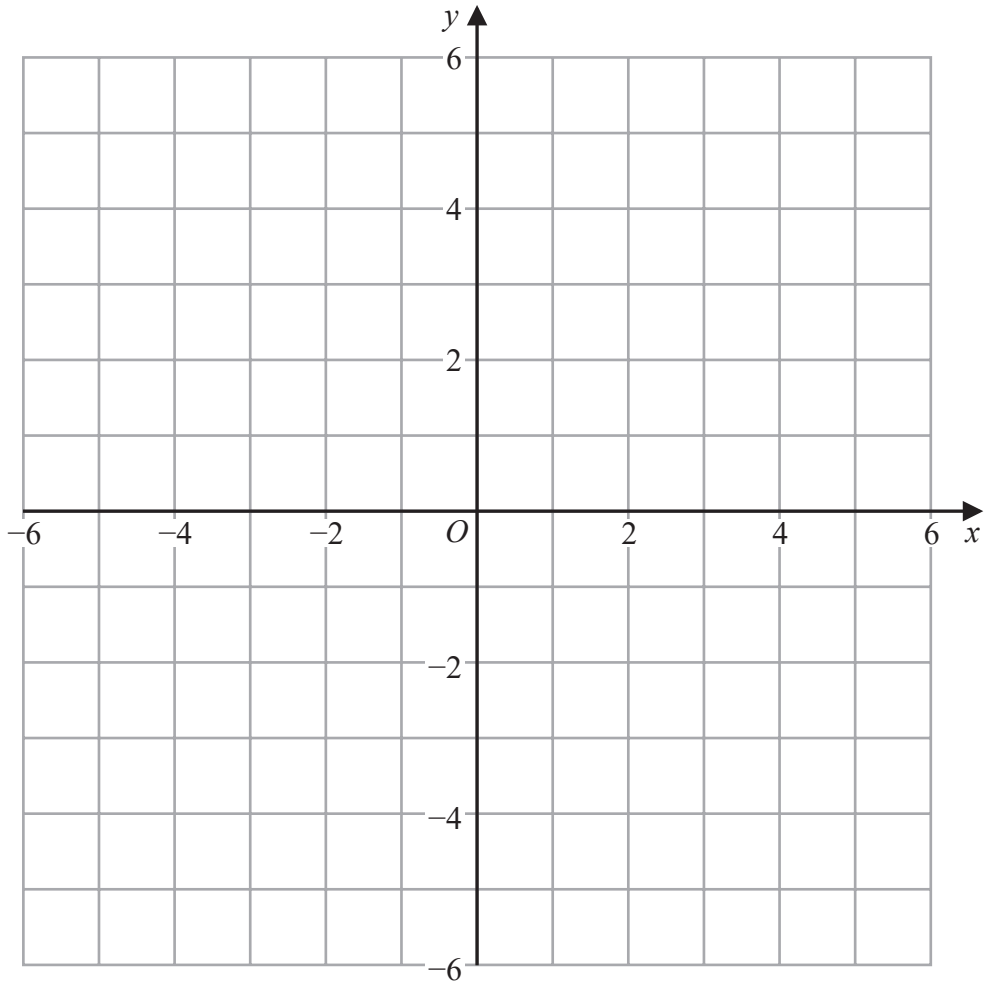
(2)

(b) Show, by shading on the grid, the region R defined by the inequalities

$4x + 5y \leqslant 20$ $3y - 4x \geqslant -12$ $y \leqslant 3$ $x \geqslant 1$

Label the region R.

(2)



Turn over for a spare grid if you need to redraw your graph.

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Question 1 continued

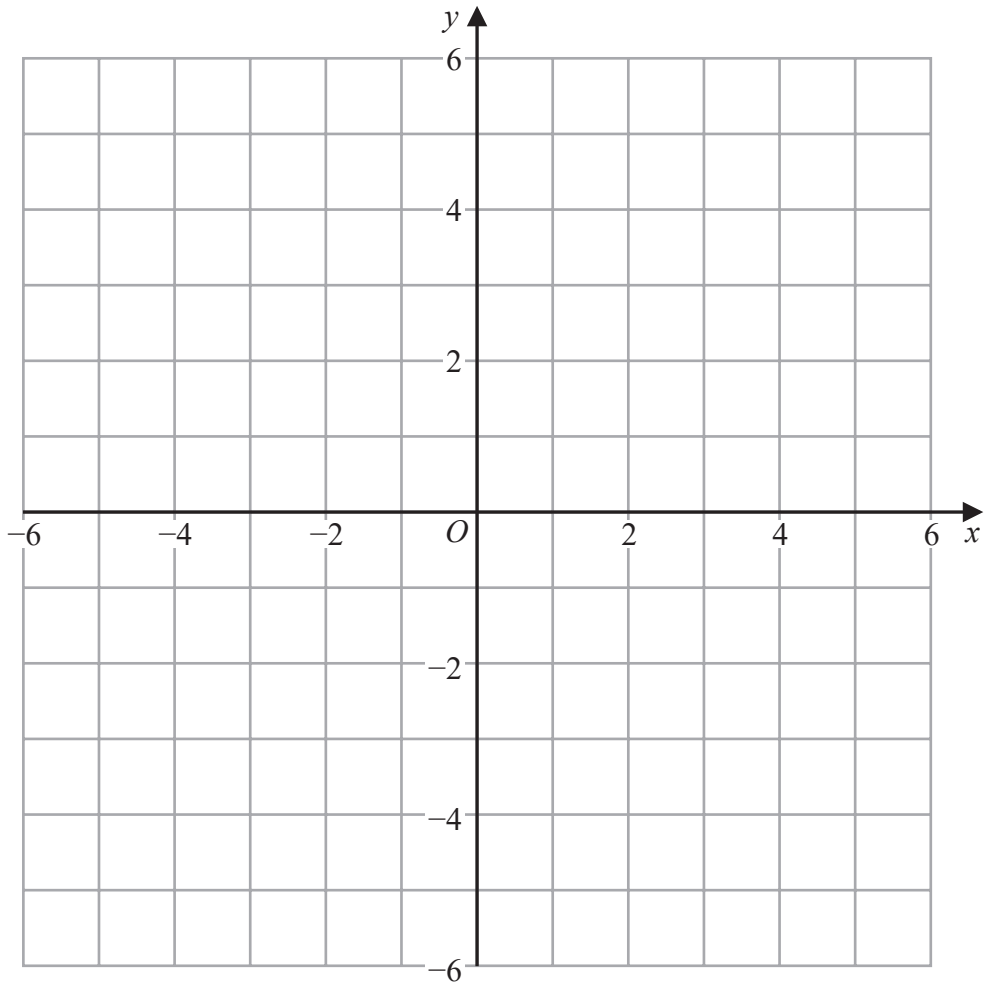
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Question 1 continued

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(Total for Question 1 is 4 marks)

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2 The nth term of an arithmetic series is u_n

Given that $u_5 = 46$ and that $u_{20} = 181$

(a) find

(i) the common difference of this series,

(ii) the first term of this series.

(4)

(b) Evaluate $\sum_{n=21}^{50} u_n$

(4)

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Question 2 continued

(Total for Question 2 is 8 marks)

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3 The point A has coordinates (1, 7) and the point B has coordinates (9, 3)

The line l is the perpendicular bisector of AB

(a) Find an equation of l

(5)

The line l crosses the x-axis at the point C

(b) Find the area of the triangle ABC

(5)

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Question 3 continued

(Total for Question 3 is 10 marks)

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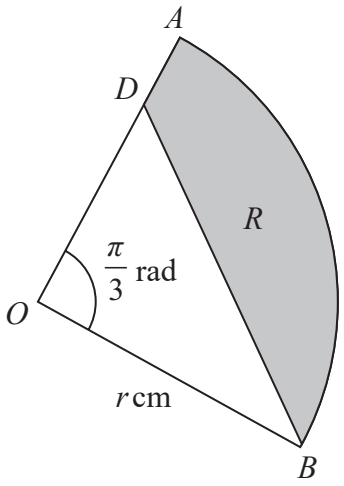


Diagram **NO** accurately drawn

Figure 1

Figure 1 shows sector OAB of a circle with centre O and radius r cm.

The angle $AOB = \frac{\pi}{3}$ radians.

The point D divides OA in the ratio $3 : 1$

The area of the region R, shown shaded in Figure 1, is $79.5\,\text{cm}^2$

(a) Calculate the value, to 2 significant figures, of r

(4)

(b) Calculate the perimeter, in cm to 2 significant figures, of the region R

(4)

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Question 4 continued

(Total for Question 4 is 8 marks)

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5

$$f(x) = (1 + ax)^n \text{ where } n > 2$$

(a) Write down the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3
Give each coefficient in terms of n and a

(2)

In the expansion of $f(x)$, the coefficient of x is 8 and the coefficient of x^2 is 30

(b) Find the value of n and the value of a

(4)

(c) Calculate the coefficient of x^3 in $f(x)$

(2)

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Question 5 continued

(Total for Question 5 is 8 marks)

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6 The points P, Q, R and S are the vertices of a quadrilateral $PQRS$ such that

$$\overrightarrow{PQ} = 2\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{PR} -\mathbf{i} + 18\mathbf{j} \qquad \overrightarrow{PS} = -3\mathbf{i} + 15\mathbf{j}$$

(a) Show that $PQRS$ is a parallelogram.

(4)

(b) Find a unit vector parallel to $\overrightarrow{QS}$ as a simplified expression in terms of **i** and **j**

(4)

The point T lies on QS such that $QT : TS = 5 : 8$

(c) Find $\overrightarrow{PT}$ as a simplified expression in terms of **i** and **j**

(2)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 10 marks)

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7

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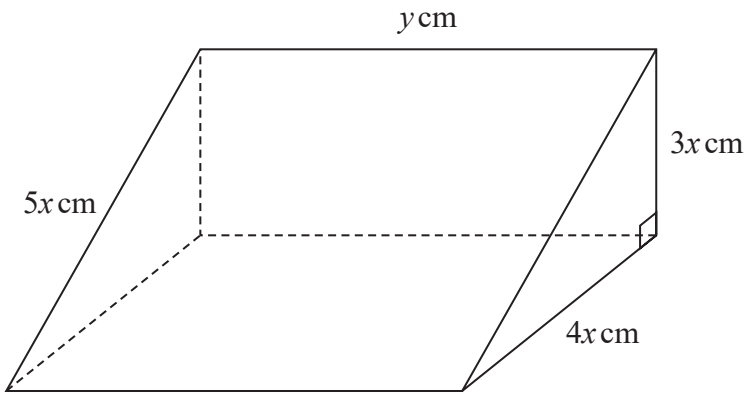


Figure 2

Figure 2 shows a block of wood in the shape of a right triangular prism.

The cross section of the prism is a right-angled triangle with sides of length $3x$ cm, $4x$ cm and $5x$ cm.

The length of the prism is y cm.

The total surface area of the five faces of the prism is $144\,\text{cm}^2$

The volume of the prism is $V\,\text{cm}^3$

(a) Show that

$$V = 72x - 6x^3$$

(5)

Given that x can vary,

(b) use calculus to find the value of x for which V is a maximum, justifying that this value gives a maximum value of V

(4)

(c) Find the maximum value of V

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

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8 The curve C has equation $y = 2x^2 - \sin x$

The point A on C has x coordinate π

Show that an equation of the normal to C at the point A is

$$x + (4\pi + 1)y - \pi(8\pi^2 + 2\pi + 1) = 0$$

(8)

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Question 8 continued

(Total for Question 8 is 8 marks)

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9 (a) Complete the table of values for

$$y = \frac{x^3 + 4}{5 - x}$$

giving your answers to 2 decimal places.

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y	−0.57	0.10	0.5		0.8		1.25		4

(2)

(b) On the grid opposite, draw the graph of

$$y = \frac{x^3 + 4}{5 - x} \qquad \text{for } -2 \leqslant x \leqslant 2$$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation $x^3 - x^2 + 8x - 11 = 0$ in the interval $-2 \leqslant x \leqslant 2$

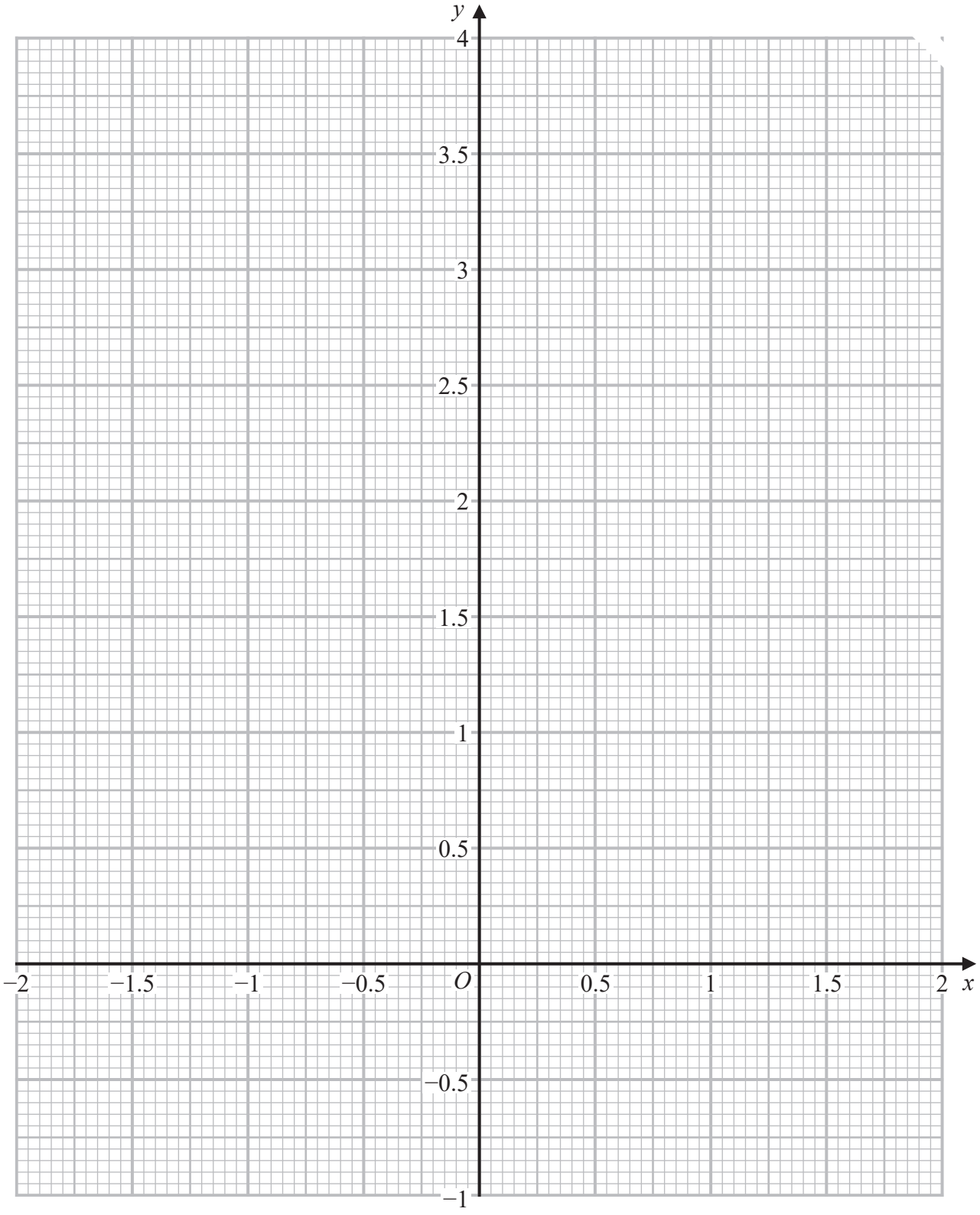
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Question 9 continued



Turn over for a spare grid if you need to redraw your graph.

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Question 9 continued

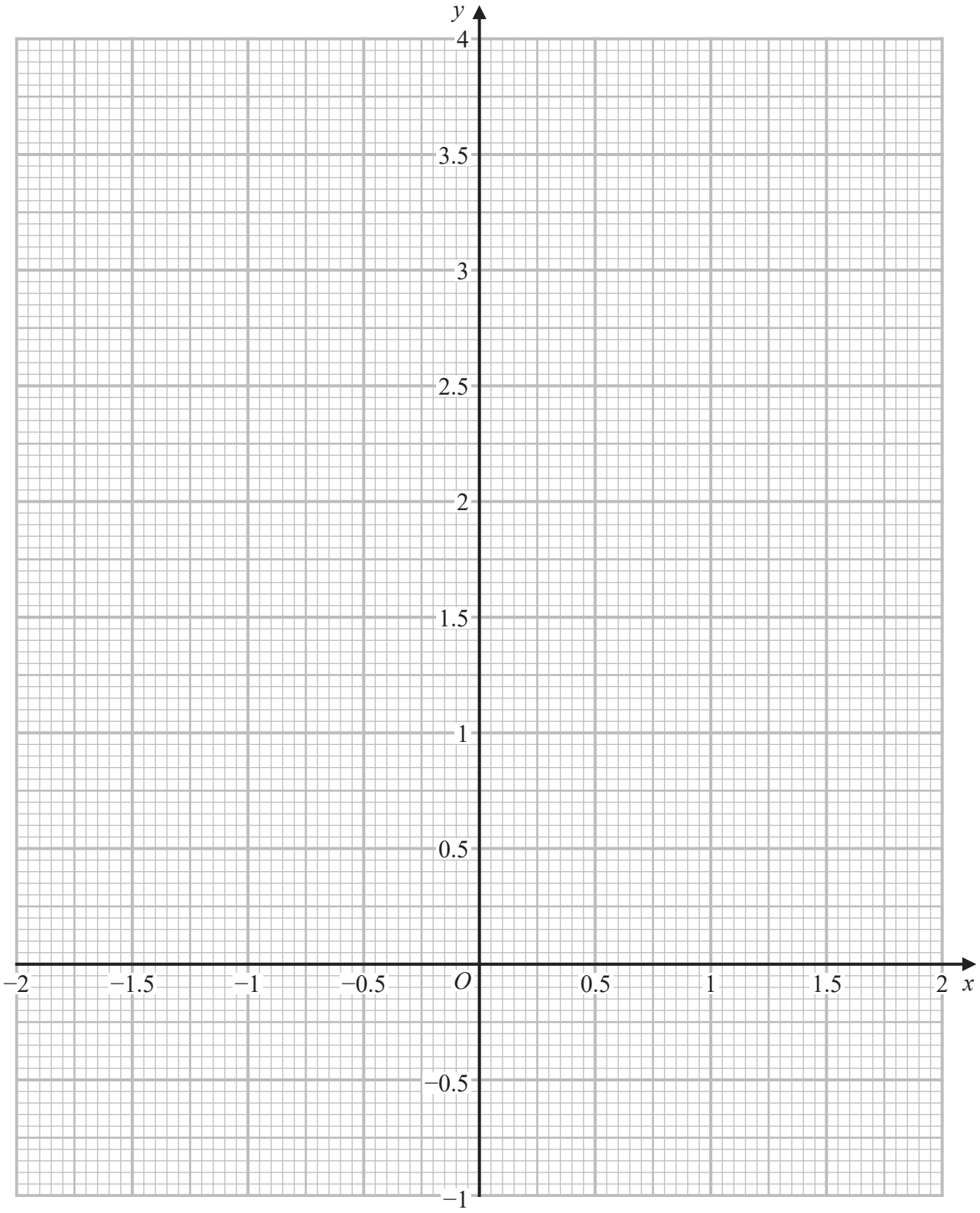
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Question 9 continued

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(Total for Question 9 is 9 marks)

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10 (a) Use the factor theorem to show that $(4x - 3)$ is a factor of

$$16x^3 + 11x - 15$$

(

(b) Using formulae given on page 2, show that

(i) $\sin 2\theta = 2 \sin \theta \cos \theta$

(ii) $\cos 2\theta = 2 \cos^2 \theta - 1$

(5)

(c) Show that the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

becomes the equation

$$16x^3 + 11x - 15 = 0$$

by using the substitution $x = \cos \theta$

(4)

(d) Hence show that any solution of the equation

$$27 \cos \theta \cos 2\theta + 19 \sin \theta \sin 2\theta - 15 = 0$$

is given by $\cos \theta = \frac{3}{4}$

(4)

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Question 10 continued

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Question 10 continued

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Question 10 continued

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11

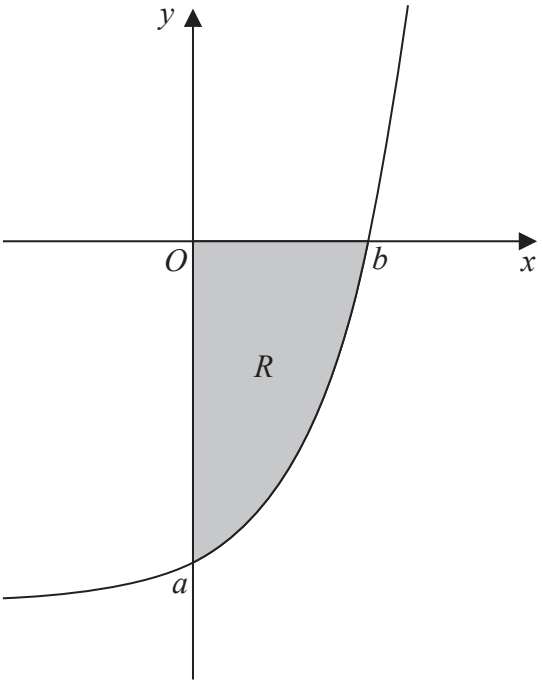


Diagram **T**
accurately n

Figure 3

The finite region R, shown shaded in Figure 3, is bounded by the curve with equation $y = e^{2x} - 9$ and the coordinate axes.

The curve crosses the coordinate axes at the points with coordinates $(0, a)$ and $(b, 0)$

(a) (i) Find the value of a

(ii) Show that $b = \ln 3$

(3)

The region R is rotated through 360° about the x-axis.

(b) Use calculus to find the volume of the solid generated.

Give your answer in the form $\pi(p \ln 3 + q)$, where p and q are integers.

(6)

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Question 11 continued

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 9 marks)

TOTAL FOR PAPER IS 100 MARKS

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