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Centre Number | Candidate Number

# Pearson Edexcel International Advanced Level

**Time** 1 hour 30 minutes

**Paper reference** **WMA13/01**

# Mathematics

## International Advanced Level
## Pure Mathematics P3

**You must have:**
Mathematical Formulae and Statistical Tables (Yellow), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

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**1.** The curve $C$ has equation

$$y = (3x - 2)^6$$

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

**(2)**

Given that the point $P\left(\dfrac{1}{3}, 1\right)$ lies on $C$,

(b) find the equation of the normal to $C$ at $P$. Write your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers to be found.

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**Question 1 continued**

**Q1**

**(Total 6 marks)**

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**2.** The functions f and g are defined by

$$f(x) = \frac{5 - x}{3x + 2} \qquad x \in \mathbb{R}, x \neq -\frac{2}{3}$$

$$g(x) = 2x - 7 \qquad x \in \mathbb{R}$$

(a) Find the value of fg(5)

**(2)**

(b) Find $f^{-1}$

**(3)**

(c) Solve the equation

$$f\left(\frac{1}{a}\right) = g(a + 3)$$

**(4)**

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**Q2**

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**3.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Given that $k$ is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, dx$$

**(2)**

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, dx = \ln 8$$

(b) find the value of $k$

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**Q3**

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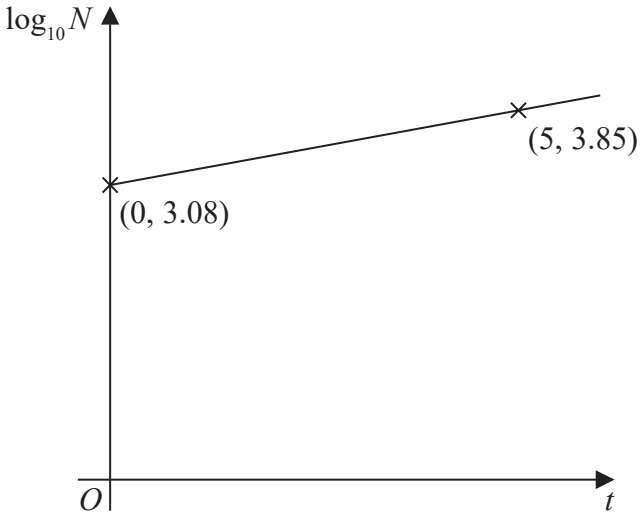


**Figure 1**

The number of subscribers to an online video streaming service, $N$, is modelled by the equation

$$N = ab^t$$

where $a$ and $b$ are constants and $t$ is the number of years since monitoring began.

The line in Figure 1 shows the linear relationship between $t$ and $\log_{10} N$

The line passes through the points (0, 3.08) and (5, 3.85)

Using this information,

(a) find an equation for this line.

**(2)**

(b) Find the value of $a$ and the value of $b$, giving your answers to 3 significant figures.

**(3)**

When $t = T$ the number of subscribers is 500 000

According to the model,

(c) find the value of $T$

**(2)**

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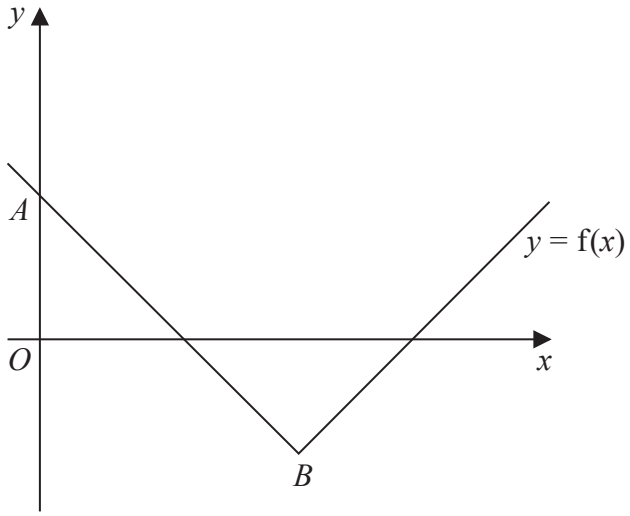


**Figure 2**

Figure 2 shows part of the graph with equation $y = f(x)$, where

$$f(x) = |kx - 9| - 2 \qquad x \in \mathbb{R}$$

and $k$ is a positive constant.

The graph intersects the $y$-axis at the point $A$ and has a minimum point at $B$ as shown.

(a) (i) Find the $y$ coordinate of $A$

(ii) Find, in terms of $k$, the $x$ coordinate of $B$

**(2)**

(b) Find, in terms of $k$, the range of values of $x$ that satisfy the inequality

$$|kx - 9| - 2 < 0$$

**(3)**

Given that the line $y = 3 - 2x$ intersects the graph $y = f(x)$ at two distinct points,

(c) find the range of possible values of $k$

**(3)**

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**Q5**

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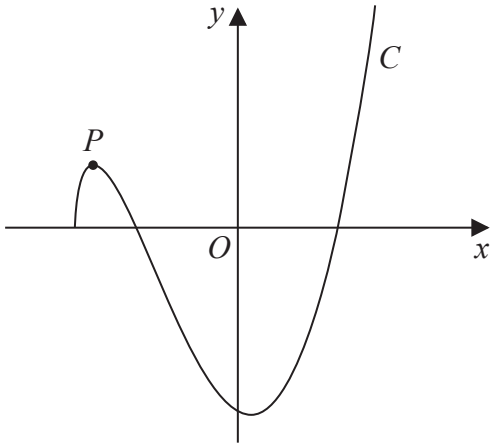


**Figure 3**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where $k$ is an integer to be found.

**(4)**

(b) Hence, find the values of $x$ for which $f'(x) = 0$

**(1)**

Figure 3 shows a sketch of the curve $C$ with equation $y = f(x)$.

The curve has a local maximum at the point $P$

(c) Find the exact coordinates of $P$

**(2)**

The function g is defined by

$$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

**(3)**

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**7.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Show that the equation

$$2\sin\theta(3\cot^2 2\theta - 7) = 13\sec\theta$$

can be written as

$$3\operatorname{cosec}^2 2\theta - 13\operatorname{cosec} 2\theta - 10 = 0$$

**(4)**

(b) Hence solve, for $0 < \theta < \frac{\pi}{2}$, the equation

$$2\sin\theta(3\cot^2 2\theta - 7) = 13\sec\theta$$

giving your answers to 3 significant figures.

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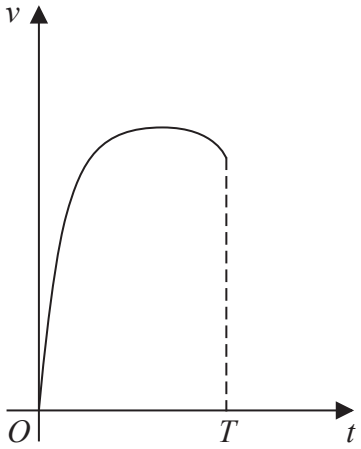


**Figure 4**

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v\,\text{m}\,\text{s}^{-1}$, is modelled by the equation

$$v = 12 - \text{e}^{t-10} - 12\text{e}^{-0.75t} \qquad t \geqslant 0$$

where $t$ seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

**(5)**

Given that the sprinter runs 100 m in $T$ seconds, such that

$$\int_0^T v\,\text{d}t = 100$$

(b) show that $T$ is a solution of the equation

$$T = \frac{1}{12}\left(116 - 16\text{e}^{-0.75T} + \text{e}^{T-10} - \text{e}^{-10}\right)$$

**(4)**

The iteration formula

$$T_{n+1} = \frac{1}{12}\left(116 - 16\text{e}^{-0.75T_n} + \text{e}^{T_n-10} - \text{e}^{-10}\right)$$

is used to find an approximate value for $T$

Using this iteration formula with $T_1 = 10$

(c) find, to 4 decimal places,

(i) the value of $T_2$

(ii) the time taken by the sprinter to run the race, according to the model.

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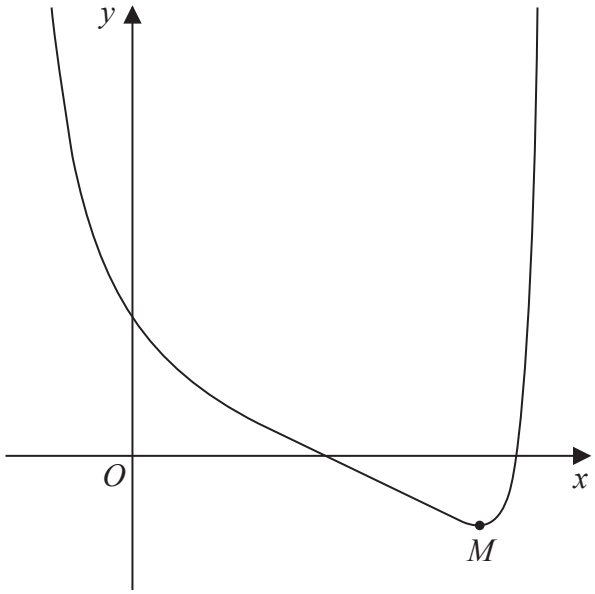


**Figure 5**

**In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

Figure 5 shows the curve with equation

$$y = \frac{1 + 2\cos x}{1 + \sin x} \qquad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point $M$, shown in Figure 5, is the minimum point on the curve.

(a) Show that the $x$ coordinate of $M$ is a solution of the equation

$$2\sin x + \cos x = -2$$

**(4)**

(b) Hence find, to 3 significant figures, the $x$ coordinate of $M$.

**(5)**

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