Pearson Edexcel

Mark Scheme (Results)

Summer 2019

Pearson Edexcel IAL Mathematics (WMA12)

Pure Mathematics

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General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Pearson Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ or ft will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - $\ast$ The answer is printed on the paper or ag- answer given
   - $\llcorner$ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		●
aA	●	
bM1		●
bA1	●	
bB	●	
bM2		●
bA2		●

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$$(x^2 + bx + c) = (x + p)(x + q), \text{ where } |pq| = |c|, \quad \text{leading to } x = \ldots$$

$$(ax^2 + bx + c) = (mx + p)(nx + q), \text{ where } |pq| = |c| \text{ and } |mn| = |a|, \quad \text{leading to } x = \ldots$$

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\quad (x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0, \quad$ leading to $x = \ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

Question Number	Scheme	Marks
1.(a)	(i) $a_2 = 1$	B1
	(ii) $a_{107} = 3$	B1
		(2)
(b)	$\sum_{n=1}^{200}(2a_n - 1) = 5+1+5+1+.......5+1 = 100\times(5+1)$	M1
	$= 600$	A1
		(2)
		(4 marks)

Notes

(a) (i)

B1 $a_2 = 1$ Accept the sight of 1. Ignore incorrect working

(a)(ii)

B1 $a_{107} = 3$ Accept sight of just 3. Ignore incorrect working
If there are lots of 1's and 3's without reference to any suffices they need to choose 3.

(b)

M1 Establishes an attempt to find the sum of a series with two distinct terms.
Look for $100\times a + 100\times b$ or $200\times a + 200\times b$ where a and b are allowable terms.
Examples of allowable terms are

$a,b = 1,5$ (which are correct)

$a,b = 1,3$ (which are the values for (a))

$a,b = 3,7$ (which is using $2a_n + 1$)

$a,b = 0,5$ (which is a slip on the first value)

Methods using AP (and GP) formulae are common and score 0 marks.

A1 600.
600 should be awarded both marks as long as no incorrect working is seen

Question Number	Scheme	Marks
2.(a)	Attempts $(x\pm 2)^2+(y\pm 5)^2.....=0$	M1
	(i) Centre $(-2,5)$	A1
	(ii) Radius $\sqrt{50}$ or $5\sqrt{2}$	B1
		(3)
(b)	Gradient of radius = $\dfrac{(5)-4}{(-2)-5}=-\dfrac{1}{7}$ which needs to be in simplest form	B1ft
	Uses $m_2=-\dfrac{1}{m_1}$ to find gradient of tangent	M1
	Equation of tangent $y-4="7"(x-5)\Rightarrow y=7x-31$	M1 A1
		(4)
		(7 marks)

Notes

(a) **Note that the epen set up here is M1 M1 B1**

M1 Attempts to complete the square on both terms or states the centre as $(\pm 2,\pm 5)$

For completing the square look for $(x\pm 2)^2+(y\pm 5)^2.....=...$

A1 Centre $(-2,5)$ Allow $x=-2, y=5$ This alone can score both marks even following incorrect lines eg $(x+2)^2..(y-5)^2=...$where … could be , for example a minus sign or blank

A1 Radius $\sqrt{50}$ or $5\sqrt{2}$ You may isw after a correct answer.

If a candidate attempts to use $x^2+y^2+2fx+2gy+c=0$ then M1 may be awarded for a centre of $(\pm 2,\pm 5)$

(b) **Note that the epen set up here is M1 M1 M1 A1**

B1ft Correct answer for the gradient of the line joining $P(5,4)$to their centre.

You may ft on their centre but the value must be fully simplified.

M1 Awarded for using $m_2=-\dfrac{1}{m_1}$ to find gradient of tangent.

Do be aware that some good candidates may do the first two marks at once so you may need to look at what value they are using for the gradient of the tangent.

M1 For an attempt to find the equation of the tangent using $P(5,4)$and a changed gradient. Condone bracketing slips only.

If the candidate uses the form $y=mx+c$ they must use *x* and *y* the correct way around and proceed as far as $c=...$

A1 $y=7x-31$ stated. It must be written in this form.

(It cannot be awarded from $y=mx+c$by just stating $c=-31$)

....

Attempts at (b) using differentiation.

B1 $x^2+y^2+4x-10y-21=0\rightarrow 2x+2y\dfrac{dy}{dx}+4-10\dfrac{dy}{dx}=0$.

M1 Substitutes $P(5,4)$into an expression of the form $ax+by\dfrac{dy}{dx}+c+d\dfrac{dy}{dx}=0$ AND finds the value of $\dfrac{dy}{dx}=(7)$. The values of *a*, *b* , *c* and *d* must be non-zero.

M1	Uses $m = \left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=5}$ with $P(5,4)$ to find equation of tangent
A1	$y = 7x - 31$

Question Number	Scheme	Marks
3. (i)	$(x-4)^2 \geqslant 2x-9 \Rightarrow x^2-10x+25 \ldots 0$	M1
	$\Rightarrow (x-5)^2 \ldots 0$	A1
	Explains that "square numbers are greater than or equal to zero" hence (as $x\in\mathbb{R}$), $\Rightarrow (x-4)^2 \geqslant 2x-9$ *	A1*
		(3)
(ii)	Shows that it is not true for a value of n Eg. When $n=3$, $2^n+1=8+1=9$ × Not prime	B1
		(1) **(4 marks)**

Notes

(i) A proof starting with the given statement

M1 Attempts to expand $(x-4)^2$ and work from form $(x-4)^2 \ldots 2x-9$ to form a 3TQ on one side of an equation or an inequality

A1 Achieves both $x^2-10x+25$ and $(x-5)^2$. Allow $(x-5)^2$ written as $(x-5)(x-5)$

A1* For a correct proof. Eg

"square numbers are **greater than or equal** to zero", hence (as $x\in\mathbb{R}$), $(x-5)^2 \geqslant 0$

$\Rightarrow (x-4)^2 \geqslant 2x-9$

This requires (1) Correct algebra throughout, (2) a correct explanation concerning square numbers and (3) a reference back to the original statement

Answers via b^2-4ac are unlikely to be correct. Whilst it is true that there is only one root and therefore it touches the x-axis, it does not show that it is always positive. The explanation could involve a sketch of $y=(x-5)^2$ but it must be accurate with a minimum on the +ve x axis with some statement alluding to why this shows $(x-5)^2 \geqslant 0$

……………………………………………………………………………………………………

Approaches via odd and even numbers will usually not score anything. They would need to proceed using the main scheme via $(2m-4)^2 \geqslant 4m-9$ and $(2m-1-4)^2 \geqslant 2(2m-1)-9$

……………………………………………………………………………………………………

Alt to (i) via contradiction

Proof by contradiction is acceptable and marks in a similar way

M1 For setting up the contradiction

'Assume that there is an x such that $(x-4)^2 < 2x-9 \Rightarrow x^2-10x+25 \ldots 0$

A1 $\Rightarrow (x-5)^2 \ldots 0$ or $(x-5)(x-5)\ldots 0$

A1* This is not true as square numbers are always greater than or equal to 0,
hence $(x-4)^2 \geqslant 2x-9$

……………………………………………………………………………………………………

Alt to part (i) States $(x-5)^2 \geqslant 0$

$\Rightarrow x^2-10x+25 \geqslant 0$

$\Rightarrow x^2-8x-16 \geqslant 2x-9$

$\Rightarrow (x-4)^2 \geqslant 2x-9$

Question Number	Scheme	Marks
M1	States $(x-5)^2 \geqslant 0$ and attempts to expand. There is no explanation required here	
A1	Rearranges to reach $x^2 - 8x - 16 \geqslant 2x - 9$	
A1*	Reaches the given answer $(x-4)^2 \geqslant 2x - 9$ with no errors	
....		
(ii)		
B1	Shows that it is not true for a value of n This requires a calculation (and value found) with a minimal statement that it is not true Eg. '$2^6 + 1 = 65$ which is not prime' or '$2^5 + 1 = 33$ ×' Condone sloppily expressed proofs. Eg. '$2^7 + 1 = \frac{\mathbf{129}}{3} = 43$ which is not prime' Condone implied proofs where candidates write $2^5 + 1 = 33$ which has a factor of 11 If there are lots of calculations mark positively. Only one value is required to be found (with the relevant statement) to score the B1 The calculation cannot be incorrect. Eg. $2^3 + 1 = 10$ which is not prime	

Question Number	Scheme	Marks
4.(a)	$\left(2-\frac{1}{4}x\right)^6 = 2^6, + {}^6C_1 2^5\left(-\frac{1}{4}x\right)^1 + {}^6C_2 2^4\left(-\frac{1}{4}x\right)^2 + {}^6C_3 2^3\left(-\frac{1}{4}x\right)^3 + ...$	B1, M1
	$= 64 - 48x + 15x^2 - 2.5x^3$	A1 A1
		(4)
(b)	$\left(2-\frac{1}{4}x\right)^6 + \left(2+\frac{1}{4}x\right)^6 = \left(64-48x+15x^2-2.5x^3\right)+\left(64+48x+15x^2+2.5x^3\right)$	M1
	$\approx 128 + 30x^2$	B1ft A1
		(3)
		(7 marks)

Notes

(a)

B1 For either 2^6 or 64. Award for an unsimplified ${}^6C_0 2^6\left(-\frac{1}{4}x\right)^0$

M1 For an attempt at the binomial expansion. Score for a correct attempt at term 2, 3 or 4.

Accept sight of ${}^6C_1 2^5\left(\pm\frac{1}{4}x\right)^1$ ${}^6C_2 2^4\left(\pm\frac{1}{4}x\right)^2$ ${}^6C_3 2^3\left(\pm\frac{1}{4}x\right)^3$ condoning omission of brackets.

Accept any coefficient appearing from Pascal's triangle. FYI 6, 15, 20

A1 For any two simplified terms of $-48x + 15x^2 - 2.5x^3$

A1 For $64 - 48x + 15x^2 - 2.5x^3$ ignoring terms with greater powers. This may be awarded in (b) if it is not fully simplified in (a). Allow the terms to be listed $64, -48x, 15x^2, -2.5x^3$. Isw after sight of correct values. The expression written out without any method can be awarded all 4 marks.

(b) **Note that this is now marked M1 B1 A1**

M1 For adding two sequences that must be of the correct form with the correct signs.

Look for $\left(A - Bx + Cx^2 - Dx^3\right) + \left(A + Bx + Cx^2 + Dx^3\right)$ but condone

$\left(A - Bx + Cx^2\right) + \left(A + Bx + Cx^2\right)$

For this to be scored there must be some negative terms in (a)

B1ft For one correct term (follow through). Usually a = 128 but accept either $a = 2\times$'their' +ve 64
or $b = 2\times$'their' $+ve$ 15

A1 For $128 + 30x^2$. CSO so must be from $\left(64-48x+15x^2-2.5x^3\right)+\left(64+48x+15x^2+2.5x^3\right)$

Allow $a = 128, b = 30$ following correct work.

This is a show that question so M1 must be awarded. It must be their final answer so do not isw here.

....

Alternative method in (a):

$$\left(2-\frac{1}{4}x\right)^6 = 2^6\left(1-\frac{1}{8}x\right)^6 = 2^6\left(1+6\left(-\frac{1}{8}x\right)+\frac{6\times5}{2}\left(-\frac{1}{8}x\right)^2+\frac{6\times5\times4}{3!}\left(-\frac{1}{8}x\right)^3+...\right)$$

B1 For sight of factor of either 2^6 or 64

M1 For an attempt at the binomial expansion seen in at least one term within the brackets.
Score for a correct attempt at term 2, 3 or 4.

Question Number	Scheme	Marks
	Accept sight of $6\left(\pm\frac{1}{8}x\right)^1$ $\frac{6\times5}{2}\left(\pm\frac{1}{8}x\right)^2$ $\frac{6\times5\times4}{3!}\left(\pm\frac{1}{8}x\right)^3$ condoning omission of brackets A1 For any two terms of $64-48x+15x^2-2.5x^3$ A1 For all four terms $64-48x+15x^2-2.5x^3$ ignoring terms with greater powers ……….. Attempts to multiply out B1 For 64 M1 Multiplies out to form $a+bx+cx^2+dx^3+...$ and gets *b*, *c* or *d* correct. A1A1 As main scheme	

Question Number	Scheme	Marks
5.(a)	$\frac{dP}{dx}=12-\frac{3}{2}x^{\frac{1}{2}}$	M1A1
	Sets $\frac{dP}{dx}=0\to 12-\frac{3}{2}x^{\frac{1}{2}}=0\to x^n=\ldots$	dM1
	$x=64$	A1
	When $x=64\Rightarrow P=12\times 64-64^{\frac{3}{2}}-120=\ldots$	M1
	Profit = (£) 136 000	A1
		(6)
(b)	$\left(\frac{d^2P}{dx^2}\right)=-\frac{3}{4}x^{-\frac{1}{2}}$ and substitutes in their $x=64$ to find its value or state its sign	M1
	At $x=64$ $\frac{d^2P}{dx^2}=-0.09375<0\Rightarrow$ maximum	A1
		(2) **(8 marks)**

Notes

You should mark parts a and b together. You may see work in (a) from (b)

(a)

M1 Attempts to differentiate $x^n\to x^{n-1}$ seen at least once. It must be an x term and **not** the $120\to 0$

A1 $\frac{dP}{dx}=12-\frac{3}{2}x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{dy}{dx}$ all of the way through part (a).

dM1 Sets their $\frac{dP}{dx}=0$ and proceeds to $x^n=k, k>0$. Dependent upon the previous M. Don't be too concerned with the mechanics of process. Condone an attempted solution of $\frac{dP}{dx}$ **...**0 where **...** could be an inequality

A1 $x=64$. Condone $x=\pm 64$ here

M1 Substitutes their solution for $\frac{dP}{dx}=0$ into P and attempts to find the value of P.
The value of x must be positive. If two values of x are found, allow this mark for any attempt using a positive value.

A1 CSO. Profit = (£) 136 000 or 136 thousand but not 136 or P = 136.
This cannot follow two values for x, eg $x=\pm 64$ Condone a lack of units or incorrect units such as $

(b)

M1 Achieves $\frac{d^2P}{dx^2}=kx^{-\frac{1}{2}}$ and attempts to find its value at $x=''64''$

Alternatively achieves $\frac{d^2P}{dx^2}=kx^{-\frac{1}{2}}$ and attempts to state its sign. Eg $\frac{d^2P}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}<0$

Allow $\frac{d^2P}{dx^2}$ appearing as $\frac{d^2y}{dx^2}$ for the both marks.

A1 Achieves $x=64$, $\frac{d^2P}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}$ and states $\frac{d^2P}{dx^2}=-\frac{3}{32}<0$ (at $x=64$) then the profit is maximised.

This requires the correct value of *x*, the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.

Alt: Achieves $x = 64$, $\frac{d^2P}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$ and states as $x > 0$ or $\sqrt{x} > 0$ means that $\frac{d^2P}{dx^2} < 0$ then the profit is maximised.

Part (b) merely requires the use of calculus so allow

M1 Attempting to find the value of $\frac{dP}{dx}$ at two values either side, but close to their 64. Eg. For 64 , allow the lower value to be $63.5 \leqslant x < 64$ and the upper value to be $64 < x \leqslant 64.5$

A1 Requires correct values, correct calculations with reason and conclusion

Question Number	Scheme	Marks
6.(a)	Sets $\mathrm{f}(3)=0\rightarrow$ equation in k Eg. $27k-135-96-12=0$	M1
	$\Rightarrow 27k=243\Rightarrow k=9$ * (= 0 must be seen)	A1*
		(2)
(b)	$9x^3-15x^2-32x-12=(x-3)(9x^2+12x+4)$	M1 A1
	$=(x-3)(3x+2)^2$	dM1 A1
		(4)
(c)	Attempts $\cos\theta=-\dfrac{2}{3}$	M1
	$\theta=131.8°, 228.2°$ (awrt)	A1
		(2)
		(8 marks)

Notes

(a)

M1 Attempts to set $\mathrm{f}(3)=0\rightarrow$ equation in k Eg. $27k-135-96-12=0$. Condone slips.
Score when you see embedded values within the equation or two correct terms on the lhs of the equation. It is implied by sight of $27k-243=0$ or $27k=135+96+12$.

A1* Completes proof with at least one intermediate "solvable" line namely $27k=243\Rightarrow k=9$ or $27k-243=0\Rightarrow k=9$. This is a given answer so there should be no errors.
It is a "show that" question so expect to see
(i) Either $\mathrm{f}(3)=0$ explicitly stated or implied by sight of $27k-135-96-12=0$ or $27k-243=0$
(ii) One solvable intermediate line followed by $k=9$

....

A candidate could use $k=9$ and start with $\mathrm{f}(x)=9x^3-15x^2-32x-12$

M1 For attempting $\mathrm{f}(3)=9\times3^3-15\times3^2-32\times3-12$.
Alt attempts to divide $\mathrm{f}(x)$ by (x-3). See below on how to score such an attempt

A1* Shows that $\mathrm{f}(3)=0$ and makes a minimal statement to the effect that "so $k=9$"
If division is attempted it must be correct and a statement is required to the effect that there is no remainder, " so $k=9$"

....

If candidates have divided (correctly) in part (a) they can be awarded the first two marks in (b) when they start factorsing the $9x^2+12x+4$ term.

(b)

M1 Attempt to divide or factorise out $(x-3)$. Condone students who use a different value of k.
For factorisation look for first and last terms $9x^3-15x^2-32x-12=(x-3)(\pm9x^2..........\pm4)$
For division look for the following line

$$\begin{array}{r|l} & 9x^2\ +.......... \\ x-3 & \overline{9x^3-15x^2-32x-12} \\ & \underline{9x^3-27x^2} \end{array}$$

A1 Correct quadratic factor $9x^2+12x+4$.
You may condone division attempts that don't quite work as long as the correct factor is seen.

Question Number	Scheme	Marks
	dM1 Attempt at factorising their $9x^2+12x+4$ Apply the usual rules for factorising A1 $(x-3)(3x+2)^2$ or $(x-3)(3x+2)(3x+2)$ on one line. Accept $9(x-3)\left(x+\frac{2}{3}\right)^2$ oe. It must be seen as a product Remember to isw for candidates who go on to give roots $f(x)=(x-3)(3x+2)^2 \Rightarrow x=....$ .... Note: Part (b) is "Hence" so take care when students **write down the answer** to (b) without method If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ score 0 0 0 0 If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=(3x+2)(3x+2)(x-3)$ they score SC 1010. If candidates state $x=-\frac{2}{3},3 \Rightarrow f(x)=9\left(x+\frac{2}{3}\right)\left(x+\frac{2}{3}\right)(x-3)$ they score SC 1010. If candidate writes down $f(x)=(3x+2)(3x+2)(x-3)$ with no working they score SC 1010. If a candidate writes down $(x-3)(3x+2)$ are factors it is 0000 .... (c) M1 A correct attempt to find one value of θ in the given range for their $\cos\theta=-\frac{2}{3}$ (You may have to use a calculator). So if (b) is factorised correctly the mark is for one of the values. This can be implied by sight of awrt 132 or 228 in degrees or awrt 2.3 which is the radian solution. A1 CSO awrt $\theta=131.8°, 228.2°$ with no additional solutions within the range $0 \leqslant \theta < 360°$ Watch for correct solutions appearing from $3\cos\theta-2=0 \Rightarrow \cos\theta=\frac{2}{3}$. This is M0 A0 .... Answers without working are acceptable. M1 For one correct answer M1 A1 For two correct answers with no additional solutions within the range.	

Question Number	Scheme	Marks
7.(a)	Attempts to use $31\,500 = 16\,200 + 9d$ to find 'd'	M1
	For 16 200 + their d $= (1700)$ where d has been found by an allowable method	M1
	Year 2 salary is (£)17 900	A1
		(3)
(b)	Attempts to use $31\,500 = 16\,200r^9$ to find 'r'	M1
	For 16 2000 × their $r = (1.077)$ where r has been found by an allowable method	M1
	Year 2 salary in the range $17440 \leqslant \mathbb{S} \leqslant 17450$	A1
		(3)
(c)	Attempts $\frac{10}{2}\{16200+31500\}$ or $\frac{16200\left(1.077^{10}-1\right)}{1.077-1}$	M1
	Finds $\pm\left(\frac{10}{2}\{16200+31500\}-\frac{16200\left('1.077'^{10}-1\right)}{'1.077'-1}\right)$	dM1
	Difference = £7480 cao	A1
		(3)
		(9 marks)

Notes

(a)

M1 Attempts to use the AP formula in an attempt to find 'd'

Accept an attempt at $31\,500 = 16\,200 + 9d$ resulting in a value for d.

Accept the calculation $\frac{31500-16200}{9}$ condoning slips on the 31500 and 16200

M1 A correct attempt to find the second term by adding 16 200 to their 'd' which must have been found via an allowable method.

Allow d to be found from an "incorrect" AP formula with 10d being used instead of 9d.

Eg $31\,500 = 16\,200 + 10d$ or more likely $\frac{31500-16200}{10} = 1530$ usually leading to an answer of 17730

A1 Year 2 salary is (£) 17 900

(b)

M1 Attempts to use the GP formula in an attempt to find 'r'

Accept an attempt at $31\,500 = 16\,200r^9 \Rightarrow r^9 = \frac{31500}{16200} \Rightarrow r = ...$ condoning numerical slips.

Accept the calculation $\sqrt[9]{\frac{31500}{16200}}$ or $\sqrt[9]{\frac{35}{18}}$ condoning slips on the 31500 and 16200.

It will most likely be implied by a value of r rounding to 1.08

Accept an attempt at $31\,500 = 16\,200r^9$ via logs condoning slips but correct log work must be seen

Question Number	Scheme	Marks

M1 A correct attempt to find the second term by multiplying 16 200 by their '*r*' which must have been found via an allowable method.
Allow *r* to be found from an "incorrect" GP formula with 10 being used instead of 9. Eg following $31\,500 = 16\,200 r^{10}$ or $\sqrt[10]{\frac{31500}{16200}}$. You may also award, condoning slips, for an attempt at 16200 $\times r$ where *r* is their solution of $31\,500 = 16\,200 r^{n}$ where $n = 9$ or 10

A1 For an answer in the range £17440 $\leqslant \mathbb{S} \leqslant$ 17450
Note that $r = 1.077 \Rightarrow 17447.40$

(c)
M1 A correct method to find the sum of either the AP or the GP
For the AP accept an attempt at either $\frac{10}{2}\{16200 + 31500\}$ or $\frac{10}{2}\{2 \times 16200 + 9 \times 'd'\}$
For the GP accept an attempt at either $\frac{16200\left('r'^{10} - 1\right)}{'r' - 1}$ or $\frac{16200\left(1 - 'r'^{10}\right)}{1 - 'r'}$

dM1 Both formulae must be attempted "correctly" (see above) and the difference taken (either way around)
FYI if *d* and *r* are correct, the sums are £238 500 and £231 019.(24)
A1 Difference = £7480 **CAO. Note that this answer is found using the unrounded value for *r*.**
Note that using the rounded value will give £7130 which is A0

If the solutions for (a) and (b) are reversed, eg GP in (a) and AP in (b) then please send to review.

(i) General approach to marking part (i) This is now marked M1 A1 M1 A1 on epen

M1 Takes log of both sides and uses the power law. Accept any base. Condone missing brackets
A1 For a correct linear equation in *x* which only involve logs of base 2 usually $\log_2 6$, $\log_2 2$ or $\log_2 8$ but sometimes $\log_2 \frac{3}{4}$ and others so read each solution carefully
M1 Attempts to use a log law to create a linear equation in $\log_2 3$
Eg. $\log_2 6 = \log_2 2 + \log_2 3$ which is implied by $\log_2 6 = 1 + \log_2 3$
Eg. $\log_2 \frac{3}{4} = \log_2 3 - \log_2 4$ which may be implied by $\log_2 3 - 2$
A1 For $x = -\frac{1}{3} + \frac{\log_2 3}{6}$ oe in the form required by the question. Note that $x = \frac{\log_2 3 - 2}{6}$ is A0

Question Number	Please read notes for 8(i) before looking at scheme		Marks
8.(i)	$8^{2x+1}=6\Rightarrow 2x+1=\log_8 6$ M1	$2^{6x+3}=6$	
	$\Rightarrow 2x+1=\dfrac{\log_2 6}{\log_2 8}$ A1	$\Rightarrow(6x+3)\log_2 2=\log_2 6$ M1 A1	
	$\Rightarrow 2x+1=\dfrac{\log_2 2+\log_2 3}{3}$ M1	$\Rightarrow(6x+3)=\log_2 2+\log_2 3$ M1	
	$\Rightarrow x=\dfrac{\log_2 3}{6}-\dfrac{1}{3}$ A1	$\Rightarrow x=\dfrac{\log_2 3}{6}-\dfrac{1}{3}$ A1	
			(4)
(ii)	$\log_5(7-2y)=2\log_5(y+1)-1$	$2\log_5(y+1)-\log_5(7-2y)=1$	
	$\log_5(7-2y)=\log_5(y+1)^2-1$	$\log_5(y+1)^2-\log_5(7-2y)=1$	M1
	$\log_5(7-2y)=\log_5(y+1)^2-\log_5 5$	$\log_5\dfrac{(y+1)^2}{(7-2y)}=1$	dM1
	$(7-2y)=\dfrac{(y+1)^2}{5}$	$\dfrac{(y+1)^2}{(7-2y)}=5$	A1
	$y^2+12y-34=0\Rightarrow y=$	$y^2+12y-34=0\Rightarrow y=$	ddM1
	$y=-6+\sqrt{70}$ oe only		A1
			(5)
			(9 marks)

Notes

There are many different ways to attempt this but essentially can be marked in a similar way.

If index work is used marks are not scored until the log work is seen

Eg 1: $8^{2x+1}=6\Rightarrow 8^{2x}\times 8=6\Rightarrow 8^{2x}=\dfrac{3}{4}$.

1ST M1 is scored for $2x=\log_8\dfrac{3}{4}$ and then 1ST A1 for $2x=\dfrac{\log_2\frac{3}{4}}{\log_2 8}$

but BOTH of these marks would be scored for $2x\log_2 8=\log_2\dfrac{3}{4}$

2nd M1 would then be awarded for $\log_2\dfrac{3}{4}=\log_2 3-\log_2 4$ which may be implied by $\log_2 3-2$

Two more examples where the candidate initially uses index work.

$8^{2x+1}=6\Rightarrow 2^{3(2x+1)}=6$	$8^{2x+1}=6\Rightarrow 64^x=\dfrac{3}{4}$
$3(2x+1)=\log_2 6$ is M1 A1	$\Rightarrow x=\log_{64}\dfrac{3}{4}$ is M1
as it is a correct linear equation in x involving a log $_2$ term	But $\Rightarrow x\log_2 64=\log_2\dfrac{3}{4}$ is M1 A1

Question Number	**Please read notes for 8(i) before looking at scheme**	Marks

(ii)

M1 Attempts a correct log law. This may include

$2\log_5(y+1) \to \log_5(y+1)^2 \qquad 1 \to \log_5 5$

You may award this following incorrect work. Eg

$1 = 2\log_5(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 2(y+1) - \log_5(7-2y) \Rightarrow 1 = \log_5 \dfrac{2(y+1)}{(7-2y)}$

dM1 Uses two correct log laws. It may not be awarded following errors (see above)

It is awarded for $2\log_5(y+1) - 1 = \log_5 \dfrac{(y+1)^2}{5}$, $2\log_5(y+1) - \log_5(7-2y) = \log_5 \dfrac{(y+1)^2}{(7-2y)}$

$1 + \log_5(7-2y) = \log_5 5(7-2y)$ or $2\log_5(y+1) - 1 = \log_5(y+1)^2 - \log_5 5$

A1 A correct equation in '*y*' not involving logs

ddM1 A correct attempt at finding at least one value of *y* from a 3TQ in *y*

All previous M's must have been awarded. It can be awarded for decimal answer(s), 2.4 and $-$14.4

A1 $y = -6 + \sqrt{70}$ or exact equivalent only.

It cannot be the decimal equivalent but award if the candidate chooses 2.4 following the exact answer. If $y = -6 \pm \sqrt{70}$ then the final A mark is withheld

....

Special case:

Candidates who write

$\log_5(y+1)^2 - \log_5(7-2y) = 1 \Rightarrow \dfrac{\log_5(y+1)^2}{\log_5(7-2y)} = 1 \Rightarrow \dfrac{(y+1)^2}{(7-2y)} = 5$

can score M1 dM0 A0 ddM1 A1 if they find the correct answer.

Question Number	Scheme	Marks
9 (a)	Uses $\tan\theta = \frac{\sin\theta}{\cos\theta} \rightarrow$ $\cos\theta - 1 = 4\sin\theta \frac{\sin\theta}{\cos\theta}$	M1
	$\cos^2\theta - \cos\theta = 4\sin^2\theta$ oe	A1
	Uses $\sin^2\theta = 1-\cos^2\theta \rightarrow$ $\cos^2\theta - \cos\theta = 4\left(1-\cos^2\theta\right)$	M1
	$5\cos^2\theta - \cos\theta - 4 = 0$ *	A1 *
		(4)
(b)	$(5\cos 2x + 4)(\cos 2x - 1) = 0$	M1
	Critical values of $-\frac{4}{5}, 1$	A1
	Correct method to find x from their $\cos 2x = -\frac{4}{5}$	dM1
	$x = 0, 1.25$	A1
		(4) **(8 marks)**

Notes

(a)

M1 Uses $\tan\theta = \frac{\sin\theta}{\cos\theta}$ oe in their $\cos\theta - 1 = 4\sin\theta\tan\theta$.
Condone slips in coefficients and the equation may have been adapted.
This may be implied by candidates who multiply by $\cos\theta$ and reach
$\cos\theta - 1 = 4\sin\theta\tan\theta \Rightarrow \cos^2\theta - \cos\theta = 4\sin^2\theta$. This would be M1 A1

A1 Correct equation, without any fractional terms, in $\sin\theta$ and $\cos\theta$
If the identity $\sin^2\theta = 1-\cos^2\theta$ is used before the multiplication by $\cos\theta$ then it will be for a correct equation, without any fractional terms, in $\cos\theta$ Condone incorrect notation $\cos\theta^2$ for $\cos^2\theta$

M1 Uses $\sin^2\theta = 1-\cos^2\theta$ to produce an equation in just $\cos\theta$

A1* Proceeds to $5\cos^2\theta - \cos\theta - 4 = 0$ with no arithmetical or notational errors. Both identities must be seen to have been applied. Candidates cannot just go from $\cos^2\theta - \cos\theta = 4\sin^2\theta$ to the answer without any evidence of the appropriate identity. No mixed variables within the lines of the "proof"
Condone incomplete lines if it is part of their working.

Eg.
$$\cos^2\theta - \cos\theta = 4\sin^2\theta$$
$$= 4\left(1-\cos^2\theta\right)$$

An example of a notational error is $\cos\theta^2$ for $\cos^2\theta$ (Note that this would only lose the A1*)

(b)

M1 Attempts to find the critical values of the given quadratic by a correct method.

A1 Critical values of $-\frac{4}{5}, 1$. Allow this to be scored even if written as $\cos x = ...$ or even x.
Allow these to be written down (from a calculator)

Question Number	Scheme	Marks
dM1 A correct method to find one value of x from their $\cos 2x = -\frac{4}{5}$ Look for correct order of operations. It is dependent upon the previous mark. This can be implied by awrt 1.5/71.6° or awrt 1.24/1.25 (rads) A1 Both $x = 0$ and awrt 1.25 with no other values in the range $0 \leqslant x < \frac{\pi}{2}$. Condone 1.25 written as 0.398π . Condone if written as $\theta = ..$ …………………………………………………………………………………………………… Answers **without working** can score all marks: Score M1 for one value and M1 A1 M1 A1 for both values and no others in the range.		

Question Number	Scheme	Marks
10 (a)	$\left(\mathrm{f}'(x)\right) = -\dfrac{72}{x^3} + 2$	M1 A1
	Attempts to solve $\mathrm{f}'(x) = 0 \Rightarrow x = \ldots$ via $x^{\pm n} = k, \quad k > 0$	dM1
	$x > \sqrt[3]{36}$ oe	A1
		(4)
(b)	$\displaystyle\int \frac{36}{x^2} + 2x - 13\,\mathrm{d}x = -\frac{36}{x} + x^2 - 13x\ (+c)$	M1 A1
	Uses limits 9 and 2 $= \left(-\dfrac{36}{9} + 9^2 - 13\times 9\right) - \left(-\dfrac{36}{2} + 2^2 - 13\times 2\right) = 0$ *	dM1 A1*
		(4)
(c)(i)	8	B1
(ii)	$\displaystyle\int_2^6 \left(\frac{36}{x^2} + 2x + k\right)\mathrm{d}x = 0 \Rightarrow \left[-\frac{36}{x} + x^2 + kx\right]_2^6 = 0 \Rightarrow (30 + 6k) - (-14 + 2k) = 0$	
	$44 + 4k = 0 \Rightarrow k = -11$	M1 A1
		(3)
		(11 marks)

Notes

(a)

M1 Attempts $\mathrm{f}'(x)$ with one index correct. Allow for $x^{-2} \to x^{-3}$ or $2x \to 2$

A1 $\mathrm{f}'(x) = -\dfrac{72}{x^3} + 2$ correct but may be unsimplified $\mathrm{f}'(x) = 36 \times -2x^{-3} + 2$

dM1 Attempts to find where $\mathrm{f}'(x) = 0$. Score for $x^n = k$ where $k > 0$ and $n \neq \pm 1$ leading to $x = \ldots$
Do not allow this to be scored from an equation that is adapted incorrectly to get a positive k.
Allow this to be scored from an attempt at solving $\mathrm{f}'(x)\ldots 0$ where **...** can be any inequality

A1 Achieves $x > \sqrt[3]{36}$ or $x > 6^{\frac{2}{3}}$ Allow $x \geqslant \sqrt[3]{36}$ or $x \geqslant 6^{\frac{2}{3}}$ but not $x > \left(\dfrac{1}{36}\right)^{-\frac{1}{3}}$

We require an exact value but remember to isw. An answer of 3.302 usually implies the first 3 marks.

(b)

M1 For $x^n \to x^{n+1}$ seen on either $\dfrac{36}{x^2}$ or $2x$. Indices must be processed. eg $x^{1+1} \to x^2$

A1 $\displaystyle\int \frac{36}{x^2} + 2x - 13\,\mathrm{d}x = -\frac{36}{x} + x^2 - 13x$ which may be unsimplified. Eg $x^2 \leftrightarrow \dfrac{2x^2}{2}$ Allow with + c

dM1 Substitutes 9 and 2 into their integral and subtracts either way around. Condone missing brackets
Dependent upon the previous M

A1* Completely correct integration with either embedded values seen or calculated values $(-40) - (-40)$
Note that this is a given answer and so the bracketing must be correct.

Question Number	Scheme	Marks

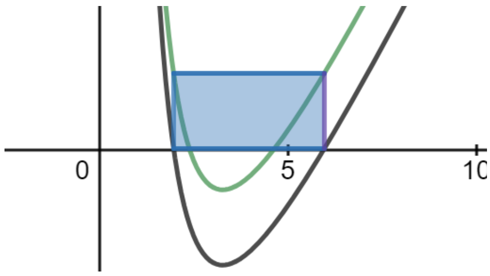
(c)(i)

B1 For sight of 8. Allow this to be scored from a restart, from a calculator or … = 8

(c)(ii)

M1 This may be awarded in a variety of ways

- A restart (See scheme). For this to be awarded all terms must be integrated with $k \to kx$, the limits 6 and 2 applied, the linear expression in k must be set equal to 0 and a solution attempted.
- An attempt at solving $\int_2^6 k+13\,\mathrm{d}x = 8$ or equivalent. Look for the linear equation $-8+4(13+k)=0$ or $4(13+k)=8$ and a solution attempted.
- Recognising that the curve needs to be moved up 2 units.
- Sight of $\frac{8}{6-2}$ or $-13+2$



A1 $k=-11$. This alone can be awarded both marks as long as no incorrect working is seen.

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