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Pearson
Edexcel

Mark Scheme (Results)

January 2021

Pearson Edexcel International Advanced Level
in Pure Mathematics P1
Paper (WMA11 / 01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

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EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes and can be used if you are using the annotation facility on ePEN.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\surd$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - * The answer is printed on the paper or ag- answer given

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- ⌐ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected. If you are using the annotation facility on ePEN, indicate this action by 'MR' in the body of the script.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Marks for each question are scored by clicking in the marking grids that appear below each student response on ePEN. The maximum mark allocation for each question/part question(item) is set out in the marking grid and you should allocate a score of '0' or '1' for each mark, or "trait", as shown:

	0	1
aM		•
aA	•	
bM1		•
bA1	•	
bB	•	
bM2		•
bA2		•

9. Be careful when scoring a response that is either all correct or all incorrect. It is very easy to click down the '0' column when it was meant to be '1' and all correct.

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General Principles for Pure Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q),\ \text{where} |pq| = |c|$, leading to x = …

$(ax^2 + bx + c) = (mx + p)(nx + q),\ \text{where} |pq| = |c| \text{ and } |mn| = |a|$, leading to x = …

2. Formula

Attempt to use correct formula (with values for *a*, *b* and *c*).

3. Completing the square

Solving $x^2 + bx + c = 0$: $(x \pm \frac{b}{2})^2 \pm q \pm c, \quad q \neq 0$, leading to x = …

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:
Method mark for quoting a correct formula and attempting to use it, even if there are small mistakes in the substitution of values.
Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Answers without working

The rubric says that these may not gain full credit. Individual mark schemes will give details of what happens in particular cases. General policy is that if it could be done "in your head", detailed working would not be required. Most candidates do show working, but there are occasional awkward cases and if the mark scheme does not cover this, please contact your team leader for advice.

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Question Number	Scheme	Marks
1. (a)	$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \Rightarrow \frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2x^2}$	M1 A1 A1 **(3)**
(b)	$x = \frac{1}{2} \Rightarrow y = 3$	B1
	Substitutes $x = \frac{1}{2}$ into their $\frac{dy}{dx} = 6x^2 - 10x + \frac{3}{2x^2} = ...\left(= \frac{5}{2}\right)$	M1
	Uses the perpendicular gradient rule Eg. $\frac{5}{2} \rightarrow -\frac{2}{5}$	dM1
	Attempts the equation of the normal at P $y - 3 = -\frac{2}{5}\left(x - \frac{1}{2}\right)$	M1
	$2x + 5y - 16 = 0$ oe	A1 **(5)**
		(8 marks)

(a)

M1 For reducing any power by 1 seen at least once $x^n \rightarrow x^{n-1}$ including $7 \rightarrow 0$ and from an incorrect index when manipulating $\frac{3}{2x}$ eg $\frac{3}{2x} \rightarrow ...x^{\frac{1}{2}} \rightarrow x^{-\frac{1}{2}}$. Allow eg $...x^2 \rightarrow ...x^1$ or $...x^3 \rightarrow ...x^{3-1}$ for this mark.

A1 For two of three correct terms (simplified) $6x^2 - 10x + \frac{3}{2x^2}$ which may appear on different lines of working. Do not allow $-10x^1$

A1 $6x^2 - 10x + \frac{3}{2x^2}$ or $6x^2 - 10x + \frac{3}{2}x^{-2}$ oe on one line (in any order) Do not accept $6x^2 + -10x + \frac{3}{2}x^{-2}$ and do not allow $-10x^1$

(b) **Be careful to check whether they are attempting to find *y* or $\frac{dy}{dx}$. Send to review if unsure**.

B1 Correct coordinates for *P* seen or used $x = \frac{1}{2} \Rightarrow y = 3$. Ignore their labelling of *y*.

M1 Attempts to find the value of $\frac{dy}{dx}$ when $x = \frac{1}{2}$ by substituting into their part (a) answer. They must proceed as far as finding a value for this mark. Ignore their labelling of $\frac{dy}{dx}$.

dM1 Finds or uses their gradient of the normal using the correct rule. Implied by $m \rightarrow -\frac{1}{m}$. It is dependent on the previous method mark.

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M1 Attempts to find the equation of the normal using a changed gradient and their $\left(\frac{1}{2}, 3\right)$. Sight of embedded values eg $y - "3" = "-\frac{2}{5}"\left(x - \frac{1}{2}\right)$ is sufficient (the bracket must be correct) or if they use $y = mx + c$ they must proceed as far as $c = ...$

This mark can only be scored if they have attempted to find y and $\frac{dy}{dx}$ when $x = \frac{1}{2}$. If only the values of these are stated or seen within the equation of the normal then they must be $y = 3$ and gradient of the normal $= -\frac{2}{5}$ (or the correct negative reciprocal value for their $\frac{dy}{dx}$ when $x = \frac{1}{2}$).

A1 $2x + 5y - 16 = 0$ oe allowing any integer multiples of this. Must have = 0. Accept $2x + 5y + -16 = 0$

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Question Number	Scheme	Marks
2 .(a)	Attempts to use the given model at least once. Eg $2^3 = p\times 3^2 + q$	M1
	Two correct simplified equations $9p+q=8$ $\quad 25p+q=13.8(24)$	A1
	Solves simultaneously to get at least one of p or q	dM1
	$p=0.364, q=4.72(4)$	A1 **(4)**
(b)	Attempts to find T when $H=5$ Eg. Calculates $\sqrt{\frac{125-"q"}{"p"}}$	M1
	$(T=)$ 18.2	A1 **(2)**
		(6 marks)

(a)

M1 For an attempt to use the model at least once. Eg. either $2^3 = p\times 3^2 + q$ or $2.4^3 = p\times 5^2 + q$

A1 Two correct simplified equations $9p+q=8$ oe $\quad 25p+q=13.8(24)$ oe

(may be implied by later work) $\quad 25p+q=13\frac{103}{125}\left(=\frac{1728}{125}\right)$

dM1 Solves simultaneously to get a value for p or a value for q. Condone slips in their working and sight of $p=...$ or $q=...$ is sufficient for this mark.

A1 $p=0.364, q=4.72(4)$ allow fractions eg $p=\frac{91}{250}$ and $q=4\frac{181}{250}\left(=\frac{1181}{250}\right)$. Correct answers with no working scores full marks. Isw after correct values for p and q.

(b)

M1 Makes $T^2=\frac{125-"q"}{"p"}$ and proceeds to $T=...$ when $H=5$ using their p and their q or values taken to be their p and their q. This can only be scored if $\frac{125-"q"}{"p"}>0$

Eg. $(T=)\sqrt{\frac{125-"q"}{"p"}}$ is sufficient or $T^2=\frac{125-"q"}{"p"}\Rightarrow T=...$

If only a value is stated you will need to check this on your calculator.

A1 $(T=)$ 18.2 cao (allow $T=18.2$ years or 18 years 2.4 months)

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Question Number	Scheme	Marks
3. (a)(i)	$P(-180,-4)$	B1, B1
(ii)	$Q(450,0)$	B1
		(3)
(b)	$R(360,7)$	B1, B1
		(2)
		(5 marks)

For all parts condone missing brackets and check the graph/next to the question for answers.

Condone use of the degree symbol for their x values eg $(-180°,...)$ instead of $(-180,...)$

(a)(i)

B1 $(-180,...)$ or $(...,-4)$ or $x=-180$ or $y=-4$ condone x in radians

B1 $(-180,-4)$ or $x=-180,\ y=-4$ Must be in degrees

SC1 $(-4,-180)$ (on EPEN this would be scored B1B0)

(a)(ii)

B1 $(450,0)$ or $x=450,\ y=0$ condone $\left(\frac{5}{2}\pi,0\right)$

(b)

B1 $(360,...)$ or $(...,7)$ $x=360$ or $y=7$ condone x in radians

B1 $(360,7)$ or $x=360,\ y=7$ Must be in degrees. Ignore any reference to $(0,7)$

SC1 $(7,\ 360)$ (on EPEN this would be scored B1B0)

Note if radians used throughout then max score:

(a)(i) $(-\pi,-4)$ B1B0 **(a)(ii)** $\left(\frac{5}{2}\pi,0\right)$ B1 **(b)** $(2\pi,7)$ B1B0

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Question Number	Scheme	Marks
4.(a)	Gradient $PQ = -3$	B1
	Attempts to find equation of l Eg. $y-13=-3(x+2)$	M1
	$y=-3x+7$	A1
		(3)
(b)	Attempts to use minimum is $(4,-5)$ Eg $y=...(x-4)^2-5$	M1
	Attempts to use $(-2,13)$ with $y=a(x-4)^2-5 \Rightarrow a=...$	dM1
	$\Rightarrow y=\frac{1}{2}(x-4)^2-5$ or $y=\frac{1}{2}x^2-4x+3$ oe	A1
		(3)
(c)	Two of $y>-3x+7$, $y<\frac{1}{2}(x-4)^2-5$ $x<-2$	M1
	All three of $y>-3x+7$, $y<\frac{1}{2}(x-4)^2-5$ $x<-2$	A1
		(2)
		(8 marks)

(a)

B1 Finds the gradient of l to be -3

If simultaneous equations are used then it would be scored for $m=-3$

M1 Attempts to find the equation of the line using their gradient for PQ and one of the points. Sight of embedded values eg $(y--5)="-3"(x-4)$ is sufficient (allow one sign slip on one of the brackets) or if they use $y=mx+c$ they must proceed as far as $c=...$

If the perpendicular gradient is used it is M0.

A simultaneous method, eg solving $13=-2m+c$ and $-5=4m+c$ must reach values for m and c.

A1 $y=-3x+7$ cso Condone $m=-3,\ c=7$

(b)

M1 Attempts to use the **minimum point** $(4,-5)$ to form a valid equation

Way One: e.g. $(y=)\ A(x-4)^2-5$ (condoning A to be 1). Condone

Way Two: Letting $y=ax^2+bx+c$ and either:

- finding $\frac{dy}{dx}$ and setting $2a\times4+b=0$
- uses the minimum point and symmetry starting with $-\frac{b}{2a}=4$ $(\Rightarrow -b=8a \Rightarrow 8a+b=0)$
- uses symmetry about the minimum point to identify the point $(10,13)$ and form the equation $13=100a+10b+c$

Do not award for only substituting the minimum point into a linear equation.

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dM1 **Way One**: Attempts to use $(-2,13)$ with $y=a(x-4)^2-5 \Rightarrow a=...$

Way Two: Attempts to use $(-2,13)$ and $(4,-5)$ in $y=ax^2+bx+c$ and using these two equations, $4a-2b+c=13$ and $16a+4b+c=-5$, with $2a\times 4+b=0$ to find *a*, *b* and *c*. Condone slips in their working.

This may also be done using a calculator or use matrices

$$\begin{pmatrix}16 & 4 & 1\\ 4 & -2 & 1\\ 8 & 1 & 0\end{pmatrix}\begin{pmatrix}a\\ b\\ c\end{pmatrix}=\begin{pmatrix}-5\\ 13\\ 0\end{pmatrix}\Rightarrow\begin{pmatrix}a\\ b\\ c\end{pmatrix}=...$$

Candidates who used symmetry with the point $(-2,13)$ and the point $(10,13)$ to form the equation $13=100a+10b+c$ attempt to solve simultaneously with two other equations from either $2a\times 4+b=0$ or $4a-2b+c=13$ and $16a+4b+c=-5$

A1 $y=\frac{1}{2}(x-4)^2-5$ or equivalent such as $y=\frac{1}{2}x^2-4x+3$ or $2y=x^2-8x+6$ (cannot be $C=...$)

(c)

M1 Two of $y>"-3x+7"$, $y<"\frac{1}{2}(x-4)^2-5"$, $x<-2$ (ignore any others for this mark)

Their line from part (a) must have a negative gradient and their curve from part (b) must be a positive quadratic.

(Allow two of $y>"-3x+7"$, $y<"\frac{1}{2}(x-4)^2-5"$, $x<k$ where *k* is any value between -2 and 4)

$"-3x+7"<y<"\frac{1}{2}(x-4)^2-5"$ also scores M1 for having two of the inequalities.

BUT $"-3x+7"<x<"\frac{1}{2}(x-4)^2-5"$ with one other inequality is M0 as this is insufficient to define the inequality $x<-2$

Ignore any use of set notation for this mark.

A1 All three of $y>-3x+7$, $y<\frac{1}{2}(x-4)^2-5$, $x<-2$ (and no others)

(Allow all three of $y>-3x+7$, $y<\frac{1}{2}(x-4)^2-5$, $x<k$ where *k* is any value between -2 and 4)

$-3x+7<y<\frac{1}{2}(x-4)^2-5$, $x<-2$ (or $x<k$ where *k* is any value between -2 and 4) is also acceptable.

If set notation is used, then they must use "$\cap$" between any of their inequalities rather than $\cup$.

Condone attempts as long as the intention is clear.

Eg $\left\{x,y\in\mathbb{R}: y>-3x+7 \cap y<\frac{1}{2}(x-4)^2-5 \cap x<-2\right\}$

$\left\{x,y\in\mathbb{R}: y>-3x+7\ , y<\frac{1}{2}(x-4)^2-5\ , x<-2\right\}$ would be acceptable

.....

Allow consistent use of $>\leftrightarrow\geqslant$ for all of their inequalities including the inequality $x<-2$

If $x<k$ or $x\leqslant k$ is stated where $(-2<k\leqslant 4)$ then only the inequalities for their "$-3x+7$" and the positive quadratic needs to be consistent with each other.

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Question Number	Scheme	Marks
5 (a)	Attempts the sine rule $\frac{\sin\alpha}{14}=\frac{\sin 0.43}{6}$	M1
	$\Rightarrow \alpha = 1.337$ (radians) Accept awrt 1.33/1.34 or awrt 76.6/76.7 ($^\circ$)	A1
	angle $AOD = \pi - 1.337 =$ awrt 1.805 (radians)	A1
		(3)
(b)	Attempts $s = r\theta$ with $r = 6$ and an allowable θ	M1
	Arc length $ABC =$ awrt 26.9 m	A1
		(2)
(c)	Attempts $\frac{1}{2}r^2\theta$ with $r = 6$ and an allowable θ in radians (= 80.6)	M1
	Attempts area $AOD = \frac{1}{2}\times 6\times 14\times \sin("0.91")$ oe (= 33.1)	M1
	Attempts sector + triangle with correct attempt at angles	dM1
	$= 113.7$ (m^2)	A1
		(4)
		(9 marks)

(a) **Note on EPEN it is M1M1A1 but we are marking this M1A1A1**

M1 Attempts the sine rule $\frac{\sin\alpha}{14}=\frac{\sin 0.43}{6}$. Sight of the values embedded in the equation or awrt 1.33/1.34 implies this mark. They may also work in degrees so sight of awrt 76.6/76.7 also implies this mark.

A1 awrt 1.33/1.34 (radians) or awrt 76.6/76.7 (degrees)

A1 awrt 1.805 (radians)

(b) Note method marks can still be awarded even if their angle from part (a) is rounded.

M1 Attempts $s = r\theta$ with $r = 6$ and an allowable angle.
Accept as an allowable their angle "(a)", $\pi - "(a)"$ or $2\pi - "(a)"$
Note if their answer was 1.805 in (a):

Angle used:	$r\theta$
1.805	10.8
$\pi - 1.805$ (1.34)	8.02
$2\pi - 1.805$ (4.48)	26.9

They may also work in degrees using an allowable angle so look for $\frac{\ldots}{360}\times 12\pi$ oe

A1 Awrt 26.9 (metres) Must come from angle $AOD =$ awrt 1.8 in (a).
Note if they use the acute angle for AOD then arc length is 29.7 (m) which scores M1A0

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(c) **Beware of different methods to find the required area. Send to review if unsure.**

M1 Attempts $\frac{1}{2}r^2\theta$ with $r=6$ and an allowable θ ($\frac{1}{2}r\times$ arc length is also acceptable)

Accept as an allowable angle their "(a)", $\pi-"(a)"$ or $2\pi-"(a)"$

Note if their answer was 1.805 in (a):

Angle used:	$\frac{1}{2}r^2\theta$
1.805	32.5
$\pi-1.805$ (1.34)	24.1
$2\pi-1.805$ (4.48)	80.6

They may also work in degrees using an allowable angle so look for $\frac{\ldots}{360}\times 36\pi$ oe

Condone use of $\frac{1}{2}r^2(\theta-\sin\theta)$ with $r=6$ and an allowable θ to score this mark.

M1 Correct method to find area of triangle AOD.

Look for the correct combination of sides and inclusive angle.

Angle OAD must be found by a correct method $\pi-0.43-"(a)"$ (or in degrees: $180-24.6-"(a)"$)

Alternatively, they may form two right angled triangles. This must be a correct method to find the area of both triangles and add them together.

Another method is to find OD eg using the sine rule $(=11.3...)$ and calculate $\frac{1}{2}\times 14\times OD\times\sin(0.43)$

dM1 Full method to find the correct area

Both areas must be found using a correct method.

Look for eg $\frac{1}{2}\times 6^2\times(2\pi-"(a)")+\frac{1}{2}\times 6\times 14\times\sin(\pi-0.43-"(a)")$

It is dependent on both of the previous method marks.

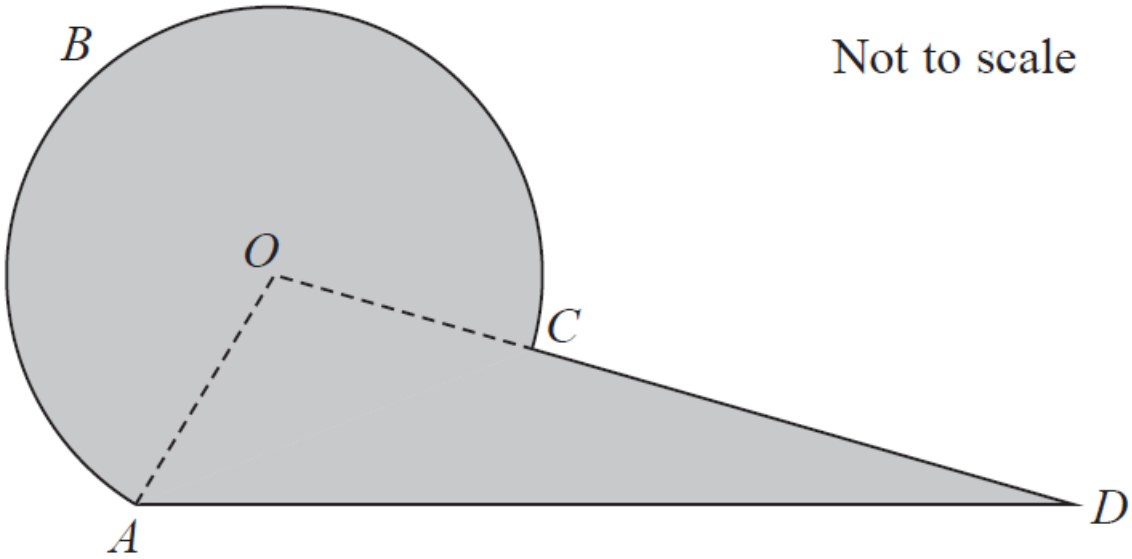
A1 awrt 113.7 (m^2)

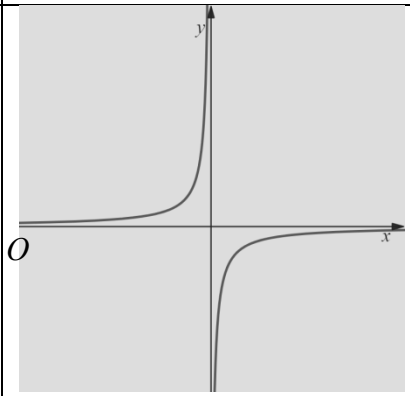
Note if they use the acute angle for AOD then total area is 130 (m^2) which scores M1M1dM1A0

If the acute angle is used then as a guide see the diagram below:

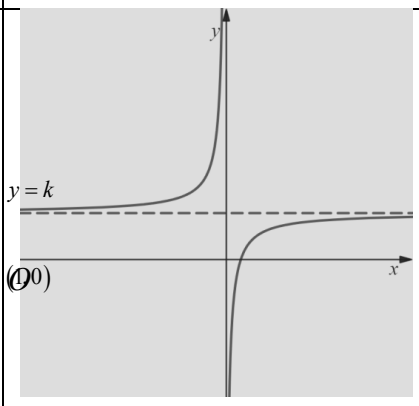
Max score is (a) M1A1A0, (b) M1A0, (c) M1M1dM1A0

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Question Number	Scheme	Marks
6.(a)	 Negative reciprocal shape Fully correct	M1 A1 **(2)**

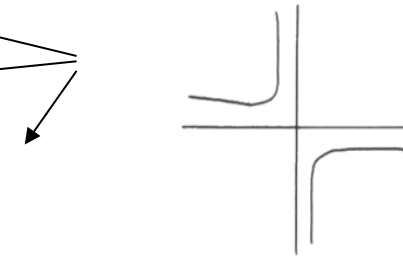
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Question Number	Scheme	Marks
(b)	 Graph is part (a) translated ↑ Correct asymptote or intercept Correct asymptote and intercept	B1ft B1 B1 **(3)**
(c)	Sets $3x+4=-\frac{k}{x}+k \Rightarrow 3x^2+(4-k)x+k=0$ Attempts use $b^2-4ac=0$ to find the critical values Uses $b^2-4ac<0$ and selects inside region for critical values $10-2\sqrt{21}<k<10+2\sqrt{21}$	M1, A1 M1 dM1 A1 **(5)** **(10 marks)**

(a)

M1 Negative reciprocal shape (top left/bottom right sections) in any position on a set of axes with no clear vertical or horizontal 'overlaps'.

A1 Correct sketch appearing in quadrants 2 and 4 only. Condone slips of the pen at the ends as long as the graph does not curve back on itself. Ignore any scaling on the axes.



This scores M1A1. We condone these as they are not clear turning points and are regarded as slips of the pen.

(b) Note on EPEN it is M1B1A1. We are marking this B1ftB1B1

B1ft For attempting to translate their graph from part (a) up (or a correct graph if (a) is incorrect)

B1 For a correct intercept stated (or allow 1, $\frac{k}{k}$ or (1,0) marked on the correct axis with the curve passing through this) OR $y=k$ stated. Do not allow just k to be marked on the line of the asymptote.

B1 For a correct intercept stated (or allow 1, $\frac{k}{k}$ or (1,0) marked on the correct axis with the curve passing through this) AND $y=k$ stated. Do not allow just k to be marked on the line of the asymptote.

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Note if they have a contradiction between what is stated and what is written on the graph then the graph takes precedence.

(c)

M1 Attempts to set $3x+4=-\frac{k}{x}+k$ and proceeds to a 3TQ in x on one side of the equation but terms do not need to be collected (=0 may be omitted).

A1 Correct quadratic seen $3x^2+(4-k)x+k=0$ with the x terms collected together or implied by their values for a, b and c

M1 Attempts to use their values in $b^2-4ac...0$ $(=k^2-20k+16)$ to find the critical values where ... is = or an inequality.

Dependent upon having $a=3$ with both b and c expressions in k.

The solution of the quadratic in k $\left(=10\pm2\sqrt{21}\right)$ must be by allowable methods but allow decimal answers. As a minimum a quadratic in k must have been produced but it is acceptable to then state the critical values from their calculator. awrt 19.2 / awrt 0.8 . You may need to check these on your calculator using their values of a, b and c.

dM1 Uses $b^2-4ac<0$ OR $b^2-4ac\leqslant0$ and selects inside region for their critical values.
It is dependent upon both previous M's. Condone if a different variable to k is used.

A1 Accept any of $10-2\sqrt{21}<k<10+2\sqrt{21}$ or exact equivalent expressions such as $10-\sqrt{84}<k<10+\sqrt{84}$ or Allow $10+2\sqrt{21}>k>10-2\sqrt{21}$ or expressions such as $'\left\{k:10-2\sqrt{21}<k<10+2\sqrt{21}\right\}'$

Must be in terms of k (Not x or any other variable)

ALLOW $'10-2\sqrt{21}<k$ AND $k<10+2\sqrt{21}'$ (or equivalent) but
DO NOT ALLOW $'10-2\sqrt{21}<k$ OR $k<10+2\sqrt{21}'$

Question Number	Scheme	Marks
7. (a)	$2x-3\sqrt{x}-5=9\Rightarrow2x-3\sqrt{x}-14=0$ and treats as quadratic equation	M1
	$\Rightarrow\left(2\sqrt{x}-7\right)\left(\sqrt{x}+2\right)=0\Rightarrow\left(\sqrt{x}=\right)\frac{7}{2},(-2)$	A1
	$\Rightarrow x=\left(\frac{7}{2}\right)^2=\frac{49}{4}$	dM1 A1
		(4)
(b)	$\left(f'(x)=\right)\ 2-\frac{3}{2}x^{-\frac{1}{2}}$	B1

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Question Number	Scheme	Marks
	$\left(f''(x)=\right)\ \frac{3}{4}x^{-\frac{3}{2}}$	M1 A1
	Attempts $\frac{3}{4}x^{-\frac{3}{2}}=6\Rightarrow x^{-\frac{3}{2}}=8\Rightarrow x=\frac{1}{4}$	dM1 A1
		(5) **(9 marks)**

(a) Note calculators in this question are not acceptable so answers on their own or roots without working score 0 marks.

M1 **Way One**: Sets $2x-3\sqrt{x}-5=9\Rightarrow 2x-3\sqrt{x}-14=0$ and attempts to solve a 3TQ quadratic in $\sqrt{x}$ or sets eg $u=\sqrt{x}$ and attempts to solve a 3TQ in u $\left(2u^2-3u-14=0\right)$. See general guidance for solving a quadratic. Condone use of other variables including $x=\sqrt{x}$.

Condone slips in their rearrangement to achieving a 3TQ quadratic and proceeding to find a value but the method must be sound. To score they must either

- show the factorised form of their quadratic eg $(2u-7)(u+2)$
- show embedded values in the quadratic formula
- show their method completing the square

Way Two: Sets $2x-14=3\sqrt{x}$ oe and attempts to square leading to a 3TQ quadratic on one side of an equation. Condone slips when multiplying out eg $\left(2x-14\right)^2$ and rearranging their equation but their method must be sound.
Note: $2x-14=3\sqrt{x}\Rightarrow 4x^2-196=9x$ would be M0.

A1 **Way One**: $\left(\sqrt{x}=\right)\frac{7}{2}$ or eg $\left(u=\right)\frac{7}{2}$ Ignore any reference to the -2. Condone $x=\frac{7}{2}$
Note the roots do not imply M1A1

Way Two: $4x^2-65x+196=0$ oe (the terms should be collected on one side of the equation, but condone lack of $=0$)

dM1 **Way One**: Attempts to find one value for x. Condone 4 or squaring -2

Way Two: Attempts to find one value for x by solving their quadratic (see general guidance for solving a quadratic). To score they must either

- show the factorised form of their quadratic eg $(4x-49)(x-4)$
- show embedded values in the quadratic formula
- show their method completing the square

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A1 $x = \frac{49}{4}$ or 12.25 or $12\frac{1}{4}$ only. If 4 is found it must be rejected

(b)

B1 $2 - \frac{3}{2}x^{-\frac{1}{2}}$ Correct differentiation. Look for two correct **terms** but may be unsimplified. The index must be processed.

M1 Differentiates again. Look for any index of their $f'(x)$ being reduced by one which may also be a "2" $\rightarrow 0$

A1 $\left(f''(x) =\right) \frac{3}{4}x^{-\frac{3}{2}}$ or exact equivalent

dM1 Proceeds to make x the subject by:

- Setting their $f''(x) = 6$ which must be of the form $Bx^k = 6$ (where k cannot be an integer)
- Achieving $x^m = A$ (where $m \neq 1$) and proceeding to $x = ...$ condoning slips in their rearrangement
  or proceeds from $Bx^k = 6$ to $x = ...$ with at least one intermediate line of working (they cannot just state the answer)

A1 $\frac{1}{4}$ cso preceded by correct working shown by an intermediate line of working (see below)

Examples:

$\Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = \frac{1}{4}$ is A1

$\Rightarrow x = \sqrt[-\frac{3}{2}]{8} = \frac{1}{4}$ is A1

$\Rightarrow x = \sqrt[-3]{64} = \frac{1}{4}$ is A1

$\Rightarrow x = \sqrt[\frac{3}{2}]{\frac{1}{8}} = \frac{1}{4}$ is A1

$\Rightarrow x^{-\frac{3}{2}} = 8 \Rightarrow x = \left(\sqrt{8}\right)^{-3} = \frac{1}{4}$ M1A0 (incorrect inverse index laws shown)

Question Number	Scheme	Marks
8 (a)	$x > 4$	B1 **(1)**
(b)	$(3x-2)^2(x-4) = (9x^2 - 12x + 4)(x-4)$ $= 9x^3 - 48x^2 + 52x - 16$	M1 A1 A1 **(3)**

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Question Number	Scheme	Marks
(c)	Sets $9x^3-48x^2+52x-16=-16\Rightarrow 9x^3-48x^2+52x\ (=0)$	B1ft
	$\Rightarrow x\left(9x^2-48x+52\right)=0\Rightarrow x=\dfrac{48\pm\sqrt{48^2-4\times9\times52}}{18}=\dfrac{16\pm4\sqrt{3}}{6}$	M1 B1
	Distance $PQ=\dfrac{16+4\sqrt{3}}{6}-\dfrac{16-4\sqrt{3}}{6}=\dfrac{4}{3}\sqrt{3}$	M1 A1
		(5) **(9 marks)**

Ignore labelling of parts in this question

(a)

B1 $x>4$ only

(b)

M1 Attempts to multiply two of the brackets and then multiplies the result by the third. This may be seen in (a)

Accept $(3x-2)^2(x-4)=\left(9x^2\pm....x\pm4\right)(x-4)=...x^3+.....$

or $(3x-2)^2(x-4)=\left(3x^2\pm....x\pm8\right)(3x-2)=...x^3+.....$

Condone invisible brackets.

Note eg $(3x-2)^2(x-4)=\left(9x^2\pm4\right)(x-4)=...x^3+.....$ is M0. They must have x terms from multiplying two brackets together.

A1 Any two correct and simplified terms of $9x^3-48x^2+52x-16$. (Be careful to check that M1 has been earned.)

A1 $9x^3-48x^2+52x-16$ (ignore any reference to = 0)

(c) **Note in EPEN it is B1ftM1A1dM1A1 we are marking this B1ftM1B1M1A1**

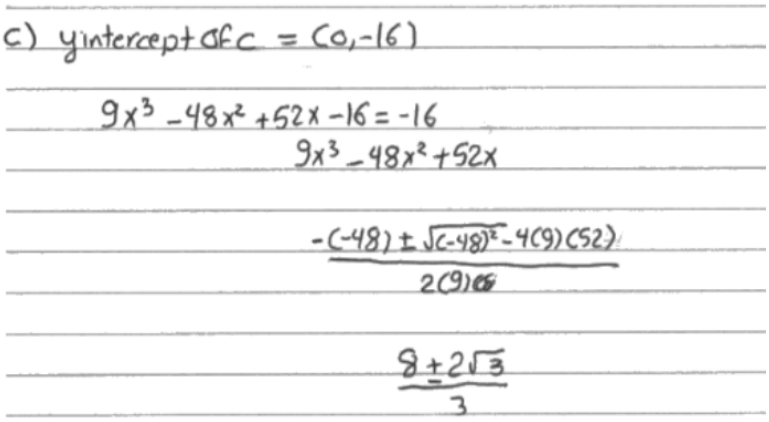
B1ft Proceeds to $9x^3-48x^2+52x\ (=0)$ but follow through on their a, b and c.

M1 **The quadratic must be seen or implied by working shown**

- **Either** factorises/cancels out the x from a cubic of the form $...x^3\pm....x^2\pm...x=0$ to produce a 3TQ. As the 3TQ is seen then using a calculator is allowed so they can proceed to just stating the roots (or they may use the quadratic formula/completing the square methods).
- **Or** states $x=0$, shows some working (eg completed square form or embedded values in the quadratic formula for their invisible quadratic) and proceeds towards at least one value for x.

If their quadratic factorises then this mark cannot be awarded.

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Eg this scores M0 because they do not state $x = 0$ Send to review if unsure.

B1 Correct roots which may be unsimplified $\frac{16 \pm 4\sqrt{3}}{6}$ ($b^2 - 4ac$ should be evaluated though)

Condone decimal answers here. Allow awrt 1.51, 3.82

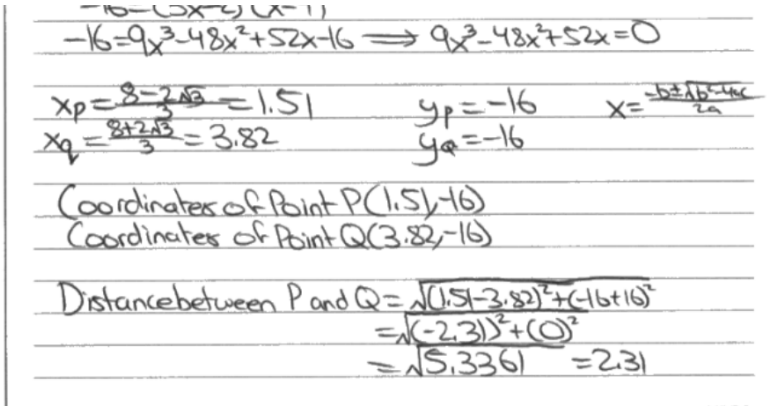
M1 Subtracts their two non-zero roots from setting their cubic $= 0$ either way round (cannot be from differentiating the cubic and setting the resulting 3TQ = 0) which may be implied by their answer. They may also use the distance formula (embedded values is sufficient, condone a sign error in the second bracket involving their "*d*" from part (b)).

$$\sqrt{\left("\frac{16+4\sqrt{3}}{6}"-"\frac{16-4\sqrt{3}}{6}"\right)^2+("d"-"d")^2}$$

A1 $\frac{4}{3}\sqrt{3}$ cso (or exact equivalent such as $\frac{16}{12}\sqrt{3}$ or $1.\dot{3}\sqrt{3}$) It must have $...\sqrt{3}$ Do not allow $1.33\sqrt{3}$

This mark can only be scored if all previous marks have been awarded.

Note candidates who solve $f(x) = 0$ will not score any marks in (c)



This example would score in (c) B1ftM0B1M1A0

Question Number	Scheme	Marks

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9. (i)	$\dfrac{(3x+2)^2}{4\sqrt{x}} = \dfrac{9x^2+12x+4}{4\sqrt{x}} = \dfrac{9}{4}x^{\frac{3}{2}} + 3x^{\frac{1}{2}} + x^{-\frac{1}{2}}$	M1
	$\int \dfrac{(3x+2)^2}{4\sqrt{x}}\,dx = \dfrac{2}{5}\times\dfrac{9}{4}x^{\frac{5}{2}} + \dfrac{2}{3}\times 3x^{\frac{3}{2}} + 2\times x^{\frac{1}{2}}(+c) = \dfrac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$	dM1 A1 A1 A1 **(5)**
(ii)	$f'(x) = x^2 + ax + b$	
	Attempts to use $f'(3) = 2 \Rightarrow$ $2 = 9 + 3a + b$	M1
	Attempts to integrate $(f(x) =)\ \dfrac{1}{3}x^3 + \dfrac{1}{2}ax^2 + bx + c$	M1
	Attempts to use y intercept $= -8$ and $(3,-2)$ in $f(x) = \dfrac{1}{3}x^3 + \dfrac{1}{2}ax^2 + bx + c$	dM1
	Correct equation in a and b $\quad -2 = 9 + \dfrac{9}{2}a + 3b - 8$	A1
	Solves simultaneously to get values for a and b	ddM1
	$a = -4, b = 5 \Rightarrow (f(x) =)\ \dfrac{1}{3}x^3 - 2x^2 + 5x - 8$	A1 **(6)** **(11 marks)**

(i)

M1 Attempts to multiply out the numerator and divide (any term) by $(4)\sqrt{x}$

Award for one correct index coming from correct working (which may be implied):

. $\dfrac{...x^2}{...\sqrt{x}} \to ...x^{\frac{3}{2}}$, $\dfrac{...x}{...\sqrt{x}} \to ...x^{\frac{1}{2}}$, $\dfrac{...}{...\sqrt{x}} \to ...x^{-\frac{1}{2}}$. Do not award this mark for $\dfrac{...}{x^{\frac{1}{2}}}$ unless implied by further work.

dM1 Raises the power of any correct index by one $...x^{\frac{3}{2}} \to ...x^{\frac{5}{2}}$, $...x^{\frac{1}{2}} \to ...x^{\frac{3}{2}}$, $...x^{-\frac{1}{2}} \to ...x^{\frac{1}{2}}$. Indices must be processed.

A1 One correct simplified term from $\dfrac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$. The term may be seen within intermediate working. Allow 0.9 instead of $\dfrac{9}{10}$

A1 Two correct simplified terms from $\dfrac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$ which do not have to be on one line.

Allow 0.9 instead of $\dfrac{9}{10}$

A1 $\dfrac{9}{10}x^{\frac{5}{2}} + 2x^{\frac{3}{2}} + 2x^{\frac{1}{2}} + c$ all on one line (or simplified equivalent including the + c).

Allow 0.9 instead of $\dfrac{9}{10}$

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Accept other simplified expressions such as $\frac{9}{10}\left(\sqrt{x}\right)^5+2\left(\sqrt{x}\right)^3+2\sqrt{x}+c$

Ignore any spurious notation including the integral sign or a dx.

(ii)

M1 Attempts to use $f'(3)=2\Rightarrow\ 2=9+3a+b$ oe. The expression does not need to be simplified so embedded values scores this mark. Condone slips when squaring but do not allow $f'(3)=-2$

M1 Attempts to integrate and achieves $\left(f(x)=\right)\ ...x^3+...ax^2+bx+(c)$ with or without the + c. $a,b\neq 0$

dM1 Uses y intercept = -8 and $(3,-2)$ in their $\left(f(x)=\right)\ ...x^3+...ax^2+bx+c$. The values embedded in the expression are sufficient. **It is dependent on the previous method mark only**.
Alternatively uses $(3,-2)$ in $\left(f(x)=\right)\ ...x^3+...ax^2+bx-8$
Beware: they may also substitute in $b=-7-3a$ at some point to achieve an equation in a only.

A1 Correct unsimplified equation in a and b $\quad -2=9+\frac{9}{2}a+3b-8$.

Note: simplified becomes $9a+6b=-6$

Alternatively they may have a correct unsimplified equation in a only : $-2=9+\frac{9}{2}a+3(-7-3a)-8$ oe

eg $4.5a=-18$

ddM1 Dependent upon all previous M's.

It is for solving their two equations with $c=-8$ to find values for a and b or solving their equation in a and then substituting in to $b=-7-3a$ to find a value for b

Don't be too concerned by the process. A calculator method is acceptable.

A1 $\frac{1}{3}x^3-2x^2+5x-8$ as their final answer. Do not isw eg $\frac{1}{3}x^3-2x^2+5x-8\rightarrow x^3-6x^2+15x-24$

$y=...$ or $f(x)=...$ are not required. Do not allow $5x^1$.

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