edexcel

Mark Scheme (Results)

Summer 2014

Pearson Edexcel International A Level
in Core Mathematics 12
(WMA01_01)

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Summer 2014
Publications Code IA038473

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 125

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - bod – benefit of doubt
   - ft – follow through
   - the symbol $\sqrt{}$ will be used for correct ft
   - cao – correct answer only
   - cso - correct solution only. There must be no errors in this part of the question to obtain this mark
   - isw – ignore subsequent working
   - awrt – answers which round to
   - SC: special case
   - oe – or equivalent (and appropriate)
   - d… or dep – dependent
   - indep – independent
   - dp decimal places
   - sf significant figures
   - ∗ The answer is printed on the paper or ag- answer given
   - ⌊ or d… The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q), \text{ where} |pq|=|c|$, leading to x = ...

$(ax^2+bx+c)=(mx+p)(nx+q), \text{ where} |pq|=|c| \text{ and } |mn|=|a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\ \ q\neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \rightarrow x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \rightarrow x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1(a)	$AC^2 = 10^2 + 8^2 - 2\times10\times8\cos65° \Rightarrow AC = ..$	M1
	$AC = 9.8...,\Rightarrow AC = 9.82\,\text{km}\,(9820\text{m})$ (to nearest 10 m)	A1,A1 **(3)**
(b)	$\dfrac{\sin A}{8} = \dfrac{\sin 65°}{'9.817..'} \Rightarrow A =$ | $\dfrac{\sin C}{10} = \dfrac{\sin 65°}{'9.817..'} \Rightarrow C =$	M1
	$\angle A = \text{awrt } 47.6°$ | $\angle C = \text{awrt } 67.4°$	A1
	$\Rightarrow \text{Bearing} = \text{awrt } 132.4°$	A1ft **(3)**
		(6 marks)
Alt (b)	$\cos A = \dfrac{10^2 + AC^2 - 8^2}{2\times10\times AC} \Rightarrow A = ...$ OR $\cos C = \dfrac{8^2 + AC^2 - 10^2}{2\times8\times AC} \Rightarrow C = ...$	M1

(a)

M1 Uses the cosine rule, or otherwise to find *AC*. The rule, if stated, must be correct. If it is not stated it must be of the correct form. Accept $AC^2 = 10^2 + 8^2 - 2\times10\times8\cos65° \Rightarrow AC = ..$
It is possible to find *AC* by other methods, eg dropping a perpendicular from *A* to a point *X* on *BC*. For M1 to be scored it must be a full method Eg A full method could be; find *AX* by using $\sin 65°$, *BX* by $\cos 65°$, and *CX* by subtraction of *BX* from 8. The M1 is finally scored after an application of Pythagoras' theorem to find *AC*.

A1 Accept answers rounding or truncating to $AC = 9.8...\,(\text{km})$

A1 Accept $AC = 9.82\,\text{km}$ *or* 9820m . Both the accuracy and the units are necessary.

(b)

M1 Uses the sine rule (or cosine rule) with their answer for *AC* to find angle *A* or angle *C*.
Accept $\dfrac{\sin A}{8} = \dfrac{\sin 65°}{'9.817..'} \Rightarrow A =$ or $\cos A = \dfrac{10^2 + '9.817'^2 - 8^2}{2\times10\times'9.817'} \Rightarrow A = ...$
Accept $\dfrac{\sin C}{10} = \dfrac{\sin 65°}{'9.817..'} \Rightarrow C =$ or $\cos C = \dfrac{8^2 + '9.817'^2 - 10^2}{2\times8\times'9.817'} \Rightarrow C = ...$
In the sine rule the sides and angles need to be correctly matched

A1 Accept $\angle A = \text{awrt } 47.6°$ or $\angle C = \text{awrt } 67.4°$
Don't be overly concerned with the labelling of the angle

A1ft Awrt $132.4°$ or follow through on their $(180 - A)°$ if they found *A* or $(65 + C)°$ if they found *C*.

Question Number	Scheme	Marks
2	$\sqrt{27}=3\sqrt{3},\ \frac{6}{\sqrt{3}}=2\sqrt{3}$	
	$x\sqrt{27}+21=\frac{6x}{\sqrt{3}}\Rightarrow 3\sqrt{3}x+21=2\sqrt{3}x$	M1 A1
	$\Rightarrow \sqrt{3}x=-21$	
	$\Rightarrow x=-\frac{21}{\sqrt{3}}\Rightarrow x=-7\sqrt{3}$	M1 A1
		(4 marks)

M1 Simplify either $\sqrt{27}=3\sqrt{3}$ or $\frac{6}{\sqrt{3}}=2\sqrt{3}=\left(\frac{6\sqrt{3}}{3}\right)$

A1 Uses both $\sqrt{27}=3\sqrt{3}$ and $\frac{6}{\sqrt{3}}=2\sqrt{3}$ to rewrite equation in a form equivalent to $3\sqrt{3}x+21=2\sqrt{3}x$

M1 Collects x terms on one side of the equation, simplifies and divides reaching x=…

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
Alt 2	$x\sqrt{27}+21=\frac{6x}{\sqrt{3}}\quad\left(\times\sqrt{3}\right)\Rightarrow\sqrt{3}\sqrt{27}x+21\sqrt{3}=6x$	M1
	$\Rightarrow 9x+21\sqrt{3}=6x$	A1
	$\Rightarrow 3x=-21\sqrt{3}\Rightarrow x=-7\sqrt{3}$	M1A1
		(4 marks)

M1 Multiply equation by $\sqrt{3}$, seen in at least two terms.

A1 $9x+21\sqrt{3}=6x$ or equivalent but the $\sqrt{81}$ must have been dealt with

M1 Collects terms in x, and proceeds to x=..

A1 Writes answer in the required form $-7\sqrt{3}$. Accept $-1\sqrt{147}$

Question Number	Scheme	Marks
3 (a)	$4^a = 20 \Rightarrow a\log 4 = \log 20 \Rightarrow a = ...$	M1
	$= \dfrac{\log 20}{\log 4} = \text{awrt } 2.16$	A1
		(2)
(b)	$3 + 2\log_2 b = \log_2 30b \Rightarrow 3 + \log_2 b^2 = \log_2 30b$	M1
	$\Rightarrow 3 = \log_2 30b - \log_2 b^2$	
	$\Rightarrow 3 = \log_2\left(\dfrac{30b}{b^2}\right)$	M1A1
	$\Rightarrow 2^3 = \dfrac{30b}{b^2} \Rightarrow b = ..$	dM1
	$b = 3.75$	A1
		(5)
		(7 marks)
Alt (b)	$3 + 2\log_2 b = \log_2(30b) \Rightarrow 3 + 2\log_2 b = \log_2 30 + \log_2 b$	2nd M1
	$\Rightarrow 3 + \log_2 b = \log_2 30$	
	$\Rightarrow \log_2 8 + \log_2 b = \log_2 30$	1st M1
	$\Rightarrow \log_2 8b = \log_2 30$	A1
	$\Rightarrow 8b = 30 \Rightarrow b = ..$	dM1
	$\Rightarrow b = 3.75$	A1
		(5)

(a)

M1 Takes logs of both sides leading to $a = ...$

Accept for this mark $\log_4 20, \dfrac{\log 20}{\log 4}$ or $a\log 4 = \log 20 \Rightarrow a = ..$

A1 awrt 2.16. Just the answer with no incorrect working scores both marks.

Do not accept answers from trial and error.

(b) Note that this part is B1M1M1dM1A1 on e pen. We are scoring it M1M1A1dM1A1

M1 Score for a correct use of the power law for logs

Accept either $2\log_2 b = \log_2 b^2$, $2\log b = \log b^2$ or $3 = \log_2 8$

M1 Score for a use of the addition or subtraction law of logs.

Examples of this would be $\log(30b) = \log 30 + \log b$ and $\log(30b) - \log(b^2) = \log\left(\dfrac{30b}{b^2}\right)$

Do not accept attempts such as $\log(30b) - 2\log(b) = \log\left(\dfrac{30b}{2b}\right)$

A1 Achieving a correct intermediate line of the form $\log_2(..) = ..$ or $\log_2 ... = \log_2 ...$

Accept exact equivalents of $3 = \log_2\left(\dfrac{30b}{b^2}\right)$, $\log_2 8b = \log_2 30$ and $\log_2 8b^2 = \log_2 30b$, $b = 2^{\log_2(30)-3}$

dM1 Dependent upon both previous M's it is for correctly undoing the logs and solving to get a value for b

A1 $b = 3.75$ or exact equivalent. Ignore any reference to $b = 0$

Question Number	Scheme	Marks
4(a)	$\mathrm{f}(x)=x^2+\dfrac{16}{x}\Rightarrow \mathrm{f}'(x)=2x-\dfrac{16}{x^2}$	M1A1 **(2)**
(b)	Setting $2x-\dfrac{16}{x^2}=0\Rightarrow x=..$	M1
	$x^3=8\Rightarrow x=2$	dM1A1
	$A=(2,12)$	A1 **(4)**
(c)(i)	$A'=(1,12)$	B1ft
(ii)	$A'=(2,6)$	B1ft **(2)**
		(8 marks)

(a)

M1 $x^n\to x^{n-1}$ for either term. Accept $x^2\to x$ or $\dfrac{1}{x}\to\dfrac{1}{x^2}\left(x^{-1}\to x^{-2}\right)$

A1 A correct unsimplified $\mathrm{f}'(x)=2x-\dfrac{16}{x^2}$. Accept versions such as $\mathrm{f}'(x)=2\times x+16\times -1x^{-2}$

(b)

M1 Sets their $\mathrm{f}'(x)=0$ and proceeds to $x=$.... Don't be overly concerned with how they get to $x=$..

dM1 Dependent upon the previous M mark. It is scored for $\times x^2$ to reach $x^3=k\Rightarrow x=\sqrt[3]{k}$
This may be implied by the correct answer to their equation.

A1 Correctly achieving $x=2$. Ignore any additional solutions.

A1 Correctly achieving $A=(2,12)$. Accept $x=2, y=12$.
If any additional solutions are given $(x>0)$ this mark will be withheld.
Accept $y=12$ appearing in part (c) as long as you are convinced that it is for $y=\mathrm{f}(x)$

(c)(i)

B1 ft $A'=(1,12)$. Accept this on a sketch graph or as $x=1, y=12$
If $A=(p,q)$ was incorrect follow through on their value from part (b), $A'=(p-1,q)$
If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

(c)(ii)

B1 ft $A'=(2,6)$ Accept this on a sketch graph or as $x=2, y=6$
If $A=(p,q)$ was incorrect follow through on their value from part (b), $A'=(p,\frac{1}{2}q)$
If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

Do Not allow multiple attempts, mark in the order given if not clearly labelled.
The isw rule is suspended for this part of the question.

Question Number	Scheme	Marks
5(a)	Gradient $PQ = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{7-4}{4-(-1)} = \dfrac{3}{5}$	M1A1
	Equation of line PQ $\quad \dfrac{3}{5} = \dfrac{y-7}{x-4}, \quad \dfrac{3}{5} = \dfrac{y-4}{x--1}$ oe	M1
	$\pm k(3x - 5y + 23 = 0)$ $\quad$ k an integer	A1
		(4)
(b)	Uses gradient PR= $-\dfrac{5}{3} \Rightarrow \dfrac{-7}{p+1} = -\dfrac{5}{3}$	M1
	$5p + 5 = 21 \Rightarrow p = \dfrac{16}{5}$ oe.	dM1, A1
		(3) **(7 marks)**
Alt 5(a)	Sub (-1,4) and (4,7) into $y = mx + c \Rightarrow 7 = 4m + c$ and $4 = -1m + c$	
	Solve simultaneously to get $m = \dfrac{3}{5}$	M1, A1
	Sub in either to get $c = ...\left(\dfrac{23}{5}\right)$	M1
	Rearranges to $\pm k(3x - 5y + 23 = 0)$	A1 (4)

(a)

M1 Attempts gradient of $PQ=\frac{\Delta y}{\Delta x}$ using both $(4,7)$ and $(-1,4)$. Accept as evidence $\frac{7-4}{4-(-1)}$ or $\frac{7-4}{4-(-1)}=\frac{y-7}{x-4}$ or similar. You may allow only one error on the sign, there must be an attempt to find a difference. For the M accept $\frac{7-4}{4-1}$ BUT NOT $\frac{7+4}{4-1}$ or $\frac{4--1}{7-4}$

A1 Achieves the gradient of $\frac{3}{5}$ or its equivalent 0.6

This may be embedded within an equation. Eg $\frac{3}{5}=\frac{y-7}{x-4}$

M1 Attempts equation of line using their gradient and a point.

Accept their'$\frac{3}{5}$'$=\frac{y-7}{x-4}$ and versions such as y-7='their'$\frac{3}{5}$'$(x-4)$ with both signs correct

Accept their'$\frac{3}{5}$'$=\frac{y-4}{x--1}$ and versions such as y - 4='their'$\frac{3}{5}$'$(x--1)$ with both signs correct

If $y=mx+c$ is used then it must be a full attempt to find c.

Accept $7='\frac{3}{5}'\times 4+c \Rightarrow c=..$

A1 $\pm k(3x-5y+23=0)$ where k is an integer

Note 1: that candidates who calculate the eqn of a line as $\frac{y-7}{7-4}=\frac{x-4}{4--1}$ oe score M1A1M1 straight away

Note 2: An alternative method can be seen when candidates express their equation in the form $y=mx+c$ or $ax+by+c=0$ or any such linear form.

M1 is scored for subbing both $(-1,4)$ and $(4,7)$ into their linear expression forming two simultaneous equations AND solving.

A1 is scored for $m=\frac{3}{5}$ in $y=mx+c$ or $\frac{a}{b}=-\frac{3}{5}$ in $ax+by+c=0$

M1 is scored for subbing their $m=\frac{3}{5}$ into $y=mx+c$ to find $c=..$ or $\frac{a}{b}=-\frac{3}{5}$ into $ax+by+c=0$ to find $\frac{c}{a}$

A1 $\pm k(3x-5y+23=0)$ where k is an integer

(b)

M1 Attempts to use the fact that the gradients are perpendicular to set up an equation in p.

Possible ways this can be done are

Attempting to use $m_1\times m_2=-1$ So accept $\frac{3}{5}\times\frac{4--3}{-1-p}=-1$, $\frac{7-4}{4--1}=-1\Big/\frac{4--3}{-1-p}$

Or attempting to find the equation of line PR using the perpendicular gradient and $P(-1,4)$ followed by a substitution of (p,-3). Accept p being interchanged with x.

Or attempting to use Pythagoras $PQ^2+PR^2=QR^2 \Rightarrow (p+1)^2+7^2+5^2+3^2=(p-4)^2+10^2$

In all cases it must be evident that a difference in the coordinates is attempted.

dM1 Dependent upon the previous M.

It is scored for proceeding to (and solving) a linear equation in p (or x).

In both methods look for . $Ap+Q=R\Rightarrow p=..$

A1 $p=\frac{16}{5}$ or equivalents such as $3.2, 3\frac{1}{5}$

Accept written as $x=$

Question Number	Scheme	Marks
6(a)	Uses $1-\sin^2 x = \cos^2 x$	M1
	$\dfrac{\cos^2 x - \sin^2 x}{1-\sin^2 x} = \dfrac{\cos^2 x - \sin^2 x}{\cos^2 x} = 1 - \dfrac{\sin^2 x}{\cos^2 x} = 1 - \tan^2 x$	A1*
	.	(2)
(b)	$\dfrac{\cos^2 x - \sin^2 x}{1-\sin^2 x} + 2 = 0 \Rightarrow 1 - \tan^2 x + 2 = 0$	M1
	$\tan^2 x = 3$	A1
	$\tan x = (\pm)\sqrt{3} \Rightarrow x = ..$	dM1
	$x = \dfrac{1}{3}\pi, \dfrac{2}{3}\pi\ \dfrac{4}{3}\pi, \dfrac{5}{3}\pi$	A1,A1
		(5)

(a)

M1 Uses $1-\sin^2 x=\cos^2 x$ in the denominator of their expression.

The usual case will be sight of $\dfrac{\cos^2 x-\sin^2 x}{1-\sin^2 x}\to\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}$

If candidate starts on the right hand side $1-\tan^2 x=1-\dfrac{\sin^2 x}{\cos^2 x}=\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}=\dfrac{\cos^2 x-\sin^2 x}{1-\sin^2 x}$ then

M1A1 is scored at the end

If the candidate multiplies both sides by $1-\sin^2 x$ and multiples out then

$\left(1-\tan^2 x\right)\left(1-\sin^2 x\right)=1-\tan^2 x-\sin^2 x+\sin^2 x\tan^2 x$

they will not get any credit until they start to use $1-\sin^2 x=\cos^2 x$

A1* Completes proof with no errors. This is a show that question.

Look for a minimum of ALL 3 steps shown below

$$\frac{\cos^2 x-\sin^2 x}{\cos^2 x}=1-\frac{\sin^2 x}{\cos^2 x}=1-\tan^2 x \quad \textbf{OR} \quad \frac{\cos^2 x-\sin^2 x}{\cos^2 x}=\frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}=1-\tan^2 x$$

(b)

M1 Scored for using part (a) $\dfrac{\cos^2 x-\sin^2 x}{1-\sin^2 x}+2=0\Rightarrow\tan^2 x=k$

A1 $\tan^2 x=3$

M1 Correct order $\tan^2 x=k$ $\tan x=(\pm)\sqrt{k}$, no need for negative, leading to at least one value of x (which may not be correct). Evidence could be the answer in degrees $x=60^{(\circ)}$ or in radians 1.047

If they achieve $\tan^2 x=k$, with $k<0$ this mark cannot be scored.

A1 Two correct answers as multiples of π. Accept versions such as $\dfrac{\pi}{1.5}$.

Degrees and answers like 1.047 etc score A0 although recovery is allowed.

A1 All four correct with no extras inside range. Ignore any extra solutions outside the range.

Special Case Although this question clearly states ‘Hence’ we will allow a maximum of 1,1,1,0,0 for a solution where candidates re start as follows

$$\frac{\cos^2 x-\sin^2 x}{1-\sin^2 x}+2=0\Rightarrow\frac{1-\sin^2 x-\sin^2 x}{1-\sin^2 x}+2=0\Rightarrow\sin^2 x=\frac{3}{4}\Rightarrow x=\frac{1}{3}\pi,\frac{2}{3}\pi\ \frac{4}{3}\pi,\frac{5}{3}\pi$$

AND similarly for

$$\frac{\cos^2 x-\sin^2 x}{1-\sin^2 x}+2=0\Rightarrow\cos^2 x=\frac{1}{4}\Rightarrow x=\frac{1}{3}\pi,\frac{2}{3}\pi\ \frac{4}{3}\pi,\frac{5}{3}\pi$$

First '1' for either $\sin^2 x=A$ OR $\cos^2 x=A$ where $0\leqslant A\leqslant 1$ followed by '1' for two correct solutions and '1' for two additional correct solutions.

Question Number	Scheme	Marks
7(i)	$\text{f}'(x)=\frac{4}{x^3}+2x-1 \Rightarrow \text{f}(x)=\frac{4}{-2}x^{-2}+x^2-1x\ (+c)$	M1A1
	Sub (2, 3) $\quad 3=-\frac{1}{2}+4-2+c \Rightarrow c=\frac{3}{2}$	dM1A1
	$\text{f}(1)=-\frac{2}{1^2}+1^2-1+\frac{3}{2}=-\frac{1}{2}$	A1
	cso	
		(5)
(ii)	$\int_1^4 \left(3\sqrt{x}+A\right)\text{d}x=\left[\frac{3x^{1.5}}{1.5}+Ax\right]_1^4=21$	M1A1
	$\left(\frac{3\times 4^{1.5}}{1.5}+4A\right)-\left(\frac{3\times 1^{1.5}}{1.5}+1A\right)=21$	M1
	$16+4A-2-A=21$	
	$\Rightarrow 3A=7 \Rightarrow A=\frac{7}{3}$	dM1A1
		(5) **(10 marks)**

(i)

M1 Score for raising the power by 1 in any of the 3 terms. Accept $x^{-3} \to x^{-2}, x \to x^2$ or $1 \to x$

A1 A correct and unsimplified form. Accept $\mathrm{f}(x) = 4\dfrac{x^{-2}}{-2} + 2\dfrac{x^2}{2} - 1x\ (+c)$. There is no requirement for $+c$

dM1 Substitutes (2, 3) into their $\mathrm{f}(x)$, which must have a $+c$ and proceeds to find a numerical value to c
It is dependent upon the previous M having been scored.

A1 $c = \dfrac{3}{2}$ or equivalent.

A1 cso $\mathrm{f}(1) = -\dfrac{1}{2}$ Accept exact equivalents such as -0.5

(ii)

M1 Score for either $3\sqrt{x} \to ....x^{\frac{3}{2}}$ or $A \to Ax$

A1 Fully correct (unsimplified) integral- There is no need to set =21or sub in any limits or have $+c$.
Accept versions of $3\dfrac{x^{1.5}}{1.5} + Ax\ (+c)$

M1 Sub in the limits 4 and 1 into a 'changed' function, subtract (either way around) and set the result =21.
Accept versions of $\left(\dfrac{3\times 4^{1.5}}{1.5} + 4A\right) - \left(\dfrac{3\times 1^{1.5}}{1.5} + 1A\right) = 21$ with their integrand with or without brackets

dM1 Proceed to solve equation in A.
This is dependent upon **both** previous M's **and having achieved** $A \to Ax$
It must end with a numerical value for *A*.

A1 $A = \dfrac{7}{3}$ or equivalents such as $2\frac{1}{3}, 2.\dot{3}$. Do not accept 2.3 or 2.33 but remember to isw.

Question Number	Scheme	Marks
8 (a)	$(1+bx)^n = 1+nbx+\left(\dfrac{n(n-1)(bx)^2}{2!}\right) = 1+12x+\left(70x^2\right)$	
	Compares x terms $\quad nb=12$	B1*
		(1)
(b)	Comparing x^2 terms $\quad \dfrac{n(n-1)b^2}{2}=70\left(\Rightarrow n(n-1)b^2=140\right)$	M1A1
	Substitutes $b=\dfrac{12}{n}$ into $n(n-1)b^2=140 \Rightarrow \dfrac{n(n-1)144}{n^2}=140$	M1
	$\Rightarrow 144(n-1)=140n \Rightarrow n=36$	dM1A1
	cso	
	Substitutes $n=36$ into $nb=12 \Rightarrow b=\dfrac{1}{3}$ cso	A1
		(6)
		(7 marks)

(a) For the purpose of marking parts (a) and (b) can be scored together

B1* Use the binomial series expansion with $'x' = bx$ and proceeding as far as the x term.
This is a show that question and you cannot just accept the given statement $nb = 12$
Accept minimal evidence however:
Accept either (including the 1) $(1+bx)^n = 1 + nbx + ... \Rightarrow nb = 12$ OR
$(1+bx)^n = 1 + {}^nC_1 bx + ... \Rightarrow nb = 12$
or a statement to the effect that 'comparing the x /second terms ${}^nC_1 1^{n-1}(bx) = 12x \Rightarrow nb = 12$'
Note: $nbx = 12x \Rightarrow nb = 12$ does not score B1 unless you see the accompanying statement 'comparing x or second terms'

(b)

M1 Attempts to set the x^2 term of the binomial expansion equal to 70. The form of the binomial must be correct so accept $\frac{n(n-1)(bx)^2}{2} = 70x^2$, ${}^nC_2(bx)^2 = 70x^2$ or $\frac{n!(bx)^2}{2!(n-2)!} = 70x^2$ with or without brackets.
It may be implied by a correct equation. It could be scored in part (a)
Condone invisible brackets, so accept for M1 just the statement $\frac{n(n-1)b}{2} = 70$

A1 Compares terms in x^2 to achieve a second correct equation in b and n.
Accept equivalent versions of $\frac{n(n-1)b^2}{2} = 70$ or $n(n-1)b^2 = 140$
Do not accept $\frac{n!b^2}{2!(n-2)!} = 70$ or ${}^nC_2(b)^2 = 70$ unless one of the above could be implied by later work.

M1 Substitutes either $b = \frac{12}{n}$ or $n = \frac{12}{b}$ or $nb = 12$ into their second equation involving both b and n to produce an equation in a single variable.

dM1 A valid method of solving their equation allowing for slips. See practice items for examples
This is dependent upon **both** previous M's

A1 Correct solution only $n = 36$ or $b = \frac{1}{3}$

A1 Correct solution only $n = 36$ and $b = \frac{1}{3}$ **and** no other solutions such as $n = 0$

Allow solutions just written down after two correct equations provided you don't see incorrect working.

Question Number	Scheme	Marks
9(i)	$\sum_{r=1}^{20}(3+5r)=8+13+18+..........+103$	M1
	Use of $S_n=\frac{n}{2}\left(2a+(n-1)d\right)$ or $S_n=\frac{n}{2}\left(a+l\right)$ with a=3or 8, n=19 or 20, d=5 and l=103	M1
	$S_{20}=\frac{20}{2}\left(8+103\right)=1110$	A1
		(3)
(ii)	$\sum_{r=0}^{\infty}\frac{a}{4^r}=16\Rightarrow\frac{a}{1}+\frac{a}{4}+\frac{a}{16}....=16 \qquad r=\frac{1}{4}$ oe	B1
	Use of $S_\infty=\frac{a}{1-r}$ with $0<\lvert r\rvert<1$ and $S_\infty=16$	M1
	$16=\frac{a}{1-'r'}\Rightarrow a=..$	dM1
	$a=12$	A1
		(4)
		(7 marks)

(i)

M1 Minimal evidence of the sum of an arithmetic sequence .
Accept as evidence the first 3 terms written out as 8+13+18.. or $8+13+..+103$
or 8,13,18 followed by the sum formula $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$
or 8,13,...103 followed by the sum formula $S_n = \frac{n}{2}\left(a+l\right)$
Do not accept on its own however $3+5\times 20$ or 103 without any reference to a sum

M1 Uses $S_n = \frac{n}{2}\left(2a+(n-1)d\right)$ with $a=3$ or 8, $d=5$ and $n=19$ or 20
or $S_n = \frac{n}{2}\left(a+l\right)$ with $n=19$ or 20, $a=3$ or 8, and $l=103$
Accept a list of 20 terms as long as all terms are written out.

A1 1110. Accept this for all 3 marks as long as no incorrect working is seen.

Alt (i)

M1 Splits the sum into two separate parts, and uses/states $\sum 1 = n$ *and* $\sum r = \frac{n(n+1)}{2}$ both

$$\sum 3+5r = \sum 3 + \sum 5r = 3\times n + 5\times\frac{n(n+1)}{2}$$

M1 sub 20 or 19 into the above

A1 1110

(ii)

B1 For stating or implying that $r=\frac{1}{4}$.
You may see a series or sequence of terms with $\times\frac{1}{4}$ or $\times 0.25$.
Accept variations on $a+\frac{a}{4}+\frac{a}{4^2}$ or even $\frac{a}{4},\frac{a}{16},\frac{a}{64},...$

M1 For using the formula $S_\infty = \frac{a}{1-r}$ with $S_\infty = 16$ and $0<|r|<1$

dM1 Dependent upon the previous M. For proceeding to $a = ..$

A1 12

Question Number	Scheme	Marks
10(a)	$kx^2+4x+k=2 \Rightarrow kx^2+4x+k-2=0$	
	Attempts to calculate b^2-4ac with $a=k, b=4$ and $c=k\pm 2$	M1
	$b^2-4ac=4^2-4\times k\times(k-2)$	A1
	Sets their $b^2-4ac>0 \Rightarrow 16-4k(k-2)>0$	dM1
	$4k^2-8k-16<0$	
	$k^2-2k-4<0$	A1*
		(4)
(b)	Solves $(k-1)^2-5=0 \Rightarrow k=1\pm\sqrt{5}$	M1
	'Insides' $1-\sqrt{5}<k<1+\sqrt{5}$	M1A1
		(3)
		(7 marks)

(a) Note that this is M1M1A1A1 on e pen. Mark in the order seen below as M1A1M1A1.

M1 Attempts to calculate b^2-4ac with $a=k, b=4$ and $c=k\pm 2$. Condone poor/incomplete bracketing.

Alternatively they may set $b^2 ... 4ac$ with ... being $=,>,<,\leqslant$ or $\geqslant$ with $a=k, b=4$ and $c=k\pm 2$.

A1 Correct (unsimplified) b^2-4ac. Accept $4^2-4k(k-2)$ oe. The bracketing must be correct.

In the alternative it is for $4^2 ... 4k(k-2)$ with ... being $=,>,<,\leqslant$ or $\geqslant$ The bracketing must be correct

dM1 Sets their $b^2-4ac>0$ with $a=k, b=4$ and $c=k\pm 2 \Rightarrow 4^2-4k(k\pm 2)>0$

In the alternative it is for just $b^2>4ac$ with $a=k, b=4$ and $c=k\pm 2 \Rightarrow 4^2>4k(k\pm 2)$

A1* Proceeds correctly to the given answer $k^2-2k-4<0$. You should expect to see the terms moved over to the other side of the inequality and a division of 4. Alternatively a division of -4 should be proceeded by the reversal of the inequality. There is no requirement for these steps to be explained but you need to check for sign errors which would be A0.

Special case: If they start with $a=kx^2, b=4x$ and $c=k\pm 2$ and proceed correctly they will score a maximum of M1A1dM1A0*. Treat candidates who start with $\sqrt{b^2-4ac}$ in a similar way.

(b)

M1 For an attempt by either the formula or completing the square to solve 3TQ=0. Do not accept factorisation. If the formula is quoted it must be correct. If it is not quoted only accept expressions of the form $\frac{-(-2)\pm\sqrt{(-2)^2-4\times 1\times -4}}{2\times 1}$ with or without either bracket. Accept awrt 3.24, -1.24 from a GC.

If completing the square is attempted accept $k^2-2k-4=0 \Rightarrow (k\pm 1)^2\pm 1\pm 4=0 \Rightarrow k=...$

M1 Chooses the inside values to their solution to the 3TQ=0. This is not dependent upon the previous M so for this mark you can accept the inside region from their roots obtained factorisation. If α is the smaller root and β is the larger root look for $\alpha<k<\beta$ **or** $k>\alpha$ and $k<\beta$ **or** $k>\alpha$ or $k<\beta$ **or** (α,β)

A1 $1-\sqrt{5}<k<1+\sqrt{5}$. Accept also $k>1-\sqrt{5}$ and $k<1+\sqrt{5}$, $k>1-\sqrt{5}\ \ k<1+\sqrt{5}$, $\left(1-\sqrt{5}, 1+\sqrt{5}\right)$

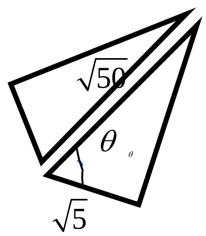
Also allow $k>1-\sqrt{5}\ \ k<1+\sqrt{5}$ with a comma between.

Accept exact equivalents like $\frac{2-\sqrt{20}}{2}<k<\frac{2+\sqrt{20}}{2}$ or $1-\frac{\sqrt{20}}{2}<k<1+\frac{\sqrt{20}}{2}$

Do not accept $k>1-\sqrt{5}$ or $k<1+\sqrt{5}$, $\left[1-\sqrt{5}, 1+\sqrt{5}\right]$ or decimals

Do not accept *x* in place of *k* for the final mark.

Special case for candidate who achieves $1-\sqrt{5}\le k\le 1+\sqrt{5}$ or $\left[1-\sqrt{5}, 1+\sqrt{5}\right]$ score M1M1A0

Question Number	Scheme	Marks
11(a)(i)	$x^2+y^2-6x+2y+5=0$ Obtains $(x\pm3)^2$ and $(y\pm1)^2$	M1
	$(x-3)^2-9+(y+1)^2-1+5=0$ Obtains $(x-3)^2$ and $(y+1)^2$	A1
	Centre = $(3,-1)$	A1
(ii)	Radius^2 = $'3'^2+'-1'^2-5=5\Rightarrow r=\sqrt{5}$	M1A1
		(5)
(b)	Calculates $TQ=\sqrt{(8-3)^2+(4--1)^2}=\sqrt{50}$	M1A1
	 Uses $\cos\theta=\dfrac{\sqrt{5}}{\sqrt{50}}\Rightarrow\theta=1.249...$	M1A1
	angle *MQN* IS 2.498 radians to 3 decimal places	A1*
		(5)
(c)	Area of sector = $'\frac{1}{2}r^2\theta'=\frac{1}{2}\times\left(\sqrt{5}\right)^2\times2.498\left(=\text{awrt } 6.24/6.25\right)$	M1A1
	Area of triangle $='\frac{1}{2}ab\sin C'=\frac{1}{2}\times\sqrt{5}\times\sqrt{50}\times\sin1.249=\left(7.50\right)$	M1
	Shaded Area = 15.0 – 6.245 = 8.76 or 8.75	dM1,A1
		(5)
		(15 marks)

Mark (a)(i) and (a)(ii) together as one part.

(a)

M1 Obtains $(x\pm3)^2$ and $(y\pm1)^2$. This could be implied by the candidate writing down a centre of $(\pm3,\pm1)$

A1 Obtains $(x-3)^2$ **and** $(y+1)^2$

A1 Centre = $(3,-1)$. Accept this without any working for M1A1A1

M1 Uses r^2 = $'3'^2+'-1'^2-5=...\Rightarrow r=...$ The value of r^2 must be positive for this to be scored.
Condone invisible bracketing. This may be implied by $r=$ awrt 2.24

A1 $r=\sqrt{5}$. Do not accept 2.24 on its own but remember to isw.
Note that the M1 A1 can be scored for correct radius following a centre of $(-3,+1)$

Alternative to part (a) using $x^2+y^2+2gx+2fy+c=0,\quad$ Centre $=(-g,-f)\quad$ Radius $=\sqrt{g^2+f^2-c}$

M1 States that $2g=\pm6$ and $2f=\pm2$. This could be implied by the candidate writing down a centre of $(\pm3,\pm1)$

A1 States that $2g=-6$ **and** $2f=2$

A1 Centre = $(3,-1)$ Accept this without any working for M1A1A1

M1 Uses Radius $=\sqrt{'g'^2+'f'^2-c}$

A1 $r=\sqrt{5}$. Do not accept 2.24 on its own but remember to isw.

(b)

M1 Calculates the distance *TQ* from points (8,4) and their *Q* using Pythagoras.
Look for $TQ^2=(8-'3')^2+(4-'(-1)')^2 \Rightarrow TQ=..$
There must be an attempt seen to find the difference between the coordinates.

A1 Calculates the length of *TQ* = $\sqrt{50}$. Accept $5\sqrt{2}$, awrt 7.07

M1 Uses a correct method to find the half angle. Usually $\cos\theta=\dfrac{\text{their } r}{\text{their } TQ} \Rightarrow \theta=...$

A1 Correct half angle calculated or implied. Accept $\theta=\text{awrt } 1.25..., 71.6°$ or $\theta=\arccos\left(\dfrac{\sqrt{5}}{\sqrt{50}}\right)$

A1* cso Angle *MQN* IS 2.498 radians to 3 decimal places.
This is a show that questions and all elements including the accuracy must be correct.
Therefore this must follow accuracy of at least 1.249 for the half angle.

(c)

M1 Uses area of a sector formula $A='\frac{1}{2}r^2\theta'$ with their value of *r* and $\theta=2.498$ or 1.249rads
If the formula is quoted it must be correct. It can be embedded within the area of a segment

A1 For the area of the sector. Accept $\frac{1}{2}\times\left(\sqrt{5}\right)^2\times 2.498$ or $\frac{1}{2}\times 5\times 2.498$ or awrt 6.24 or awrt 6.25
This may be scored on the penultimate line for $2\times\frac{1}{2}\times 5\times 1.249$

M1 A correct method of finding the area of the triangle *TQN* or triangle *TQM* or the kite *TNQM.*
Accept use of $'\frac{1}{2}ab\sin C'$ with their *r*, their *TQ* and 1.249
Alternatively candidates could find *TN* from $\sqrt{TQ^2-r^2}$ followed by $\dfrac{TN\times r}{2}$
The kite *TNQM* could be found by $\frac{1}{2}NM\times QT$ with *NM* being found by the cosine rule

dM1 A correct method to find shaded area *R* by finding
$2\times\text{Triangles}-\text{Whole sector}$ **or** $2\times(\text{Triangle}-\text{Half sector})$
or Kite - Sector **o**r Kite – Segment – Isosceles triangle

It is dependent upon having scored **both** previous method marks.

A1 Accept awrt 8.76 or 8.75

Question Number	Scheme	Marks
12(a)	Sub $x = 3$ in $y = x^2 - \frac{1}{3}x^3 = 9 - 9 = 0$	B1*
		(1)
(b)	$y = x^2 - \frac{1}{3}x^3 \Rightarrow \frac{dy}{dx} = 2x - x^2$	M1A1
	Subs $x = 3$ in $\frac{dy}{dx} = 2x - x^2 = 6 - 9 = (-3)$	dM1
	Equation of tangent is $-3 = \frac{y - 0}{x - 3} \Rightarrow y = -3x + 9$	ddM1A1*
		(5)
(c)	Sets $x^2 - \frac{1}{3}x^3 = -3x + 9 \Rightarrow x^3 - 3x^2 - 9x + 27 = 0$ oe	M1, A1
	Solves $x^3 - 3x^2 - 9x + 27 = 0 \Rightarrow (x-3)^2(x+3) = 0 \Rightarrow x = -3$	dM1, A1
		(4)
(d)	Area under curve = $\int x^2 - \frac{1}{3}x^3 dx = \left[\frac{1}{3}x^3 - \frac{1}{12}x^4\right]$	M1A1
	Area of triangle = $\frac{1}{2} \times (3 - x_B) \times y_B = (54)$	M1
	Shaded area = Triangle – area under curve = $'54' - \left(\left(\frac{1}{3}\times 3^3 - \frac{1}{12}\times 3^4\right) - \left(\frac{1}{3}\times(-3)^3 - \frac{1}{12}\times(-3)^4\right)\right)$ =36	dM1A1
		(5) **(15 marks)**
	Alternative to (d) by integration Area = $\int_{x=-3}^{x=3} (-3x+9) - \left(x^2 - \frac{1}{3}x^3\right) dx$ Either way around $\left[-3\frac{x^2}{2} + 9x - \frac{1}{3}x^3 + \frac{1}{3}\frac{x^4}{4}\right]_{x=-3}^{x=3}$ $= \left(-3\times\frac{3^2}{2} + 9\times 3 - \frac{1}{3}\times 3^3 + \frac{1}{3}\times\frac{3^4}{4}\right) - \left(-3\times\frac{(-3)^2}{2} + 9\times(-3) - \frac{1}{3}\times(-3)^3 + \frac{1}{3}\times\frac{(-3)^4}{4}\right)$ =36	

(a)

B1* Substitutes $x = 3$ in $y = x^2 - \frac{1}{3}x^3$ to get $y = 9 - \frac{1}{3}\times 27 = 0$

Alternatively substitutes $y = 0$ in $y = x^2 - \frac{1}{3}x^3$ to get $0 = x^2 - \frac{1}{3}x^3 \Rightarrow 0 = x^2\left(1 - \frac{1}{3}x\right) = 0 \Rightarrow x = 3$

Or substitutes $x = 3$ and $y = 0$ in $y = x^2 - \frac{1}{3}x^3$ to get $0 = 9 - 9$

In all cases you should expect to see a minimum of one intermediate line

(b)

M1 For any term correct (unsimplified). Look for either $x^2 \to 2x$ or $-\frac{1}{3}x^3 \to -\frac{1}{3}\times 3x^2$

A1 Gradient completely correct (unsimplified). Accept $\left(\frac{dy}{dx}\right) = 2\times x - \frac{1}{3}\times 3x^2$

dM1 substitutes $x = 3$ into their derivative. The previous M must have been awarded.

ddM1 Full method to find a numerical equation of the tangent at (3, 0). It is dependent upon **both** previous Method marks having been scored. Accept as evidence $'-3' = \frac{y-0}{x-3}$

If the form $y = mx + c$ is used it must be a full method to find the numerical value of c.

So (3,0) must be subbed into $y = '-3'x + c$ to find a value for c

A1* cso $y = -3x + 9$. This is a given answer and all aspects must be correct.

Note: It is possible to gain marks in part (b) by using roots of equations. It must be clearly part (b) and not part

(c) **Minimal evidence is required to part (c) but the requirement of the question is to use algebra**

M1 Setting equations equal to each other. Look for $x^2 - \frac{1}{3}x^3 = -3x + 9$

It may be implied by $\frac{1}{3}x^3 - x^2 - 3x + 9 = 0$

A1 Correct cubic equation = 0. Accept either $\frac{1}{3}x^3 - x^2 - 3x + 9 = 0$ or multiples of $x^3 - 3x^2 - 9x + 27 = 0$

The ‘= 0’ may be implied by subsequent working such as factorisation etc

dM1 Solving by factorisation to find another value of x other than 3.

This is dependent upon the previous M.

Accept as evidence division of cubic by $(x-3)^2$ - check first and last terms only

Accept as evidence division of cubic by $(x-3)$ followed by factorisation or use of formula on resulting quadratic. Accept the solution 'appearing' from a graphical calculator

A1 cso $x = -3$.

Note: We are going to allow a special case, scored 1,1,0,0 where the candidate writes down $x^2 - \frac{1}{3}x^3 = -3x + 9$ followed by $x = -3$

(d)

M1 Attempts the integral of $x^2 - \frac{1}{3}x^3$. One term must be correct, (un simplified) accept either $\frac{1}{3}x^3$ or $-\frac{x^4}{12}$

.

A1 Correct simplified expression $\frac{1}{3}x^3 - \frac{1}{12}x^4 (+c)$.

However the correct answer of 18 for this area, or 36 for the required area would imply this mark.

M1 Correct method for the area of the triangle. Look for $\frac{1}{2}\times(3 - x_B)\times y_B = 54$

dM1 For subtracting the area under the curve (with their 'correct' limits used) from area of the triangle.

It is dependent upon both M’s and the subtraction of the areas can be either way around.

A1 cso 36

The alternative version can be marked in an equivalent way. M1A1 scored for integrating the $\int x^2 - \frac{1}{3}x^3$ part correctly. Condone sign errors made before the integration takes place. 2nd M1 scored for using integration to find the area of the triangle. The integration must be correct $\left[-\frac{3x^2}{2}+9x\right]$ and evaluated with limits of 3 and their '-3' dM1 For subbing in their 'correct' limits and subtracting the functions (either way around). It is dependent upon both M's. A1 cso 36 Subtracting the wrong way around would score A0

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Question Number	Scheme	Marks
13(a)	$h = 3.7 + 2.5\cos(30t - 40)°, \qquad 0 \le t < 24$	
	Max = 3.7+2.5=6.2m	B1
	Occurs when $30t - 40 = 0 \Rightarrow t = \dfrac{40}{30}, = 1:20am\left(01:20\right)$	M1A1,A1
		(4)
(b)	$3.7 + 2.5\cos(30t - 40)° = 3 \Rightarrow \cos(30t - 40)° = -\dfrac{0.7}{2.5} = \left(-0.28\right)$	M1
	$30t - 40 = 106.3, (253.7)$	A1
	$t =$ awrt 4.9 or 9.8	A1
	2nd Value $30t - 40 = 253.7 \Rightarrow t = ...$	M1
	$t =$ awrt 4.9 and 9.8	A1
	Boat cannot enter the harbour between 04:53 and 09:47	A1
		(6)
		(10 marks)

(a)

B1 $h_{\max} = 6.2\ (m)$. The units are not important.

M1 Solves either $30t - 40 = 0 \Rightarrow t = ..$ or $30t - 40 = 360 \Rightarrow t = ..$. It may be implied by $t = \dfrac{40}{30}, \dfrac{400}{30}$ oe

A1 $t = \dfrac{40}{30}$, or exact equivalents like $\dfrac{4}{3}, 1.\dot{3}$.

A1 1:20am or $\left(01:20\right)$ The exact time of day is required. 1:20 or 1:20 pm is incorrect.

Special case: Candidates who solve this part by differentiation can be allowed full marks

B1 $h_{\max} = 6.2\ (m)$. The units are not important

M1 For $h' = A\sin(30t - 40)° = 0 \Rightarrow t = ..$

A1 $t = \dfrac{40}{30}$, or exact equivalents like $\dfrac{4}{3}, 1.\dot{3}$.

A1 1:20am Accept 01:20 or 0120 The exact time of day is required. 1:20 or 1:20 pm is incorrect

(b)

M1 Sets h=3 and proceeds to $\cos(30t - 40)° = ..$ Accept inequalities in place of = sign

A1 Proceeds by taking invcos to reach either $30t - 40 =$ awrt 106.3, or $30t - 40 =$ awrt 253.7
Accept inequalities in place of = sign. This may be implied by a correct answer for $t = awrt$ 4.9, (9.8)

A1 One value for t correct. Accept either awrt 1 dp $t = 4.9$ or 9.8

M1 For the correct method of finding a second value of t.
Accept $30t - 40 = (360 - \alpha) \Rightarrow t = ...$ where α is their principal value

A1 Both values of t correct awrt 1dp. $t = 4.9$ and 9.8. Ignore any values where $t \geqslant 12$ but withhold this mark for extra values in the range. These may be implied by 293 minutes and 587 minutes

A1 cso Both 04:53 and 09:47. Accept both 0453 and 0947
Accept 4:53 and 9:47 without the am as the question requires morning times.
Accept 293 and 587 minutes
If they state between 0 and 293 minutes and 587 and … minutes it is A0

Question Number	Scheme	Marks
14(a)	Area of triangle = $'\frac{1}{2}ab\sin C' = \frac{1}{2}\times 2x\times 2x\times\sin 60 = \sqrt{3}x^2$	M1
	$S = 2\times\sqrt{3}x^2 + 3\times 2xl = 2x^2\sqrt{3} + 6xl$	dM1A1*
		(3)
(b)	$960 = 2x^2\sqrt{3} + 6xl \Rightarrow l = \dfrac{960 - 2x^2\sqrt{3}}{6x}$	M1A1
	$V = x^2\sqrt{3}\,l$	B1
	Substitute $l = \dfrac{960 - 2x^2\sqrt{3}}{6x}$ into $V = x^2\sqrt{3}\,l$	
	$\Rightarrow V = x^2\sqrt{3}\times\left(\dfrac{960 - 2x^2\sqrt{3}}{6x}\right) = 160x\sqrt{3} - x^3$	dM1A1*
		(5)
(c)	$\dfrac{\mathrm{d}V}{\mathrm{d}x} = 160\sqrt{3} - 3x^2 = 0$	M1A1
	$\Rightarrow x =$ awrt 9.6	A1
	$\Rightarrow V = 160\times 9.611\times\sqrt{3} - 9.611^3 = 1776$	dM1 A1
		(5)
(d)	$\dfrac{\mathrm{d}^2V}{\mathrm{d}x^2} = -6x < 0 \Rightarrow$ Maximum	M1A1
		(2)
		(15 marks)

(a) E pen has this part marked M1 A1 A1*. We are going to score it M1dM1A1*

M1 Score for an acceptable method for finding the area of the triangle. You may have to look at the diagram for evidence.

Using $\frac{1}{2}ab\sin C$ sight of $\frac{1}{2}\times 2x\times 2x\times \sin 60$ is sufficient or $\frac{1}{2}\times 2x\times 2x\times\frac{\sqrt{3}}{2}$ with $60°$ on the diagram.

Use of $\frac{1}{2}bh$ is fine with Pythagoras' theorem being used to find h using $h^2+x^2=(2x)^2$

Accept $\frac{1}{2}\times 2x\times\sqrt{3}x$ with $\sqrt{3}x$ marked as a perpendicular height on the diagram

dM1 For adding the area of 3 rectangles and two triangles. It is dependent upon the previous M mark.

Look for something like $3\times 2xl+2\times\frac{1}{2}\times 2x\times 2x\times\sin 60$ before the given answer

A1* For completing the proof with no errors. Accept it written in a different order. Eg $S=2\sqrt{3}x^2+6xl$

This is a proof so the expectation is that they show that the area of the triangle is $\sqrt{3}x^2$

But accept for special case 1,0,0 $S=3\times 2xl+2\times\frac{1}{2}\times 2x\times 2x\times\frac{\sqrt{3}}{2}$ or $S=3\times 2xl+2\times\frac{1}{2}\times 2x\times\sqrt{3}x$

(b)

M1 Substitutes $S=960$ into $S=2x^2\sqrt{3}+6xl$ and attempts to make l or lx the subject of the formula.

Accept as evidence $960=2x^2\sqrt{3}+6xl\Rightarrow l=...$ or $960=2x^2\sqrt{3}+6xl\Rightarrow lx=...$

A1 $l=\frac{960-2x^2\sqrt{3}}{6x}$ oe $l=\frac{160}{x}-\frac{x\sqrt{3}}{3}$ or $lx=\frac{960-2x^2\sqrt{3}}{6}$ oe $lx=160-\frac{x^2\sqrt{3}}{3}$

B1 States that the volume V is $V=x^2\sqrt{3}\,l$. The order of the terms is not important.

It may be implied by $V=x^2\sqrt{3}\times$ *their l* or $V=x\sqrt{3}\times$ *their lx*

dM1 Substitute their expression for $l=...$(or $lx=...$) into their expression for $V=..$ to obtain an expression for V just in terms of x. It is dependent upon the previous M having been awarded.

A1* This is a given answer. All aspects of the proof must be correct. $V=160x\sqrt{3}-x^3$

(c) For the purpose of marking parts(c) and (d) can be scored together

M1 For differentiating with at least one term correct and setting =0.

A1 $160\sqrt{3}-3x^2=0$. Accept awrt $277-3x^2=0$

A1 $x=$ awrt 9.6 . This appearing on its own is A0. Accept versions of $x=\sqrt{\frac{160\sqrt{3}}{3}}$

The question states that calculus must be used and a correct expression for $\frac{dV}{dx}$ must be seen.

dM1 Dependent upon the previous M, this is for subbing their answer to $\frac{dV}{dx}=0$ in $V=160x\sqrt{3}-x^3$ to find V.

A1 $V=$ awrt 1776

(d)

M1 Either attempts to find the value of $\frac{d^2V}{dx^2}=-6x$ (or kx following a slip), **and** substitutes the value of x found in part (c),

Or attempts the statement $\frac{d^2V}{dx^2}=-6x$ **and** states it is less than 0

A1 Either $\frac{d^2V}{dx^2} = -6x \Rightarrow \frac{d^2V}{dx^2} < 0$ when x=.., hence a maximum. Allow on any (positive) value of x found in (c). For this to be scored $\frac{d^2V}{dx^2} = -6x$ must be correct, it must be calculated (or implied to be calculated) at the value of x found in part (c), and there must be a reason $\left(\frac{d^2V}{dx^2} < 0\right)$ and a conclusion, hence a maximum.

Or also allow $\frac{d^2V}{dx^2} = -6x = -6 \times \textit{positve} = \textit{negative}$ followed by a reason $\left(\frac{d^2V}{dx^2} < 0\right)$ and a conclusion, hence a maximum.

There are alternatives using values of the gradient and values of the function.

Using the gradient

M1 Attempts to find the numerical value of the gradient $\frac{dV}{dx}$ either side of the x value found in part (c)

A1 $\frac{dV}{dx} = 160\sqrt{3} - 3x^2$ must be correct, it must be calculated either side, there must be a reason referring to gradient being +ve, 0 and –ve, followed by a conclusion, hence maximum

Using the function

M1 Attempts to find the value of V either side of the value of x found in part (c), **and** starts to compare these with the value of V found in part (c).

A1 Correct numerical values Eg $V_{9.60} = 1775.694, V_{9.61} = 1775.698, V_{9.62} = 1775.695$ followed by a reason which could include a sketch and a conclusion, hence maximum.

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