

Mark Scheme (Results)

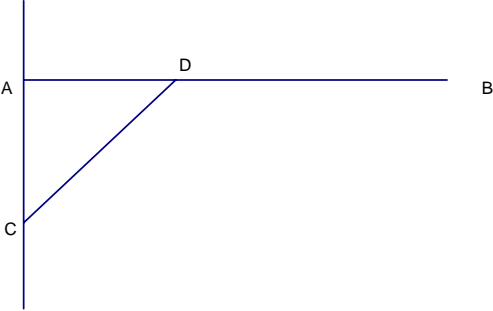
Summer 2009

GCE

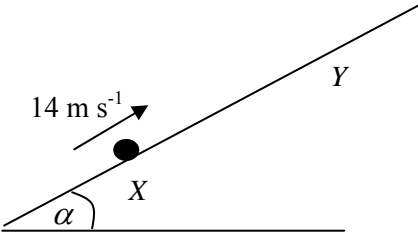
GCE Mathematics (6678/01)

June 2009
 6678 Mechanics M2
 Mark Scheme

Question Number	Scheme	Marks
Q1	$\mathbf{I} = m\mathbf{v} - m\mathbf{u}$ $5\mathbf{i} - 3\mathbf{j} = \frac{1}{4}\mathbf{v} - \frac{1}{4}(3\mathbf{i} + 7\mathbf{j})$ $\mathbf{v} = 23\mathbf{i} - 5\mathbf{j}$ $ \mathbf{v} = \sqrt{23^2 + 5^2} = 23.5$	M1A1 A1 M1A1 [5]
Q2	<p>(a)</p> $\frac{dv}{dt} = 8 - 2t$ $8 - 2t = 0$ $\text{Max } v = 8 \times 4 - 4^2 = 16 \text{ (ms}^{-1}\text{)}$ <p>(b)</p> $\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$ <p>($t=0$, displacement = 0 $\Rightarrow c=0$)</p> $4T^2 - \frac{1}{3}T^3 = 0$ $T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$ $T = 12 \text{ (seconds)}$	M1 M1 M1A1 (4) M1A1 DM1 DM1 A1 (5) [9]
Q3	<p>(a) Constant $v \Rightarrow$ driving force = resistance $\Rightarrow F=120 \text{ (N)}$ $\Rightarrow P=120 \times 10 = 1200\text{W}$</p> <p>(b) Resolving parallel to the slope, zero acceleration: $\frac{P}{v} = 120 + 300g \sin \theta (= 330)$ $\Rightarrow v = \frac{1200}{330} = 3.6 \text{ (ms}^{-1}\text{)}$</p>	M1 M1 (2) M1A1A1 A1 (4) [6]

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<p>Q4 (a)</p>  <p>(b)</p>	<p>Taking moments about A:</p> $3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$ $T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$ <p> $\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$ </p> <p> $\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7N)$ </p> <p> $\Rightarrow R = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$ </p> <p> at angle $\tan^{-1} \frac{1}{3} = 18.4^\circ$ (0.322 radians) below the line of BA 161.6° (2.82 radians) below the line of AB $(108.4^\circ$ or 1.89 radians to upward vertical) </p>	<p>M1A1A1</p> <p>A1</p> <p>(4)</p> <p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>M1A1</p> <p>(7) [11]</p>
<p>Q5 (a)</p> <p>(b)</p>	<p>Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800) Centre of mass of the triangle is 20cm down from AD (seen or implied)</p> <p> $\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$ $\bar{y} = 50cm$ </p> <p>Distance of centre of mass from AB is 60cm</p> <p>Required angle is $\tan^{-1} \frac{60}{50}$ $= 50.2^\circ$ (0.876 rads)</p> <p>(their values)</p>	<p>B1 B1</p> <p>M1A1 A1</p> <p>(5)</p> <p>B1</p> <p>M1A1ft A1</p> <p>(4) [9]</p>

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Q6 (a)	$\rightarrow x = u \cos \alpha t = 10$ $\uparrow y = u \sin \alpha t - \frac{1}{2} g t^2 = 2$ $\Rightarrow t = \frac{10}{u \cos \alpha}$ $2 = u \sin \alpha \times \frac{10}{u \cos \alpha} - \frac{g}{2} \times \frac{100}{u^2 \cos^2 \alpha}$ $= 10 \tan \alpha - \frac{50g}{u^2 \cos^2 \alpha} \text{ (given answer)}$	<p>M1A1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>(6)</p>
(b)	$2 = 10 \times 1 - \frac{100g \times 2}{2u^2 \times 1}$ $u^2 = \frac{100g}{8}, u = \sqrt{\frac{100g}{8}} = 11.1 \text{ (m s}^{-1}\text{)}$ $\frac{1}{2} m u^2 = m \times 9.8 \times 2 + \frac{1}{2} m v^2$ $v = 9.1 \text{ m s}^{-1}$	<p>M1A1</p> <p>A1</p> <p>M1A1</p> <p>A1</p> <p>(6)</p> <p>[12]</p>

Question Number	Scheme	Marks
Q7 (a)	 <p style="text-align: right;">KE at X = $\frac{1}{2}mv^2 = \frac{1}{2} \times 2 \times 14^2$</p> <p style="text-align: right;">GPE at Y = $mgd \sin \alpha \left(= 2 \times g \times d \times \frac{7}{25} \right)$</p> <p style="text-align: right;">Normal reaction $R = mg \cos \alpha$</p> <p style="text-align: right;">Friction = $\mu \times R = \frac{1}{8} \times 2g \times \frac{24}{25}$</p> <p>Work Energy: $\frac{1}{2}mv^2 - mgd \sin \alpha = \mu \times R \times d$ or equivalent</p> $196 = \frac{14gd}{25} + \frac{6gd}{25} = \frac{20gd}{25}$ $d = 25 \text{ m}$	<p>B1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>M1A1</p> <p>A1</p> <p style="text-align: right;">(7)</p>
(b)	<p>Work Energy</p> <p>First time at X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2$</p> <p>Work done = $\mu \times R \times 2d = \frac{1}{8} \times 2g \times \frac{24}{25} \times 2d$</p> <p>Return to X: $\frac{1}{2}mv^2 = \frac{1}{2}m14^2 - \frac{1}{8} \times 2g \times \frac{24}{25} \times 50$</p> $v = 8.9 \text{ ms}^{-1} \quad (\text{accept } 8.85 \text{ ms}^{-1})$ <p>OR: Resolve parallel to XY to find the acceleration and use of $v^2 = u^2 + 2as$</p> $2a = 2g \sin \alpha - F_{\max} = 2g \times \frac{7}{25} - \frac{6g}{25} = \frac{8g}{25}$ $v^2 = (0+)^2 + 2 \times a \times s = 8g ; v = 8.9 \quad (\text{accept } 8.85 \text{ ms}^{-1})$	<p>M1A1</p> <p>DM1A1</p> <p style="text-align: right;">(4)</p> <p>M1A1</p> <p>DM1;A1</p> <p style="text-align: right;">[11]</p>

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Q8 (a)	<div style="text-align: center; margin-bottom: 20px;"> </div> <p>Conservation of momentum: $4mu - 3mv = 3mkv$</p> <p>Impact law: $kv = \frac{3}{4}(u + v)$</p> <p>Eliminate k: $4mu - 3mv = 3m \times \frac{3}{4}(u + v)$</p> <p style="text-align: center;">$u = 3v$ (Answer given)</p> <p>(b) $kv = \frac{3}{4}(3v + v), k = 3$</p> <p>(c) Impact law: $(kv + 2v)e = v_C - v_B$ ($5ve = v_C - v_B$) Conservation of momentum: $3 \times kv - 1 \times 2v = 3v_B + v_C$ ($7v = 3v_B + v_C$) Eliminate v_C: $v_B = \frac{v}{4}(7 - 5e) > 0$ hence no further collision with A.</p>	<p>M1A1</p> <p>M1A1</p> <p>DM1</p> <p>A1</p> <p style="text-align: right;">(6)</p> <p>M1,A1</p> <p style="text-align: right;">(2)</p> <p>B1</p> <p>B1</p> <p>M1 A1</p> <p style="text-align: right;">(4)</p> <p style="text-align: right;">[12]</p>