

Mark Scheme (Results) January 2007

GCE

GCE Mathematics

Mechanics M2 (6678)



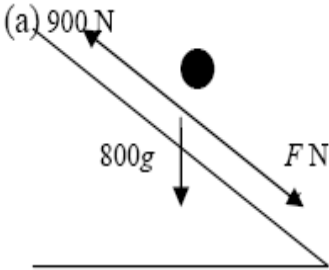
January 2007
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
1.	<p>(a) $\frac{1}{2}0.8(15^2 - 10^2) = 50 \text{ (J)}$</p> <p>(b) $F = \mu R = \mu 0.8g$ Work-energy $\mu 0.8g \times 20 = 50$ $\mu \approx 0.32$</p>	<p>M1 A1 <u>2</u></p> <p>M1 M1 A1ft A1 <u>4</u> 6</p>

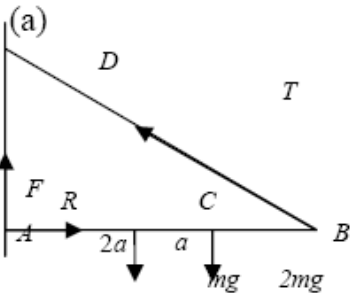
	<p><i>Alternative for (b)</i></p> <p>$v^2 = u^2 + 2as \Rightarrow a = \frac{15^2 - 10^2}{2 \times 20} = 3.125$</p> <p>N2L $F = \mu mg = ma = 3.125m$ $\mu \approx 0.32$</p>	<p>M1 M1 A1ft A1 <u>4</u></p>

	<p><i>Alternative for (b)</i></p> <p>WE $F = \frac{50}{20} (= 2.5)$</p> <p>$F = \mu R \Rightarrow \frac{50}{20} = \mu 0.8g$ $\mu \approx 0.32$</p>	<p>M1 M1 A1 ft A1 4</p>

	<p>The first M1 for (b) could be scored in (a):</p> <p>$v^2 = u^2 + 2as \Rightarrow 10^2 = 15^2 - 2 \times 20 \times (-)a \Rightarrow a = (-)\frac{125}{40}$</p> <p>$F = ma \Rightarrow F = 2.5$ $WD = F \times d \Rightarrow 2.5 \times 20 = 50J$</p>	<p>(b)M1 (a)M1A1</p>

2.		$F + 800g \sin \alpha = 900$ $F = 573\frac{1}{3}$ $W = 573\frac{1}{3} \times 15 = 8600$ $= 8.6 \text{ kW}$	<p>M1 A1 M1 A1 <u>4</u></p>
NB. Going up hill is an error, not a Misread			
(b)	N2L	$800 \times 9.8 \times \frac{1}{24} - 900 = 800a \quad *$ $a = -\frac{43}{60} \quad \text{awrt } -0.72$ $0 = 15 - \frac{43}{60}T$ $T \approx 21 \quad \text{accept } 20.9$	<p>M1 A1 M1 Alcso <u>4</u> 8</p>
* If they are using their F from (a) then they need to have scored the M1 in (a) in order to score the M1 here.			
Alternative for (b)			
WD: $573\frac{1}{3}s = \frac{1}{2} \times 800 \times 15^2$			
$s = 157$			
Use of $v^2 = u^2 + 2as$ M1 for getting as far as an equation in a .			
$a = 0.72$ A1 finish as above.			
2 nd Alternative for (b)			
$Ft = \text{Change in momentum:}$			
M1 Using the correct F			
M1 Use of the method to form an equation			
A1 Equation correct unsimplified but fully substituted			
A1 $T \approx 21$			

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3.	<p>(a) Large Small Template Mass Ratios 24^2 8^2, 512 anything in ratio 9 : 1 : 8 (c.1810 c.200 c.1610) $M(A)$ $9 \times 24 = 16 \times 1 + 8\bar{x}$ $\bar{x} = 25$ (cm) exact</p> <p>(b) M(axis) $11M = 12 \times \frac{1}{4}m$ ft their \bar{x} $(36 - \bar{x})M = 12 \times \frac{1}{4}m$ $M = \frac{3}{11}m$ (o.e.e.)</p>	<p>B1, B1ft M1* A1 DM1* A1 <u>6</u> M1 † A1ft DM1 † A1 <u>4</u> 10</p>
4. (a)	<p>NEL $3v - (-v) = eu$ $u = 8v$</p> <p>(b) LM $8mv = -mv + 3kmv$ ft their u $(m \times (u) = -mv + 3kmv)$ $k = 3$</p> <p>(c) LM $9mv = -3my + 11my$ ft their k NEL $2y = e \times 3v$ $y = \frac{9}{8}v \Rightarrow e = \frac{3}{4}$ * cso</p> <p>(d) $y = \frac{9}{8}v > v \Rightarrow$ further collision between P and Q A1 is cso – watch out for incorrect statements re. velocity</p>	<p>M1 A1 A1 <u>3</u> M1 A1ft A1 <u>3</u> M1 A1ft M1 A1 <u>4</u> M1 A1 <u>2</u> 12</p>

Question Number	Scheme	Marks
5.	<p>(a) </p> <p>M(A) $T \sin \theta \times 4a = mg \times 2a + 2mg \times 3a$</p> $T = \frac{8mg}{4} \times \frac{5}{3} = \frac{10}{3}mg$ <p>Accept 32.7m, 33m</p> <p>(b) $\rightarrow R = T \cos \theta = \frac{10}{3}mg \times \frac{4}{5}; = \frac{8}{3}mg$ *</p> <p>(c) $\uparrow F + T \sin \theta = 3mg \Rightarrow F = mg$ ft their T Or: M(B) $F \times 4a = mg \times 2a + 2mg \times a \Rightarrow F = mg$</p> $F = \mu R \Rightarrow \mu = \frac{3}{8}$ <p>(a) Alternative approach: $\rightarrow R = T \cos \theta$ $\uparrow F + T \sin \theta = 3mg$ M(B) $F \times 4a = mg \times 2a + 2mg \times a (\Rightarrow F = mg)$ $\Rightarrow mg + T \sin \theta = 3mg \Rightarrow T = \frac{2mg}{\sin \theta} = \frac{10mg}{3}$</p> <p>If they use this method, watch out for F=mg just quoted in (c): M1A1</p>	<p>M1* A1=A1 DM1* A1 <u>5</u></p> <p>M1 A1ft; A1 <u>3</u></p> <p>M1 A1ft</p> <p>M1 A1 <u>4</u> 12</p>

6.	<p>(a) N2L $(1.5t^2 - 3)\mathbf{i} + 2t\mathbf{j} = 0.5\mathbf{a}$ $\mathbf{a} = (3t^2 - 6)\mathbf{i} + 4t\mathbf{j}$</p> <p>(b) $\mathbf{v} = (t^3 - 6t)\mathbf{i} + 2t^2\mathbf{j} \quad (+\mathbf{c})$ $t = 2 \quad -4\mathbf{i} + 5\mathbf{j} = -4\mathbf{i} + 8\mathbf{j} + \mathbf{c} \quad (\mathbf{c} = -3\mathbf{j})$ $\mathbf{v} = (t^3 - 6t)\mathbf{i} + (2t^2 - 3)\mathbf{j} \quad (\text{ms}^{-1})$ $t = 3 \quad \mathbf{v} = 9\mathbf{i} + 15\mathbf{j} \quad (\text{ms}^{-1}) \quad *$</p> <p>(c) $\mathbf{Q} = 0.5(-3\mathbf{i} + 20\mathbf{j} - (9\mathbf{i} + 15\mathbf{j})) \quad (= 0.5(-12\mathbf{i} + 5\mathbf{j}))$ $\mathbf{Q} = 0.5\sqrt{(5^2 + 12^2)} = 6.5$</p> <p>(d) acute angle is $\arctan \frac{5}{12} \approx 23^\circ$ or required angle is $\arctan \frac{-5}{12}$ or acute angle is $\arccos \frac{12}{13} \approx 23^\circ$ or required angle is $\arccos \frac{-12}{13}$ required angle is 157° awrt $157^\circ, 203^\circ$</p>	<p>M1 A1 <u>2</u></p> <p>M1 A1 M1 A1 A1 <u>5</u> cso</p> <p>M1 M1 A1 <u>3</u></p> <p>M1 A1</p> <p>A1 <u>3</u> 13</p>
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Question Number	Scheme	Marks
7.	<p>(a) Energy $\frac{1}{2}m(24.5^2 - u^2) = mg \times 15$ $u^2 = 24.5^2 - 30g = 306.25$ $u = \sqrt{306.25} = 17.5$ ★</p> <p>(b) $\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$ $\psi = \arccos \frac{14}{24.5} \approx 55^\circ$ accept 55.2° <small>(0.96 rads, or 0.963 rads)</small></p> <p>(c) $\uparrow u_y = u \sin \theta = 17.5 \times 0.6 = 10.5$ $s = ut + \frac{1}{2}at^2 \Rightarrow -45 = 10.5t - 4.9t^2$ leading to $t = 4.3$, awrt $t = 4.3$ or $t = 4\frac{2}{7}$ $\rightarrow BD = 14 \times 4\frac{2}{7}$ (14 x t) ft their t = 60 (m) only</p>	<p>M1 A1=A1</p> <p>A1 <u>4</u></p> <p>B1</p> <p>M1 A1 <u>3</u></p> <p>B1</p> <p>M1 A1</p> <p>A1</p> <p>M1 A1ft</p> <p>A1 <u>7</u> 14</p>
	<p><i>Alternative for (a)</i> $\rightarrow u_x = u \cos \theta = 0.8u$, $\uparrow u_y = u \sin \theta = 0.6u$ $v_y^2 = 0.36u^2 + 2 \times 9.8 \times 15 = 0.36u^2 + 294$ $24.5^2 = u_x^2 + v_y^2 = 0.64u^2 + 0.36u^2 + 294$ $u^2 = 306.25 \Rightarrow u = 17.5$ ★ cso</p> <p><i>Alternative for (b)</i> $\rightarrow u_x = u \cos \theta = 17.5 \times 0.8 = 14$ $\uparrow v_y^2 = u^2 \sin^2 \theta + 2 \times 9.8 \times 15 = 404.25$ $\psi = \arctan \frac{\sqrt{404.25}}{14} \approx 55^\circ$ accept 55.2°</p> <p><i>Alternative for (c)</i> Use of $y = x \tan \theta - \frac{g \sec^2 \theta}{2u^2} x^2$ $-45 = \frac{3}{4}x$, $-\frac{g}{2 \times 17.5^2} \times \frac{25}{16} x^2$ $x^2 - 30x - 1800 = 0$ o.e. Factors or quadratic formula BD = 60 (m)</p>	<p>M1 A1,A1</p> <p>A1 <u>4</u></p> <p>B1</p> <p>M1 A1 <u>3</u></p> <p>M1</p> <p>B1,A1</p> <p>A1</p> <p>M1 A1ft</p> <p>A1</p>