# Assign 1

1) $xy\,dx + (x+1)\,dy = 0$ Separation

$$xy\,dx = -(x+1)\,dy$$

$$\int \frac{x\,dx}{x+1} = -\int \frac{dy}{y}$$

$$\int \frac{x+1-1}{x+1}\,dx = -\ln|y| + C$$

$$\int 1 - \frac{1}{x+1}\,dx = -\ln|y| + C$$

$$x - \ln|x+1| = -\ln|y| + C$$

2) $x y' + y = y^2$ Separation

$$xy' = y^2 - y$$

$$x\frac{dy}{dx} = y^2 - y$$

$$\int \frac{dy}{y^2 - y} = \int \frac{dx}{x}$$

$$\int \frac{dy}{y(y-1)} = \ln|x| + C$$

$$\int \frac{A}{y} + \frac{B}{y-1}\,dy = \ln|x| + C$$

$y=0 \quad A=-1 \qquad y=1 \quad B=1$

$$\int -\frac{1}{y} + \frac{1}{y-1}\,dy = \ln|x| + \ln|C|$$

$$-\ln|y| + \ln|y-1| = \ln xC$$

$$\ln\frac{y-1}{y} = \ln xC$$

$$\frac{y-1}{y} = xC$$

$$y-1 = xyC$$

3) $2y + (xy + 3x)y = 0$ Separation

$2y + x(y+3)\frac{dy}{dx} = 0$

$2y\,dx = -x(y+3)\,dy.$

$-\int \frac{2}{x}\,dx = \int \frac{y+3}{y}\,dy.$

$-2\ln|x| = \int 1 + \frac{3}{y}\,dy$

$-2\ln|x| = y + 3\ln|y| + C$

4) $(x^2-1)y' + 2xy^2 = 0$ Separation

$(x^2-1)\frac{dy}{dx} = -2xy^2$

$(x^2-1)\,dy = -2xy^2\,dx$

$\int \frac{dy}{y^2} = -\int \frac{2x}{x^2-1}\,dx.$

$-\frac{1}{y} = -\ln|x^2-1| + C$

$\frac{1}{y} = \ln|x^2-1| + C$

5) $y' - xy^2 = 2xy$ Separation.

$\frac{dy}{dx} - 2xy = xy^2$.

$\frac{dy}{dx} = 2xy + xy^2$

$\frac{dy}{dx} = xy(2+y)$

$\int \frac{dy}{y(y+2)} = \int x\,dx$

$\int \frac{A}{y} + \frac{B}{y+2}\,dy = \frac{x^2}{2} + C$

$y=0 \quad A = \frac{1}{2}$ $\quad y=-2 \quad y=-\frac{1}{2}$

$\int \frac{1}{2}\cdot\frac{1}{y} - \frac{1}{2}\cdot\frac{1}{y+2}\,dy = \frac{x^2}{2} + C$

$\frac{1}{2}\ln|y| - \frac{1}{2}\ln|y+2| = \frac{x^2}{2} + C$

$\ln\frac{y}{y+2} = x^2 + C$

6) $y' = \cos(y-x)$

Separación

$y - x = t$

$\frac{dt}{dx} + 1 = \cos t$

$\frac{dy}{dx} - 1 = \frac{dt}{dx}$

$\frac{dt}{dx} = \cos t - 1$

$\frac{dy}{dx} = 1 + \frac{dt}{dx}$

$\int \frac{dt}{1-\cos t} = -\int dx$

8)

$\int \frac{dt}{2\sin^2 \frac{t}{2}} = -x + C$

$\int \frac{1}{2} \operatorname{cosec}^2 \frac{t}{2}\, dt = -x + C$

$-\cot t = -x + C$

9)

$\cot t = x + C$

7) $y' - y = 2x - 3.$

$z = y + 2x - 3$

$y' = y + 2x - 3$

$\frac{dz}{dx} = \frac{dy}{dx} + 2$

$-2 + \frac{dz}{dx} = z$

$\frac{dz}{dx} - 2 = \frac{dy}{dx}$

$$\frac{dz}{dx} = z+2$$

$$\int \frac{dz}{z+2} = \int dx$$

$$\ln|z+2| = x + C$$

8) $(x+2)\,y' = 1$

$$(x+2)\frac{dy}{dx} = 1$$

$$\int dy = \int \frac{dx}{x+2}$$

$$y = \ln|x+2| + C$$

9) $(x-y)\,dx + (x+y)\,dy = 0$ Homogeneous

$$\left(1-\frac{y}{x}\right) + \left(1+\frac{y}{x}\right)\frac{dy}{dx} = 0$$

$$(1-z) + (1+z)\left(z + x\frac{dz}{dx}\right) = 0$$

$$1 - z + z + z^2 + x(1+z)\frac{dz}{dx} = 0$$

$$z^2 + 1 = -x(1+z)\frac{dz}{dx}$$

$\frac{y}{x} = z$

$y = xz$

$y' = z + x\frac{dz}{dx}$

$\frac{dy}{dx} = \frac{1-\frac{y}{x}}{1+\frac{y}{x}}$

$$\int \frac{1+z}{z^2+1}\,dz = -\int \frac{dx}{x}$$

$$\int \frac{1}{z^2+1} + \frac{z}{z^2+1}\,dz = -\ln|x| + C$$

$$\tan^{-1} z + \frac{1}{2}\ln|z^2+1| = -\ln|x| + C$$

$$\tan^{-1}\frac{y}{x} + \frac{1}{2}\ln\left|\frac{y^2}{x^2}+1\right| = -\ln|x| + C$$

10) $y' = \sqrt{4x+2y-1}$

$4x+2y-1 = z^2$

$4 + 2y' = 2z\frac{dz}{dx}$

$2 + y' = z\frac{dz}{dx}$

$y' = z\frac{dz}{dx} - 2$

$z\frac{dz}{dx} - 2 = \sqrt{z^2}$

$z\frac{dz}{dx} = 2 + z$

$\int \frac{z}{z+2}\,dz = \int dx$

$\int \frac{z+2-2}{z+2}\,dz = \int dx$

$\int 1 - \frac{2}{z+2}\,dz = x + C$

$z - 2\ln|z+2| = x + C$

$\sqrt{4x+2y-1} - 2\ln\left|\sqrt{4x+2y-1}+2\right| = x + C$

11) $xy' = \sqrt{x^2 - y^2} + y$ Homojen

$xy' = x\sqrt{1 - \left(\frac{y}{x}\right)^2} + y$

$y' = \sqrt{1 - \left(\frac{y}{x}\right)^2} + \frac{y}{x}$

Let $\frac{y}{x} = z$

$y = xz$

$y' = z + x\frac{dz}{dx}$

$z + x\frac{dz}{dx} = \sqrt{1 - z^2} + z$

$x\frac{dz}{dx} = \sqrt{1 - z^2}$

$\int \frac{dz}{\sqrt{1-z^2}} = \int \frac{dx}{x}$

$\sin^{-1} z = \ln|x| + C$

$\sin^{-1}\frac{y}{x} = \ln|x| + C$

or

$y' = \sqrt{1-v^2} + v = f(v)$

$\int \frac{dv}{f(v) - v} = \ln|x| + C$

$\int \frac{dv}{\sqrt{1-v^2}+v-v} = \ln|x| + C$

12) $(x+1)y' = x\sqrt{y+1}$ ; $y(0) = 0$

$(x+1)\frac{dy}{dx} = x\sqrt{y+1}$

$\int \frac{dy}{\sqrt{y+1}} = \int \frac{x}{x+1}\,dx$

$$\int (y+1)^{-\frac{1}{2}}\,dy = \int \frac{x+1-1}{x+1}\,dx.$$

$$2(y+1)^{\frac{1}{2}} = \int 1 - \frac{1}{x+1}\,dx.$$

$$2\sqrt{y+1} = x - \ln|x+1| + C$$

$$2 = C \qquad 2\sqrt{y+1} = x - \ln|x+1| + 2$$

13) $(y-x-1)y' = y-x$ $\quad y(-5)=5.$

$y-x=z$

$y'-1=\frac{dz}{dx}$

$y' = \frac{dz}{dx}+1$

$$(z-1)\left(\frac{dz}{dx}+1\right) = z$$

$$(-1+z)\frac{dz}{dx} + z - 1 = z$$

$$(z-1)\frac{dz}{dx} = 1$$

$$\int (z-1)\,dz = \int dx$$

$$\frac{z^2}{2} - z = x + C$$

$$\frac{(y-x)^2}{2} - (y-x) = x + C$$

$x=-5 \quad y=5$

$$\frac{100}{2} - 10 = -5 + C$$

$$C = 45$$

$$\frac{(y-x)^2}{2} - (y-x) = x + 45$$

14) $(x^2+1)y' = y^2+1 \quad ; y(0)=1$

$$(x^2+1)\frac{dy}{dx} = y^2+1$$

$$\int \frac{dy}{y^2+1} = \int \frac{dx}{x^2+1}$$

$$\tan^{-1} y = \tan^{-1} x + C$$

$$\tan^{-1} 1 = \tan^{-1} 0 + C$$

$$\frac{\pi}{4} = C$$

$$\tan^{-1} y = \tan^{-1} x + \frac{\pi}{4}$$

# Assignment 2

1) $x y' - 2y = 2x^4$

$y' - \frac{2}{x} y = 2x^3$ Linear

$y' + P(x) y = Q(x)$

$\mu = e^{\int P(x) dx} = e^{-2\int \frac{dx}{x}} = e^{-2\ln x}$

$= e^{\ln x^{-2}} = x^{-2}$

$\mu y = \int \mu Q(x)\, dx + C$

$x^{-2} y = \int x^{-2} 2x^3\, dx + C$

$\frac{y}{x^2} = 2\int x\, dx + C$

$\frac{y}{x^2} = x^2 + C$

2) $(2x+1)y' = 4x + 2y$

$(2x+1)y' - 2y = 4x$

$y' - \frac{2}{2x+1}y = \frac{4x}{2x+1}$ Lineal

$y' + P(x)y = Q(x)$

$\mu = e^{-2\int \frac{dx}{2x+1}} = e^{\ln|2x+1|}$

$= e^{\ln(2x+1)^{-1}} = (2x+1)^{-1} = \frac{1}{2x+1}$

$\mu y = \int \mu Q(x)\, dx + C$

$\frac{y}{2x+1} = \int \frac{1}{2x+1}\frac{4x}{(2x+1)}\, dx + C$

$\frac{y}{2x+1} = 2\int \frac{2x+1-1}{(2x+1)^2}\, dx + C$

$\frac{y}{2x+1} = 2\int \frac{dx}{2x+1} - (2x+1)^{-2}dx + C$

$\frac{y}{2x+1} = \ln|2x+1| + \frac{1}{2x+1} + C$

3) $y' - y\tan x = \sec x$ Linear

$y' + P(x)y = Q(x)$

$P(x) = \tan x \quad Q(x) = \sec x$

$\mu = e^{\int P(x)\,dx} = e^{-\int \tan x\,dx}$

$\mu = e^{\ln|\cos x|} = \cos x$

$\mu y = \int \mu Q(x)\,dx + C$

$\cos x\, y = \int \cos x \sec x\,dx + C$

$\cos x\, y = x + C$

4) $x(y' - y) = e^x$

$y' - y = \frac{e^x}{x}$ Linear

$y' + P(x)y = Q(x)$

$\mu = e^{-\int dx} = e^{-x}$

$$\mu y = \int \mu Q(x)\,dx + C$$

$$e^{-x} y = \int e^{-x} \frac{e^x}{x}\,dx + C$$

$$e^{-x} y = \int \frac{dx}{x} + C$$

$$e^{-x} y = \ln|x| + C$$

5) $y' = 2x(x^2 + y)$

$$y' - 2xy = 2x^3 \quad \text{Linear}$$

$$y' + P(x)y = Q(x)$$

$$P(x) = -2x \qquad Q(x) = 2x^3$$

$$\mu = e^{\int P(x)\,dx} = e^{-\int 2x\,dx} = e^{-x^2}$$

$$\mu y = \int \mu Q(x)\,dx + C$$

$$e^{-x^2} y = \int e^{-x^2} 2x^3\,dx + C \qquad x^2 = t,\ 2x\,dx = dt$$

$$e^{-x^2} y = \frac{2}{2}\int e^{-t} \cdot t\,dt + C$$

$$e^{-x^2} y = t\frac{e^{-t}}{(-1)} - 1\frac{e^{-t}}{(-1)^2} + C$$

$$e^{-x^2} y = -x^2 e^{-x^2} - e^{-x^2} + C$$

6) $(x+y^2)\,dy = y\,dx$.

$$x+y^2 = y\frac{dx}{dy}$$

$$\frac{x}{y} + y = \frac{dx}{dy}$$

$$\frac{dx}{dy} - \frac{x}{y} = y$$

$$\frac{dx}{dy} + P(y)x = Q(y)$$

$P(y) = -\frac{1}{y}$ $\quad Q(y) = y$

$$\mu = e^{\int P(y)\,dy} = e^{-\int \frac{dy}{y}}$$

$$= e^{-\ln y} = e^{\ln y^{-1}} = y^{-1}$$

$$\mu x = \int \mu Q(y)\,dy + C$$

$$y^{-1}x = \int y^{-1}\cdot y\,dy + C$$

$$\frac{x}{y} = y + C$$

7) $y' - y\tan x = y^4 \cot x$ Bernoulli's

$y' + P(x)y = Q(x)y^n$

$P(x) = -\tan x \quad Q(x) = \cot x \quad n = 4$

$\mu = e^{\int (1-n) P(x)\, dx}$

$= e^{\int (1-4)(-\tan x)\, dx} \quad = e^{3\int \tan x\, dx}$

$= e^{3 \ln|\sec x|} = \sec^3 x.$

$\mu y^{1-n} = \int \mu Q(x) (1-n)\, dx + C$

$\sec^3 x\, y^{1-4} = \int \sec^3 x \cot x \cdot (1-4)\, dx + C$

$\frac{\sec^3 x}{y^3} = -3 \int \sec^3 x \frac{\cos x}{\sin x}\, dx + C$

$\frac{\sec^3 x}{y^3} = -3 \int \frac{\sec^2 x}{\sin x}\, dx + C$

$\frac{\sec^3 x}{y^3} = -3 \int \frac{1 + \tan^2 x}{\sin x}\, dx + C$

$\frac{\sec^3 x}{y^3} = -3 \int \operatorname{cosec} x + \frac{\sin x}{\cos^2 x}\, dx + C$

$$\frac{\sec^3 x}{y^3} = -3\int \csc x + \tan x \sec x \, dx + C$$

$$\frac{\sec^3 x}{y^3} = -3\left[\ln|\csc x - \cot x| + \sec x + C\right]$$

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8) $xy' - 2x^2\sqrt{y} = 4y$

$$y' - 2x\sqrt{y} = \frac{4y}{x}$$

$$y' - \frac{4y}{x} = 2x\sqrt{y}$$

$$y' + P(x)y = Q(x)y^n$$

$P(x) = -\frac{4}{x}$ $\quad Q(x) = 2x$ $\quad n = \frac{1}{2}$

$$\mu = e^{\int (1-n) P(x)\, dx}$$

$$= e^{\int (1-\frac{1}{2})(-\frac{4}{x})\, dx}$$

$$= e^{\frac{1}{2}\int -\frac{4}{x}\, dx} = e^{-2\int \frac{dx}{x}}$$

$$= e^{-2\ln x} = x^{-2}$$

$$\mu y^{1-n} = \int \mu(1-n) Q(x)\, dx + C$$

$$x^{-2} y^{1-\frac{1}{2}} = \int x^{-2}\left(1-\frac{1}{2}\right)(2x)\, dx + C$$

$$x^{-2} y^{\frac{1}{2}} = \frac{1}{2}(2) \int x^{-1} dx + C$$

$$\frac{y^{\frac{1}{2}}}{x^2} = \ln|x| + C$$

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9) $(e^y + y\cos x)\, dx + (xe^y + \sin x)\, dy = 0$ Exact

$M\, dx + N\, dy = 0$

$M = e^y + y\cos x \qquad N = xe^y + \sin x$

$\frac{\partial M}{\partial y} = e^y + \cos x = \frac{\partial N}{\partial x} = e^y + \cos x$

Exact

$$\int_0^x e^y + y\cos x\, dx + \int_0^y 0\, dy = C$$

$$xe^y + y\sin x \Big]_0^x = C$$

$$xe^y + y\sin x = C$$

10) $x(x^2+2y^2)dx + y(2x^2+y^2)dy = 0$

$M = x^3 + 2y^2x \qquad N = 2x^2y + y^3$

$\frac{\partial M}{\partial y} = 4yx = \frac{\partial N}{\partial x} = 4xy$ Exact.

$\int_0^x (x^3 + 2y^2x)\,dx + \int_0^y y^3\,dy = C$

$\frac{x^4}{4} + y^2x^2 + \frac{y^4}{4} = C$

11) $(x+y)dx + (x+y^2)dy = 0$

$M\,dx + N\,dy = 0$

$M = x+y \qquad N = x+y^2$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x} = 1$ . Exact.

$\int_0^x (x+y)\,dx + \int_0^y y^2\,dy = C$

$\frac{x^2}{2} + yx + \frac{y^3}{3} = C$

12) $(x^2+2x)\,dx - x\,dy = 0$

$M = x^2+2x \qquad N = -x$

$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = -1$

$$f(x) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = \frac{0+1}{-x} = -\frac{1}{x}$$

$$\mu(x) = e^{-\int \frac{dx}{x}} = e^{-\ln x} = e^{\ln x^{-1}} = \frac{1}{x}$$

$$\frac{1}{x}(x^2+2x)\,dx - \frac{x}{x}\,dy = 0$$

$$(x+2)\,dx - 1\,dy = 0$$

$M = x+2 \qquad N = -1$

$\frac{\partial M}{\partial y} = 0 \qquad \frac{\partial N}{\partial x} = 0$ Exact.

$$\int_0^x (x+2)\,dx - \int_0^y dy = C$$

$$\frac{x^2}{2} + 2x - y = C$$

12) $(x^2+2x)\,dx - x\,dy = 0$ Separable

$(x^2+2x)\,dx = x\,dy$

$\int (x+2)\,dx = \int dy$

$\frac{x^2}{2} + 2x = y + C$

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13) $2y^2(x+y^2)\,dx + xy(x+6y^2)\,dy = 0$

$M = 2y^2(x+y^2) \qquad N = xy(x+6y^2)$

$\frac{\partial M}{\partial y} = 4y(x+y^2) + 2y^2(2y)$

$\frac{\partial N}{\partial x} = y(x+6y^2) + xy(1)$

$f(y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{4yx + 4y^3 + 4y^3 - yx - 6y^3 - 2x}{4}$

$= \frac{+2xy + 2y^3}{2y^2(x+y^2)} = \frac{2y(x+y^2)}{2y^2(x+y^2)} = \frac{1}{y}$

$$\mu(y) = e^{-\int \frac{dy}{y}} = e^{-\ln y} = y^{-1} = \frac{1}{y}$$

$$2y(x+y^2)\,dx + x(x+6y^2)\,dy = 0$$

$$M = 2y(x+y^2) \qquad N = x(x+6y^2)$$

$$\frac{\partial M}{\partial y} = 2(x+y^2) + 4y^2 \qquad \frac{\partial N}{\partial x} = 2x + 6y^2$$

$$= 2x + 6y^2 = \frac{\partial N}{\partial x} \quad \therefore \text{exact.}$$

$$\int_0^x 2y(x+y^2)\,dx + \int_0^y 0\,dy = C$$

$$2y\left(\frac{x^2}{2} + y^2 x\right) = C$$

# Assign 3

1) $yy'' = y'^2$ x absar

$y' = z \quad y'' = z\frac{dz}{dy}$

$y z \frac{dz}{dy} = z^2$

$z\left[y\frac{dz}{dy} - z\right) = 0$

$z = 0$ | $y\frac{dz}{dy} - z = 0$

$\frac{dy}{dx} = 0$

$y = c$

$\int \frac{dz}{z} = \int \frac{dy}{y}$

$\ln|z| = \ln|y| + \ln|C|$

$z = yC$

$\frac{dy}{dx} = yC$

$\int \frac{dy}{y} = \int C\,dx$

$\ln|y| = Cx + C_1$

2) $xy'' - y' = 0$ $\quad y' = z$, $\quad y'' = \frac{dz}{dx}$

$$x\frac{dz}{dx} - z = 0$$

$$\int \frac{dz}{z} = \int \frac{dx}{x}$$

$$\ln|z| = \ln|x| + \ln|C|$$

$$\ln|z| = \ln|xC|$$

$$z = xC$$

$$\frac{dy}{dx} = xC$$

$$\int dy = \int xC\,dx$$

$$y = C\frac{x^2}{2} + C_1$$

3) $y'' + 2x(1+y')^2 = 0$

y absent

$y' = z \qquad y'' = \frac{dz}{dx}$

$\frac{dz}{dx} + 2x(1+z)^2 = 0$

$\int \frac{dz}{(1+z)^2} = -\int 2x\,dx$

$-\frac{1}{1+z} = -x^2 + C$

$1+z = \frac{1}{x^2+C}$

$z = \frac{1}{x^2+C^2} - 1$

$\frac{dy}{dx} = \frac{1}{x^2+C^2} - 1$

$\int dy = \int \frac{dx}{x^2+C^2} - \int dx$

$y = \frac{1}{C}\tan^{-1}\frac{x}{C} - x + C_1$

4) $yy'' = y'^2 - y^3$.

$y' = z$

$y'' = z\frac{dz}{dy}$

$yz\frac{dz}{dy} = z^2 - z^3$.

$y\frac{dz}{dy} = z - z^2$.

$\frac{dz}{z - z^2} = \frac{dy}{y}$

$\int \frac{dz}{z^2 - z} = -\int \frac{dy}{y}$

$z = 0$

$\frac{dy}{dx} = 0$

$y = C$

$\int \frac{dz}{z(z-1)} = -\ln|y| + \ln C$

$\int -\frac{1}{z} + \frac{1}{z-1}\,dz = -\ln|y| + \ln|C|$

$-\ln|z| + \ln|z-1| = \ln\frac{C}{y}$.

$\ln\frac{z-1}{z} = \ln\frac{C}{y}$

$\frac{z-1}{z} = \frac{C}{y}$

$1 - \frac{dx}{dy} = \frac{C}{y}$

$\frac{dx}{dy} = 1 - \frac{C}{y}$

$dx = \left(1 - \frac{C}{y}\right)dy$

$x = y - C\ln|y| + C_1$

5) $y'' = 1 + y'^2$ , $y(0) = 0$ , $y'(0) = 0$

6)

yardımcı

$y' = z \quad y'' = \frac{dz}{dx}$

$\frac{dz}{dx} = 1 + z^2$

$\int \frac{dz}{1+z^2} = \int dx$

$\tan^{-1} z = x + C$

$z = \tan(x + C)$ $\quad x = 0,\ z = 0,\ C = 0$

$\frac{dy}{dx} = \tan(x + C)$

$\int dy = \int \tan(x + C)\, dx$

$y = \ln|\sec(x + C)| + C_1$

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$y = \ln|\sec x| + C$

$0 = \ln|\sec 0| + C$

$C = 0$

$y = \ln|\sec x|$

6) $2xy'y'' = y'^2 - 1$ yabsent

$y' = z \quad y'' = \frac{dz}{dx}$.

$2xz\frac{dz}{dx} = z^2 - 1$

$\int \frac{2z\,dz}{z^2-1} = \int \frac{dx}{x}$

$\ln|z^2-1| = \ln|x| + \ln|C|$

$\ln|z^2-1| = \ln Cx$

$z^2 - 1 = Cx$

$z^2 = Cx + 1$

$z = \pm\sqrt{Cx+1}$

$\int dy = \pm\int \sqrt{Cx+1}\,dx$.

$y = \pm\frac{2}{3}\frac{(Cx+1)^{\frac{3}{2}}}{C} + C_1$

4) $y'' = xy'^2$ , $y(0) = 1$ , $y'(0) = -2$

y absent

$y' = z \quad y'' = \frac{dz}{dx}$

$\frac{dz}{dx} = xz^2$

$\int \frac{dz}{z^2} = \int x\,dx$

$-\frac{1}{z} = \frac{x^2}{2} + C$

$2 = C$

$-\frac{1}{z} = \frac{x^2}{2} + 2$

$-\frac{dx}{dy} = \frac{x^2}{2} + 2$

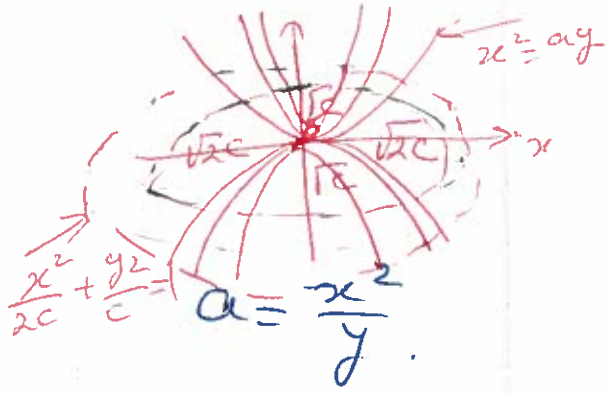
$-\frac{dx}{dy} = \frac{x^2+4}{2}$

$\int \frac{dx}{x^2+4} = -\frac{1}{2}\int dy$

$\tan^{-1}\frac{x}{2} = -\frac{1}{2}y + C$

$0 = -\frac{1}{2} + C \quad C = \frac{1}{2}$

$\tan^{-1}\frac{x}{2} = -\frac{1}{2}y + \frac{1}{2}$




$a = -2$

1) $x^2 = ay$.

$2x = ay'$

$a = \frac{x^2}{y}$.

$2x = \frac{x^2}{y} y'$

$y' \xrightarrow{1} -\frac{1}{y'}$

$2x = \frac{x^2}{y}\left(-\frac{dx}{dy}\right)$

$2 = -\frac{x}{y}\frac{dx}{dy}$.

$\int 2y\,dy = \int -x\,dx$.

$y^2 = -\frac{x^2}{2} + C \Rightarrow \frac{x^2}{2C} + \frac{y^2}{C} = 1$

2) $y = \frac{ax}{1+x}$.

$y' = \frac{a(1+x) - ax}{(1+x)^2} = \frac{a}{(1+x)^2}$

$a = \frac{y(1+x)}{x}$

$y' = \frac{y(1+x)}{x(1+x)^2} = \frac{y}{x(1+x)}$

$$y' = \frac{y}{x(1+x)}$$

$$+ \; y' \longrightarrow -\frac{1}{y'}$$

$$-\frac{dx}{dy} = \frac{y}{x(1+x)}$$

$$\int x(1+x)\,dx = -\int y\,dy$$

$$\frac{x^2}{2} + \frac{x^3}{3} = -\frac{y^2}{2} + C$$

---

3) $2x^2 + y^2 = 4ax$. $\qquad 4a = 2x + \frac{y^2}{x}$

$$4x + 2yy' = 4a.$$

$$4x + 2yy' = 2x + \frac{y^2}{x}$$

$$2x + 2yy' = \frac{y^2}{x}$$

$$\frac{2x}{2y} + y' = \frac{y^2}{x(2y)}$$

$$\frac{x}{y} + y' = \frac{y}{2x}.$$

$$y' = \frac{y}{2x} - \frac{x}{y}.$$

$$\perp \; y' \longrightarrow -\frac{1}{y'}$$

$$-\frac{1}{y'} = \frac{y}{2x} - \frac{x}{y}.$$

$$-\frac{dx}{dy} = \frac{y}{2x} - \frac{x}{y}.$$

$$\frac{dx}{dy} - \frac{x}{y} = -\frac{y}{2x}$$

$$\frac{dx}{dy} + P(y)\,x = Q(y)\,x^n$$

$2x + \frac{y^2}{x}$

$$P(y) = -\frac{1}{y} \qquad Q(y) = -\frac{y}{2} \qquad n = -1$$

$$\mu = e^{\int (1-n) P(y)\, dy} = e^{\int (1+1)(-\frac{1}{y})\, dy} = e^{-\frac{y^2}{2}}$$

$$\mu x^{1-n} = \int \mu (1-n) Q(y)\, dy + C$$

$$e^{-\frac{y^2}{2}} x^{1+1} = \int e^{-\frac{y^2}{2}} (1+1)\left(-\frac{y}{2}\right) dy + C$$

$$x^2 e^{-\frac{y^2}{2}} = \int -y\, e^{-\frac{y^2}{2}}\, dy + C$$

$$x e^{-\frac{y^2}{2}} = e^{-\frac{y^2}{2}} + C$$

$$x = 1 + C e^{\frac{y^2}{2}}$$

4) $e^x \cos y = a$

$e^x \cos y - e^x \operatorname{sen} y \, y' = 0$

$\operatorname{sen} y \, y' - \cos y = 0$

$y' = + \cot y$

$\perp y' \longrightarrow -\frac{1}{y'}$

$-\frac{1}{y'} = \cot y$

$-\frac{dx}{dy} = \cot y$

$-\int dx = \int \cot y \, dy$

$-x = \ln|\operatorname{sen} y| + C$

5) $y + x + 1 = ae^x$ ae

$y' + 1 = ae^x$

$y + x + 1 = y' + 1$

$y' = y + x$

$+y' \longrightarrow -\frac{1}{y'}$

$-\frac{1}{y'} = y + x$

$-\frac{dx}{dy} = y + x.$

$\frac{dx}{dy} + x = -y.$

$\frac{dx}{dy} + P(y)\,x = Q(y)$ Linear

$P(y) = 1 \qquad Q(y) = -y.$

$\mu = e^{\int P(y)\,dy} = e^{\int 1\,dy} = e^{y}.$

$\mu x = \int \mu Q(y)\,dy + C$

$xe^{y} = -\int ye^{y}\,dy + C$

$xe^{y} = -ye^{y} + e^{y} + C$

$x = -y + 1 + Ce^{-y}$

# Assign 4.

Find the general solution

1) $y'' + 4y = \sec 2x$.

A.E

$m^2 + 4 = 0$

$m = \pm 2i$

$y_{C.F} = A\cos 2x + B\sin 2x$

Variation of Parameter

$$y_{P.I} = -y_1 \int \frac{y_2 f(x)}{w}\,dx + y_2 \int \frac{y_1 f(x)}{w}\,dy$$

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \qquad y_1 = \cos 2x \quad y_2 = \sin 2x$$

$$w = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2\cos^2 2x + 2\sin^2 2x$$

$$= \underline{\underline{2}}$$

$f(x) = \sec 2x$

$$y_{P.I} = -\cos 2x \int \frac{\tan 2x \sec 2x}{2}\, dx + \sin 2x \int \frac{\cos 2x \sec 2x}{2}\, dx.$$

$$y_{P.I} = -\frac{\cos 2x}{2} \int \tan 2x\, dx + \frac{\sin 2x}{2} \int dx.$$

$$y_{P.I} = -\frac{\cos 2x}{2} \ln|\sec 2x| + \frac{x}{2} \sin 2x$$

$$y_{G.S} = y_{C.F} + y_{P.I}$$

$$y_{G.S} = A\cos 2x + B \sin 2x - \frac{\cos 2x}{2} \ln|\sec 2x| + \frac{x}{2} \sin 2x$$

2) $y'' + y = \sec x \tan x$.

A.E $m^2 + 1 = 0$

$m = \pm i$

$y_{C.F} = A\cos x + B \sin x$

Variation of Parameter

$y_1 = \cos x \qquad y_2 = \sin x \qquad f(x) = \sec x \tan x$

$y_1' = -\sin x \qquad y_2' = \cos x$

$$W = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1$$

$$y_{P.I} = -y_1 \int \frac{y_2 f(x)}{W} dx + y_2 \int \frac{y_1 f(x)}{W} dx$$

$$= -\cos x \int \frac{\sin x \sec x \tan x}{1} dx + \sin x \int \frac{\cos x \sec x \tan x}{1} dx$$

$$y_{P.I} = -\cos x \int \frac{\sin^2 x}{\cos^2 x}\, dx + \sin x \int \tan x\, dx$$

$$= -\cos x \int \frac{1-\cos^2 x}{\cos^2 x}\, dx + \sin x \int \tan x\, dx$$

$$= -\cos x \int (\sec^2 x - 1)\, dx$$

$$+ \sin x \int \tan x\, dx$$

$$y_{P.I} = -\cos x \left[\tan x - x\right] + \sin x \ln|\sec x|$$

$$y_{G.S} = A\cos x + B\sin x$$

$$-\cos x \left[\tan x - x\right]$$

$$+ \sin x \ln|\sec x|$$

3) $y'' - 2y' + y = \frac{e^x}{x}$.

A.E

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1, 1$

$y_{C.F} = e^x (Ax + B)$

Variation of Parameter

& operator D

$y_{P.I} = \frac{1}{D^2 - 2D + 1} \frac{e^x}{x}$.

$= \frac{1}{(D-1)^2} \frac{e^x}{x}$

$= \frac{e^x}{[(D+1)-1]^2} \frac{1}{x}$.

$= \frac{e^x}{D^2} \left(\frac{1}{x}\right) = e^x \int \ln x \, dx$.

$\frac{1}{D}\left(\frac{1}{x}\right) = \int \frac{dx}{x} = \ln|x|$

$= e^x \left[x \ln x - \int x \frac{dx}{x}\right]$

$= e^x [x \ln x - x]$

$y_{G.S} = e^x (Ax + B) + e^x [x \ln x - x]$

## 02 Variation of Parameter

$y_1 = xe^x \qquad y_2 = e^x \qquad f(x) = \frac{e^x}{x}$

$y_1' = e^x + xe^x \qquad y_2' = e^x$

$$W = \begin{vmatrix} xe^x & e^x \\ e^x + xe^x & e^x \end{vmatrix}$$

$$= xe^{2x} - e^{2x} - xe^{2x}$$

$$= -e^{2x}$$

$$y_{P.I} = -y_1 \int \frac{y_2 f(x)}{W} dx + y_2 \int \frac{y_1 f(x)}{W} dx$$

$$= -xe^x \int \frac{e^x e^x}{x(-e^{2x})} dx + e^x \int \frac{x e^x e^x}{x(-e^{2x})} dx$$

$$= -xe^x \int -\frac{dx}{x} + e^x \int -dx$$

$$= xe^x \ln|x| - xe^x$$

$$y_{G.S} = e^x(Ax + B) + xe^x \ln|x| - xe^x$$

4) $y'' + 9y = \operatorname{cosec} 3x$.

A.E $\quad m^2 + 9 = 0$

$m = \pm 3i$

$y_{C.F} = A\cos 3x + B \operatorname{sen} 3x$

V.A

$y_1 = \cos 3x \qquad y_2 = \operatorname{sen} 3x$

$y_1' = -3\operatorname{sen} 3x \qquad y_2' = 3\cos 3x.$

$$W = \begin{vmatrix} \cos 3x & \operatorname{sen} 3x \\ -3\operatorname{sen} 3x & 3\cos 3x \end{vmatrix}$$

$= 3.$

$$y_{P.I} = -y_1 \int \frac{y_2 f(x)}{W}\, dx + y_2 \int \frac{y_1 f(x)}{W}\, dx$$

$$= -\cos 3x \int \frac{\operatorname{sen} 3x \operatorname{cosec} 3x}{3}\, dx.$$

$$+ \operatorname{sen} 3x \int \frac{\cos 3x \operatorname{cosec} 3x}{3}\, dx$$

$$y_{P.I} = -\frac{\cos 3x}{3}\int dx + \frac{\sin 3x}{3}\int \cot 3x\, dx$$

$$= -\frac{\cos 3x}{3}\, x + \frac{\sin 3x}{3}\cdot\frac{1}{3}\ln|\sin 3x|$$

$$= -\frac{x\cos 3x}{3} + \frac{1}{9}\sin 3x \ln|\sin 3x|$$

$$y_{G.S} = A\cos 3x + B\sin 3x - \frac{x}{3}\cos 3x + \frac{1}{9}\sin 3x \ln|\sin 3x|$$

---

5) $y'' + 2y' + y = \frac{e^{-x}}{x^2}$.

A.E $m^2 + 2m + 1 = 0$

$(m+1)^2 = 0$

$m = -1, -1$

$y_{G.F} = e^{-x}(Ax + B)$

$\frac{y_1 P(x)}{W} dx$

V.P

$y_1 = x e^{-x}$ $\quad y_2 = e^{-x}$

$y_1' = e^{-x} - x e^{-x}$ $\quad y_2' = -e^{-x}$.

$$W = \begin{vmatrix} xe^{-x} & e^{-x} \\ e^{-x} - xe^{-x} & -e^{-x} \end{vmatrix} = -xe^{-2x} - e^{-2x} + xe^{-2x}$$

$$= -e^{-2x}$$

$$W = \begin{vmatrix} e^{-x} - xe^{-x} & -e^{-x} \\ xe^{-x} & e^{-x} \end{vmatrix}$$

$$W = e^{-2x} - xe^{-2x} + xe^{-2x}$$

$$W = e^{-2x}$$

$$y_{P.I} = -y_1 \int \frac{y_2 f(x)}{W} dx + y_2 \int \frac{y_1 f(x)}{W} dx$$

$$= +xe^{-x} \int \frac{e^{-x} e^{-x}}{x^2 e^{-2x}} dx - e^{-x} \int \frac{xe^{-x} e^{-x}}{x^2 e^{-2x}} dx$$

$$= xe^{-x}\left(-\frac{1}{x}\right) - e^{-x} \int \frac{dx}{x}$$

$$= -e^{-x} - e^{-x} \ln|x|$$

$$y_{G.S} = e^{-x}(Ax + B) - e^{-x}(1 + \ln|x|)$$

$e^{-2x}$

$\frac{1}{\omega}\int_L f(x)\,dx$

$\frac{xe^{-x}-e^{-x}}{x^2e^{-2x}}\,dx$

$\frac{dx}{x}$

$+\ln|x|$

# Assign 5

1) $y'' - 7y' + 6y = e^{4x}$

A.E $m^2 - 7m + 6 = 0$

$(m-6)(m-1) = 0$

$m = 1, 6$

$y_{C.F} = Ae^x + Be^{6x}$

$(D^2 - 7D + 6)\,y = e^{4x}$

$y_{P.I} = \frac{1}{D^2 - 7D + 6}\,e^{4x}$

$y_{P.I} = \frac{e^{4x}}{16 - 28 + 6} = -\frac{e^{4x}}{6}$ $\quad D \to 4$

$y_{G.S} = Ae^x + Be^{6x} - \frac{e^{4x}}{6}$

2) $y'' + 4y' + 4y = e^{5x}$

A.E $m^2 + 4m + 4 = 0$

$(m+2)^2 = 0$

$m = -2, -2$

$y_{C.F} = e^{-2x}(Ax + B)$

$(D^2 + 4D + 4)y = e^{5x}$

$y_{P.I} = \frac{1}{D^2 + 4D + 4} e^{5x}$

$y_{P.I} = \frac{e^{5x}}{25 + 20 + 4} = \frac{e^{5x}}{49}$

$y_{G.S} = e^{-2x}(Ax + B) + \frac{e^{5x}}{49}$

3) $y'' + 25y = \cos 3x$

A.E

$m^2 + 25 = 0$

$m = \pm 5i$

$y_{C.F} = A\cos 5x + B\sin 5x$

$y_{P.I} = \frac{1}{D^2 + 25}\cos 3x$

$y_{P.I} = \frac{\cos 3x}{-(3)^2 + 25} = \frac{\cos 3x}{16}$

$y_{G.S} = A\cos 5x + B\sin 5x + \frac{\cos 3x}{16}$

4) $y'' + 16y = \sin 2x$.

A.E

$m^2 + 16 = 0$

$m = \pm 4i$

$y_{C.F} = A\cos 4x + B\sin 4x$

$(D^2 + 16)\, y = \sin 2x$

$y_{P.I} = \frac{1}{D^2 + 16}\sin 2x$

$$y_{PI} = \frac{\sin 2x}{-4+16} = \frac{\sin 2x}{12}$$

$$y_{GS} = A\cos 4x + B\sin 4x + \frac{\sin 2x}{12}$$

5) $y'' + 2y' + 2y = \cos 3x$.

AE $m^2 + 2m + 2 = 0$

$$m = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$m = -1 \pm i$$

$$y_{C.F} = e^{-x}(A\cos x + B\sin x)$$

$(D^2 + 2D + 2)\, y = \cos 3x$.

$$y_{P.I} = \frac{1}{D^2+2D+2}\cos 3x$$

$$y_{P.I} = \frac{1}{-9+2D+2}\cos 3x$$

$$y_{P.I} = \frac{1}{2D-7}\cos 3x$$

$$y_{P.I} = \frac{2D+7}{(2D-7)(2D+7)}\cos 3x$$

Sen 2x
12

$$y_{P.I} = \frac{2D+7}{4D^2-49} \cos 3x$$

$$y_{P.I} = \frac{2D+7}{4(-9)-49} \cos 3x.$$

$$y_{P.I} = \frac{2D+7}{-36-49} \cos 3x$$

$$y_{P.I} = -\frac{1}{85}(2D+7)\cos 3x.$$

$$y_{P.I} = -\frac{1}{85}\left[-6\operatorname{Sen} 3x + 7\cos 3x\right]$$

$$y_{G.S} = e^{-x}(A\cos x + B \operatorname{sen} x)$$

$$+\frac{6}{85}\operatorname{Sen} 3x - \frac{7}{85}\cos 3x$$

6) $y'' - 5y' + 4y = \sinh 2x$

A.E $m^2 - 5m + 4 = 0$

$(m-1)(m-4) = 0$

$m = 1, 4$

$y_{C.F} = Ae^x + Be^{4x}$

$y_{P.I} = \frac{1}{D^2 - 5D + 4} \sinh 2x$

$= \frac{1}{+4 - 5D + 4} \sinh 2x$

$= \frac{1}{8 - 5D} \sinh 2x$

$= \frac{8 + 5D}{(8-5D)(8+5D)} \sinh 2x$

$= \frac{8 + 5D}{64 - 25D^2} \sinh 2x$

$= \frac{8 + 5D}{64 - 25(4)} \sinh 2x$

$= \frac{8 + 5D}{64 - 100} \sinh 2x$

$$y_{P.I} = \frac{8+5D}{-36} \sinh 2x.$$

$$y_{P.I} = -\frac{1}{36}\left[8\sinh 2x + 10\cosh 2x\right]$$

$$y_{G.S} = Ae^{x} + Be^{4x} - \frac{1}{36}\left[8\sinh 2x + 10\cosh 2x\right]$$

---

7) $y'' + y' - 2y = x^3$

A.E $\quad m^2 + m - 2 = 0$

$(m+2)(m-1) = 0$

$m = 1, -2$

$$y_{C.F} = Ae^{x} + Be^{-2x}$$

$$y_{P.I} = \frac{1}{D^2 + D - 2} x^3$$

$$y_{P.I} = -\frac{1}{2}\frac{1}{\left(1 - \frac{D^2+D}{2}\right)} x^3$$

$$= -\frac{1}{2}\left[1 - \frac{D^2+D}{2}\right]^{-1} x^3$$

$$= -\frac{1}{2}\left[1 + \frac{D^2+D}{2} + \frac{(D^2+D)^2}{4}\right] x^3$$

$$y_{P.I} = -\frac{1}{2}\left[1 + \frac{D^2+D}{2} + \frac{1}{4}(D^4+2D^3+D^2)\right]x^3$$

$$= -\frac{1}{2}\left[x^3 + \frac{1}{2}(6x+3x^2) + \frac{1}{4}(2\times 6 + 6x)\right]$$

$$= -\frac{1}{2}\left[x^3 + 3x + \frac{3}{2}x^2 + 3 + \frac{3}{2}x\right]$$

$$= -\frac{1}{2}\left[x^3 + \frac{9}{2}x + \frac{3}{2}x^2 + 3\right]$$

$$y_{G.S} = Ae^x + Be^{-2x} - \frac{1}{2}\left[x^3 + \frac{3}{2}x^2 + \frac{9}{2}x + 3\right]$$

8) $y'' + 4y' + 3y = xe^{-3x}$

A.E $m^2 + 4m + 3 = 0$

$(m+3)(m+1) = 0$

$m = -1, -3$

$y_{C.F} = Ae^{-x} + Be^{-3x}$

$(D^2 + 4D + 3)y = xe^{-3x}$

$$y_{PI} = \frac{1}{D^2+4D+3} e^{-3x} \cdot x$$

$$= \frac{e^{-3x}}{(D-3)^2+4(D-3)+3} x$$

$$= \frac{e^{-3x}}{D^2-6D+9+4D-12+3} x$$

$$= \frac{e^{-3x}}{D^2-2D} x$$

$$= \frac{e^{-3x}}{-2D\left(1-\frac{D}{2}\right)} x.$$

$$= \frac{e^{-3x}}{-2D}\left(1-\frac{D}{2}\right)^{-1} x$$

$$= \frac{e^{-3x}}{-2D}\left[1+\frac{D}{2}\right] x$$

$$= \frac{e^{-3x}}{-2D}\left[x+\frac{1}{2}\right]$$

$$= \frac{e^{-3x}}{-2}\left[\frac{x^2}{2}+\frac{1}{2}x\right]$$

$$y_{G.S} = Ae^{-x} + Be^{-3x} - \frac{e^{-3x}}{2}\left(\frac{x^2}{2}+\frac{1}{2}x\right)$$

9) $y'' + y = \sin^2 x$

A.E $m^2 + 1 = 0$

$m = \pm i$

$y_{C.F} = A\cos x + B\sin x$

$y'' + y = \frac{1}{2} - \frac{1}{2}\cos 2x.$

$(D^2+1)y = \frac{1}{2} - \frac{1}{2}\cos 2x.$

$y_{P.I} = \frac{1}{D^2+1} e^{0x} - \frac{1}{2}\frac{1}{D^2+1}\cos 2x$

$D \to 0$

$y_{P.I} = \frac{1}{2} - \frac{1}{2}\frac{\cos 2x}{-4+1}$

$y_{P.I} = \frac{1}{2} + \frac{1}{6}\cos 2x.$

$y_{G.S} = A\cos x + B\sin x + \frac{1}{2} + \frac{1}{6}\cos 2x$

10) $y'' + 25y = \cos 5x$

A.E

$m^2 + 25 = 0$

$m = \pm 5i$

$y_{C.F} = A\cos 5x + B\sin 5x$

$(D^2 + 25)y = \cos 5x$

$y_{P.I} = \frac{Re}{(D+5i)(D-5i)} e^{i5x} \cdot 1$

$5i \qquad D+5i$

$y_{P.I} = \frac{Re\ e^{i5x}}{10i\ (D+5i-5i)}\ 1$

$= Re\ \frac{(\cos 5x + i\sin 5x)}{10i} \frac{1}{D}\ 1$

$= Re\ \frac{\cos 5x + i\sin 5x}{10i}\ x$

$= Re\left[-i\frac{\cos 5x}{10} + \frac{\sin 5x}{10}\right] x$

$= \frac{x\sin 5x}{10}$

$y_{G.S} = A\cos 5x + B\sin 5x + \frac{x}{10}\sin 5x$

11) $y'' + 4y' + 3y = e^{-3x}$.

A.E $m^2 + 4m + 3 = 0$

$(m+3)(m+1) = 0$

$m = -1, -3$.

$y_{C.F} = Ae^{-x} + Be^{-3x}$

$(D^2 + 4D + 3)y = e^{-3x}$.

$y_{P.I} = \frac{1}{(D+3)(D+1)} e^{-3x} \cdot 1$

$y_{P.I} = \frac{e^{-3x}}{(D-3+3)(D-3+1)} 1.$

$y_{P.I} = \frac{e^{-3x}}{D(D-2)} 1.$

$y_{P.I} = \frac{e^{-3x}}{-2D\left(1 - \frac{D}{2}\right)} 1.$

$y_{P.I} = \frac{e^{-3x}}{-2D} \left(1 - \frac{D}{2}\right)^{-1} 1$

$$y_{P.I} = \frac{e^{-3x}}{-2D} 1 = \frac{e^{-3x}}{-2} x$$

$$y_{G.S} = Ae^{-x} + Be^{-3x} - \frac{e^{-3x}}{2} x$$

OR

$$y_{P.I} = \frac{1}{(D+3)(D+1)} e^{-3x} \cdot 1$$

$D \to D-3$, $-3$

$$= \frac{e^{-3x}}{(D-3+3)(-3+1)} 1$$

$$= \frac{e^{-3x}}{-2} \cdot \frac{1}{D} 1$$

$$= \frac{e^{-3x}}{-2} \int 1 \, dx.$$

$$= -\frac{x e^{-3x}}{2}$$

$$y_{G.S} = Ae^{-x} + Be^{-3x} - \frac{x e^{-3x}}{2}$$

12) $y'' + 16y = \sin 4x$

A.E $m^2 + 16 = 0$

$m = \pm 4i$

$y_{C.F} = A\cos 4x + B\sin 4x$

$(D^2 + 16)y = \sin 4x$

$y_{P.I} = \frac{I}{(D+4i)(D-4i)} e^{i4x} \cdot 1$

$y_{P.I} = \frac{I\, e^{i4x}}{(4i+4i)(D+4i-4i)} 1$

$= \frac{I\, e^{i4x}}{8i\ D} 1.$

$= I \frac{(\cos 4x + i\sin 4x)}{8i} \int 1\, dx$

$= I \frac{(\cos 4x + i\sin 4x)}{8i} x.$

$= I \left(-\frac{i}{8}\cos 4x + \frac{\sin 4x}{8}\right) x$

$$y_{p.I} = -\frac{x \cos 4x}{8}$$

$$y_{G.S} = A\cos 4x + B\sin 4x - \frac{x\cos 4x}{8}$$

$\sin 4x$

$e^{i4x} \cdot 1$

$\frac{}{4c)}\ 1$

$\frac{}{-4x)} \int 1\, dx$

$\frac{}{4x)}\ x.$

$\frac{\sin 4x}{8}\ )\ x$

# Problem 1

1) $\mathcal{L}\{8\} = \frac{8}{s}$

2) $\mathcal{L}\{4e^{5t}\} = \frac{4}{s-5}$

3) $\mathcal{L}\{2t+t^2\} = \frac{2}{s^2} + \frac{2!}{s^3}$

4) $\mathcal{L}\{t\sin 6t\} = -\frac{d}{ds}\left(\frac{6}{s^2+36}\right)$

$= \frac{6(2s)}{(s^2+36)^2} = \frac{12s}{(s^2+36)^2}$

5) $\mathcal{L}\{e^{-5t}(\cosh 2t + \sinh 2t)\}$

$= \frac{s+5}{(s+5)^2-4} + \frac{2}{(s+5)^2-4} = \frac{s+7}{(s+5)^2-4}$

6) $\mathcal{L}\{e^{-t}\sin^2 t\} = \mathcal{L}\left\{\frac{e^{-t}}{2}(1-\cos 2t)\right\}$

$= \frac{1}{2}\left[\frac{1}{s+1} - \frac{(s+1)}{(s+1)^2+4}\right]$

7) $\mathcal{L}\{e^{4t}(t^4+3t^2-2)\}$

$= \frac{4!}{(s-4)^5} + \frac{3.2!}{(s-4)^3} - \frac{2}{(s-4)}$

8) $\mathcal{L}\{t^n e^{-5t}\} = \frac{n!}{(s+5)^{n+1}}$

9) $\lim_{t\to 0} \frac{\sinh t}{t} = \frac{0}{0}$

$\lim_{t\to 0} \frac{\cosh t}{1} = 1$ exist.

$$\mathcal{L}\left\{\frac{\sinh t}{t}\right\} = \int_s^\infty \frac{ds}{s^2-1} = \int_s^\infty \frac{ds}{(s-1)(s+1)}$$

$$= \frac{1}{2}\int_s^\infty \frac{ds}{s-1} - \frac{ds}{s+1}$$

$$= \frac{1}{2} \ln \frac{s-1}{s+1}\Big]_s^\infty$$

$$= \frac{1}{2}\ln 1 - \frac{1}{2}\ln\left|\frac{s-1}{s+1}\right|$$

$$= -\frac{1}{2}\ln\left|\frac{s-1}{s+1}\right|$$

10) $\mathcal{L}\{e^{-4t}\cos 3t\} = \frac{s+4}{(s+4)^2+9}$

11) $\lim_{t\to 0} \frac{e^{-2t} - e^{-3t}}{t} = \frac{0}{0}$

$\lim_{t\to 0} \frac{-2e^{-2t} + 3e^{-3t}}{1} = 1$ exist.

$$\mathcal{L}\left\{\frac{e^{-2t} - e^{-3t}}{t}\right\} = \int_s^\infty \frac{1}{s+2} - \frac{1}{s+3}\, ds$$

$$\ln|s+2| - \ln|s+3| = \ln\frac{s+2}{s+3}\Big]_s^\infty = -\ln\left|\frac{s+2}{s+3}\right|$$

12) $\lim_{t \to 0} \frac{\cos t - \cos 3t}{t} = \frac{0}{0}$

$\lim_{t \to 0} \frac{-\sin t + 3\sin 3t}{1} = 0$ exist.

$$\mathcal{L}\left\{\frac{\cos t - \cos 3t}{t}\right\}$$

$$= \int_s^\infty \frac{s}{s^2+1} - \frac{s}{s^2+9}\, ds$$

$$= \frac{1}{2}\left[\ln|s^2+1| - \ln|s^2+9|\right]_s^\infty$$

$$= \frac{1}{2}\left[\ln\left|\frac{s^2+1}{s^2+9}\right|\right]_s^\infty$$

$$= \frac{1}{2}\ln|1| - \frac{1}{2}\ln\left|\frac{s^2+1}{s^2+9}\right|$$

13) $\mathcal{L}\{e^{-t}(\sin t - \cos t)^2$

$\mathcal{L}\{e^{-t}(\sin^2 t + \cos^2 t - 2\sin t\cos t\}$

$\mathcal{L}\{e^{-t}(1 - \sin 2t) = \frac{1}{s+1} - \frac{2}{(s+1)^2+4}$

14) $\mathcal{L}\{t e^{-2t} \sin 3t\}$.

$= \mathcal{L}\{\sin 3t\} = \frac{3}{s^2+9}$

$\mathcal{L}\{t \sin 3t\} = -\frac{d}{ds}\frac{3}{(s^2+9)}$

$= \frac{3(2s)}{(s^2+9)^2} = \frac{6s}{(s^2+9)^2}$

$\mathcal{L}\{t e^{-2t} \sin 3t\} = \frac{6(s+2)}{[(s+2)^2+9]^2}$

15) $\mathcal{L}\{e^{2t}(t-1)^2$

$= \mathcal{L}\{e^{2t}(t^2-2t+1)\}$.

$= \frac{2!}{(s-2)^3} - \frac{2}{(s-2)^2} + \frac{1}{s-2}$

16) $\mathcal{L}\{t^2 \cos^2 t\} = \mathcal{L}\{\frac{t^2}{2}(1+\cos 2t)\}$

$= \frac{1}{2}\frac{2!}{s^3} + \frac{d^2}{ds^2}\frac{s}{s^2+4}$

$= \frac{1}{s^3} + \frac{d}{ds}\left(\frac{s^2+4-2s^2}{(s^2+4)^2}\right) = \frac{1}{s^3} + \frac{-2s(s^2+4)^2}{(s^2+4)^4}$

$\rightarrow (4-s^2)(2)(s^2+4)(2s)$

# Model Answer

## Revision Sheet

2)

1) $y' \tan x = 2y - 8$

$$\frac{dy}{dx} - \frac{2}{\tan x} y = -8 \cot x$$

3)

Linear

$$\frac{dy}{dx} \underbrace{- 2\cot x}_{P(x)} y = \underbrace{-8 \cot x}_{Q(x)}$$

$$\frac{dy}{dx} + P(x)\, y = Q(x)$$

$P(x) = -2\cot x \qquad Q(x) = -8\cot x$

$$\mu = e^{-2\int \cot x\, dx} = e^{-2 \ln|\sin x|}$$

$$= e^{\ln (\sin x)^{-2}} = \frac{1}{\sin^2 x} = \operatorname{cosec}^2 x$$

$$\mu y = \int \mu Q(x)\, dx + C$$

$$\operatorname{cosec}^2 x\, y = \int \operatorname{cosec}^2 x (-8 \cot x)\, dx + C$$

$$\operatorname{cosec}^2 x\, y = 8 \frac{\cot^2 x}{2} + C$$

$$\operatorname{cosec}^2 x\, y = 4 \cot^2 x + C$$

2) $(1+e^y)y' = 4x$ Separation

$(1+e^y)\frac{dy}{dx} = 4x$

$\int (1+e^y)\,dy = \int 4x\,dx$

$y + e^y = 2x^2 + C$

3) $x^2y' - xy = y^2$ $\div x^2$

$\frac{dy}{dx} - \frac{1}{x}y = \frac{1}{x^2}y^2$ Bernoulli's

$\frac{dy}{dx} + P(x)y = Q(x)y^n$

$P(x) = -\frac{1}{x}$ $Q(x) = \frac{1}{x^2}$ $n = 2$.

$\mu = e^{\int (1-n)P(x)\,dx}$.

$= e^{\int (1-2)(-\frac{1}{x})\,dx} = e^{\ln x} = x$.

$\mu y^{1-n} = \int \mu(1-n)Q(x)\,dx + C$

$xy^{1-2} = \int x(1-2)\frac{1}{x^2}\,dx + C$

$\frac{x}{y} = -\int \frac{dx}{x} + C$

$\frac{x}{y} = -\ln|x| + C$

4) $x\frac{dy}{dx} + y = x^3 y^3 e^{2x}$ $\div x$

$\frac{dy}{dx} + \frac{y}{x} = x^2 y^3 e^{2x}$

Bernoulli's $\frac{dy}{dx} + P(x) = Q(x) y^n$

$P(x) = \frac{1}{x}$ $Q(x) = x^3 e^{2x}$ $n = 3$.

$\mu = e^{\int (1-n) P(x)\, dx}$

$= e^{\int (1-3)(\frac{1}{x})\, dx}$ $= e^{-2\int \frac{dx}{x}}$

$= e^{-2\ln|x|}$ $= e^{\ln x^{-2}}$ $= x^{-2}$

$\mu y^{1-n} = \int \frac{1}{\mu}(1-n) Q(x)\, dx + C$

$x^{-2} y^{1-2} = \int \frac{1}{x^2} x^2 e^{2x} (1-3)\, dx + C$

$\frac{1}{x^2 y} = -2\int e^{2x}\, dx + C$

$\frac{1}{x^2 y} = -e^{2x} + C$.

÷ x

5) $(y^2 - x)\,dy + y\,dx = 0$

$N$ $M$

$M\,dx + N\,dy = 0$

$M = y \qquad N = y^2 - x$

$\frac{\partial M}{\partial y} = 1 \neq \frac{\partial N}{\partial x} = -1$ Not exact.

$n = 3.$

$\int \frac{dx}{x}$

$x^{-2}$

$\ln + C$

$\ln x + C$

$f(y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{1+1}{y} = \frac{2}{y}$

$\mu = e^{-\int f(y)\,dy}$

$= e^{-\int \frac{2}{y}\,dy} = e^{-2\ln y} \quad e^{\ln y^{-2}} = y^{-2}$

$y^{-2} y\,dx + y^{-2}(y^2 - x)\,dy = 0$

$y^{-1}\,dx + (1 - x y^{-2})\,dy = 0$

$M = y^{-1} \qquad N = 1 - x y^{-2}$

$\frac{\partial M}{\partial y} = -y^{-2} = \frac{\partial N}{\partial x} = -y^{-2}$ Exact.

$$\int y^{-1}\,dx = xy^{-1}$$

$$\int (1 - xy^{-2})\,dy = y + xy^{-1}$$

$$\therefore \frac{x}{y} + y = C$$

6) $\left(x \operatorname{sen}\frac{y}{x} - y\cos\frac{y}{x}\right)dx + x\cos\frac{y}{x}\,dy = 0 \quad \frac{1}{x}$

Homogeneous

$$\operatorname{sen}\frac{y}{x} - \frac{y}{x}\cos\frac{y}{x} = -\cos\frac{y}{x}\,\frac{dy}{dx}$$

$$-\frac{dy}{dx} = \frac{\operatorname{sen}\frac{y}{x} - \frac{y}{x}\cos\frac{y}{x}}{\cos\frac{y}{x}}$$

$$\text{let } \frac{y}{x} = v$$

$$\frac{dy}{dx} = -\left(\frac{\operatorname{sen} v - v\cos v}{\cos v}\right) = f(v)$$

$$f(v) = \frac{v\cos v - \sin v}{\cos v} = v - \tan v.$$

$$\int \frac{dv}{f(v) - v} = \ln|x| + C$$

$$\int \frac{dv}{v - \tan v - v} = \ln|x| + C$$

$$-\int \cot v \, dv = \ln|x| + \ln|C|$$

$$-\ln|\sin v| = \ln|Cx|$$

$$-\ln \sin\frac{y}{x} = \ln|Cx|$$

$$\ln\left(\operatorname{cosec}\frac{y}{x}\right) = \ln|Cx|$$

$$\operatorname{cosec}\frac{y}{x} = Cx$$

7/ $(2x-4y+5)y' + (x-2y+3) = 0$

Separation

$[2(x-2y)+5]\frac{dy}{dx} + x-2y+3 = 0$

let $x-2y = z$

$1-2y' = \frac{dz}{dx}$

$2y' = 1 - \frac{dz}{dx}$

$y' = \frac{1}{2}\left(1-\frac{dz}{dx}\right)$

$\frac{1}{2}(2z+5)\left(1-\frac{dz}{dx}\right) + z = 0$

$\left(z+\frac{5}{2}\right)\left(1-\frac{dz}{dx}\right) + z = 0$

$z + \frac{5}{2} - \left(z+\frac{5}{2}\right)\frac{dz}{dx} + z = 0$

$\left(z+\frac{5}{2}\right)\frac{dz}{dx} = 2z + \frac{5}{2}$

$(2z+5)\frac{dz}{dx} = 2z + \frac{5}{2}$

$\frac{2z+5}{2z+\frac{5}{2}}\,dz = dx$

$\int \frac{2z+\frac{5}{2}+\frac{5}{2}}{2z+\frac{5}{2}} dz = \int dx$

$\int 1 + \frac{\frac{5}{2}}{2z+\frac{5}{2}}\,dz = x + C$

8)

$$z + \left(\frac{5}{2}\right)\left(\frac{1}{2}\right) \ln\left|2z + \frac{5}{2}\right| = x + C$$

$$x + 2y + \frac{5}{4} \ln\left|2x + 4y + \frac{5}{2}\right| = x + C$$

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8) $x y' + x^2 + xy - y = 0 \quad \div x$

$$y' + x + y - \frac{y}{x} = 0$$

$$y' + \left(1 - \frac{1}{x}\right) y = -x$$

Linear

$$\frac{dy}{dx} + P(x) y = Q(x)$$

$P(x) = 1 - \frac{1}{x} \qquad Q(x) = -x$

$$\mu = e^{\int P(x)\,dx} = e^{\int \left(1 - \frac{1}{x}\right) dx}$$

$$= e^{x - \ln x} = e^x e^{\ln x^{-1}}$$

$$= x^{-1} e^x = \frac{e^x}{x}$$

$$\mu y = \int \mu Q(x)\,dx + C$$

$$\frac{e^x}{x} y = \int \frac{e^x}{x} (-x)\,dx + C$$

$$\frac{e^x}{x} y = e^x + C$$

9) $y - y' = y^2 + xy'$ separation

$y - y^2 = y'(x+1)$

$y - y^2 = (x+1)\frac{dy}{dx}$

$\int \frac{dy}{y^2 - y} = -\int \frac{dx}{x+1}$

$\int \frac{dy}{y(y-1)} = -\ln|x+1| + C$

$\int -\frac{1}{y} + \frac{1}{y-1}\, dy = -\ln|x+1| + \ln|C|$

$-\ln|y| + \ln|y-1| = \ln C - \ln|x+1|$

$\ln \frac{y-1}{y} = \ln \frac{C}{x+1}$

$\frac{y-1}{y} = \frac{C}{x+1}$

ation

10) $(\sin x + y)\,dy + (y\cos x - 2x)\,dx = 0$

$M = y\cos x - 2x \qquad N = \sin x + y$

$\frac{\partial M}{\partial y} = \cos x = \frac{\partial N}{\partial x} = \cos x$

Exact

$\int (\sin x + y)\,dy = y\sin x + \frac{y^2}{2}$

C

$\int (y\cos x - 2x)\,dx = y\sin x - x^2$

$y\sin x + \frac{y^2}{2} - x^2 = C$

11) $(\cos x - x\sin x)\,y\,dx + (x\cos x - 2y)\,dy = 0$

$M = (\cos x - x\sin x)\,y \qquad N = x\cos x - 2y$

$\frac{\partial M}{\partial y} = \cos x - x\sin x \qquad \frac{\partial N}{\partial x} = \cos x - x\sin x$

Exact.

$\int (\cos x - x\sin x)\,y\,dx = \sin x\,y - y\int x\sin x\,dx$

$= y\sin x - y\left[x\frac{\cos x}{(-1)} - 1\frac{\sin x}{(-1)}\right]$

$= yx\cos x.$

$\int (x\cos x - 2y)\,dy = yx\cos x - y^2$

$\therefore xy\cos x - y^2 = C$

12) $xy' = e^{-xy} - y$

$xy = z$

$y + xy' = \frac{dz}{dx}$

$y' = \frac{1}{x}\left(\frac{dz}{dx} - y\right)$

$\frac{dz}{dx} - y = e^{-z} - y$

$\frac{dz}{dx} = e^{-z}$

$\int e^{z} dz = \int dx$

$e^{z} = x + C$

$e^{xy} = x + C$

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19) $y' = (x + e^{y} - 1)e^{-y}$

$x + e^{y} = z$

$1 + e^{y} y' = \frac{dz}{dx}$

$e^{y} y' = \frac{dz}{dx} - 1$

$y' = e^{-y}\left(\frac{dz}{dx} - 1\right)$

$y' = (z - 1)e^{-y}$

$e^{-y}\left(\frac{dz}{dx} - 1\right) = (z-1)e^{-y}$

$\frac{dz}{dx} - 1 = z - 1$

$\frac{dz}{dx} = z$

$\int \frac{dz}{z} = \int dx$

$\ln|z| = x + C$

$\ln|x + e^{y}| = x + C$

14) $y^2 dx + (xy+1) dy = 0$

$M = y^2 \qquad N = xy + 1$

$\frac{\partial M}{\partial y} = 2y \qquad \frac{\partial N}{\partial x} = y$

$$F(y) = \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{M} = \frac{2y - y}{y^2} = \frac{1}{y}.$$

$$\mu(y) = e^{-\int \frac{dy}{y}} = e^{-\ln|y|} = e^{\ln y^{-1}}$$

$$\mu(y) = \frac{1}{y}$$

$$\frac{1}{y}\left(y^2 dx + (xy+1) dy\right) = 0$$

$$y\,dx + \left(x + \frac{1}{y}\right) dy = 0$$

$M = y \qquad N = x + \frac{1}{y}$

$\frac{\partial M}{\partial y} = 1 = \frac{\partial N}{\partial x} = 1$ Exact.

$\int y\,dx = xy \qquad \int \left(x + \frac{1}{y}\right) dy = xy + \ln|y|$

$xy + \ln|y| = C$

15) $L\frac{di}{dt} + Ri = E$ separarlim

$$L\frac{di}{dt} = E - Ri$$

$$L\int \frac{di}{E-Ri} = \int dt$$

$$-\frac{L}{R}\ln|E-Ri| = t + C$$

for $i=0$ $t=0$

$$-\frac{L}{R}\ln E = C$$

$$-\frac{L}{R}\ln(E-Ri) = t - \frac{L}{R}\ln E$$

$$\frac{L}{R}\left[\ln E - \ln(E-Ri)\right] = t$$

$$\ln\frac{E}{E-Ri} = \frac{Rt}{L}$$

$$\frac{E}{E-Ri} = e^{\frac{Rt}{L}}$$

$$E - Ri = Ee^{-\frac{Rt}{L}}$$

$$i = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

16) $yy'' = y'^2 - y'^3$ · x absent

$y' = z$ · $y'' = z\frac{dz}{dy}$

$yz\frac{dz}{dy} = z^2 - z^3$

$z = 0$ or $y\frac{dz}{dy} = z - z^2$.

$y = c$

$-\int \frac{dz}{z^2 - z} = \int \frac{dy}{y}$

$-\int \frac{dz}{z(z-1)} = \ln|y| + \ln c$.

$-\int \frac{1}{z-1} - \frac{1}{z}\, dz = \ln yc$

$-\ln|z-1| - \ln|z| = \ln yc$

$-\ln\frac{z-1}{z} = \ln yc$.

$\ln\frac{z}{z-1} = \ln yc$.

$yc = \frac{z}{z-1}$ $\qquad \frac{z-1}{z} = \frac{1}{yc}$.

$1 - \frac{1}{z} = \frac{1}{yc}$ $\qquad 1 - \frac{dx}{dy} = \frac{1}{yc}$.

$\frac{dx}{dy} = 1 - \frac{1}{yc} = \frac{yc-1}{yc}$

$$\int dx = \int 1 - \frac{1}{yc}\, dy$$

$$x = y - \frac{1}{c}\ln|y| + C_1 \quad \text{or} \quad y = c.$$

17) $2yy'' = 1 + (y')^2$ — $x$ absent. $y(0) = 2$, $y'(0) = -1$

$y' = z \qquad y'' = z\frac{dz}{dy}$

$$2yz\frac{dz}{dy} = 1 + z^2$$

$$\int \frac{2z}{1+z^2} = \int \frac{dy}{y}$$

$$\ln|1+z^2| = \ln|y| + \ln|c|$$

$$\ln|1+z^2| = \ln|yc|$$

$1 + z^2 = yc$ — $x = 0$, $y = 2$, $z = -1$

$1 + 1 = 2c$

$c = 1$

$1 + z^2 = y \qquad z^2 = y - 1$

$z = \pm\sqrt{y-1}$

$\frac{dy}{dx} = \pm\sqrt{y-1} \qquad \int \frac{dy}{\sqrt{y-1}} = \pm\int dx$

$2(y-1)^{\frac{1}{2}} = \pm x + C$

$2 = C$

$\therefore$

$2\sqrt{y-1} = \pm x + 2$

18) $y'' = (y')^2 e^{-x}$ $y(0)=1$, $y'(0)=\frac{1}{2}$.

$y$ absent.

$y' = z$ $\quad y'' = \frac{dz}{dx}$.

$\frac{dz}{dx} = z^2 e^{-x}$.

$\int z^{-2} dz = \int e^{-x} dx$.

$-\frac{1}{z} = -e^{-x} - C$

$\frac{1}{z} = e^{-x} + C$ $\quad z = \frac{1}{2}$ $x = 0$ $y = 1$

$2 = 1 + C$

$C = 1$

$\frac{1}{z} = e^{-x} + 1$

$\frac{dx}{dy} = e^{-x} + 1$

$\int \frac{e^x dx}{e^x(e^{-x}+1)} = \int dy$

$\ln|1+e^x| = y + C_1$

$\ln 2 = 1 + C_1$

$C_1 = \ln 2 - 1$

$\therefore \ln|1+e^x| = y + \ln 2 - 1$

19) $y'' + 2x(1+y')^2 = 0$.

$y$ absent.

$y' = z \qquad y'' = \frac{dz}{dx}$

$\frac{dz}{dx} + 2x(1+z)^2 = 0$

$\int \frac{dz}{(1+z)^2} = -\int 2x\,dx$

$-\frac{1}{1+z} = -x^2 + C$

$\frac{1}{1+z} = x^2 + C$

$1+z = \frac{1}{x^2+C}$

$z = \frac{1}{x^2+C} - 1$

$\frac{dy}{dx} = \frac{1}{x^2+C} - 1$

$z \neq \frac{\#+x^2+C}{x^2+C}$

$\frac{dy}{dx} = \frac{-x^2+C+1}{x^2+C}$

$\int dy = \int \frac{1}{x^2+C^2} - 1\,dx$

$y = \frac{1}{C} + \frac{x}{C} - x + C$

20) a $y = (x-c)^2$.

$y' = 2(x-c)$ $\quad x - c = \frac{y'}{2}$.

$y = \left(\frac{y'}{2}\right)^2$.

$(y'^2) = 4y$.

$y' = \pm 2\sqrt{y}$.

$\frac{y'}{1} = \frac{1}{-\frac{1}{y}}$,

$-\frac{1}{y'} = \pm 2\sqrt{y}$.

$-\frac{dx}{dy} = \pm 2\sqrt{y}$.

$\int dx = \pm \int 2\sqrt{y}\, dy$.

$x = \pm \frac{4}{3} y^{\frac{3}{2}} + C$.

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$\frac{dy}{dx} = \frac{1}{x^2 \pm c^2} \cdot 1$

$y = \int \frac{1}{x^2 \pm c^2} \cdot 1 \, dx$

$\pm C$

b) $e^x \cos y = C$

$e^x \cos y - e^x \operatorname{sen} y \, y' = 0$.

$y' = \cot y$.

$+ \quad y' \longrightarrow -\frac{1}{y'}$

$-\frac{1}{y'} = \cot y$.

$-\frac{dx}{dy} = \cot y$

$\int -dx = \int \cot y \, dy$

$-x = \ln|\operatorname{sen} y| + \ln C$.

$-x = \ln C \operatorname{sen} y$.

$C \operatorname{sen} y = e^{-x}$.

$\boxed{e^x \operatorname{sen} y = C}$

c) $y^2 = ax^3$

$2yy' = 3ax^2$.

$\frac{2yy'}{y^2} = \frac{3ax^2}{ax^3}$.

$\frac{2y'}{y} = \frac{3}{x}$

$\frac{2}{y} y' = \frac{3}{x}$ ~~+~~ $y' \longrightarrow -\frac{1}{y_1}$

$-\frac{2}{y} \frac{1}{y_1} = \frac{3}{x}$.

$-\frac{2}{y} \frac{dx}{dy} = \frac{3}{x}$.

$\int -\frac{2}{3} x \, dx = \int y \, dy$.

$-\frac{x^2}{3} = \frac{y^2}{2} + C$