



Cambridge International AS & A Level

CANDIDATE
NAME

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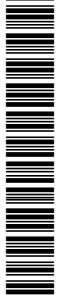
CENTRE
NUMBER

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

1 Solve the equation $4|5^x - 1| = 5^x$, giving your answers correct to 3 decimal places. [4]

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- 2 (a) Express $5 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

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- (b) Hence state the greatest and least possible values of $(5 \sin x - 3 \cos x)^2$. [2]

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3 The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point. [4]

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(b) Determine whether the stationary point is a maximum or a minimum. [2]

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5 (a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$.

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(b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places.

[3]

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- 6 When $(a + bx)\sqrt{1 + 4x}$, where a and b are constants, is expanded in ascending powers of x , the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b .

[6]

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7 (a) Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}. \quad [1]$$

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The variables x and t satisfy the differential equation

$$x \ln x + t \frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t , simplifying your answer. [7]

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(b) Verify by calculation that a lies between 9 and 11. [2]

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(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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9 Two lines l and m have equations $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + s(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ respectively.

(a) Show that l and m are perpendicular. [2]

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(b) Show that l and m intersect and state the position vector of the point of intersection. [5]

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(c) Show that the length of the perpendicular from the origin to the line m is $\frac{1}{3}\sqrt{5}$. [4]

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10 The complex number $1 + 2i$ is denoted by u . The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by $p(x)$. It is given that u is a root of the equation $p(x) = 0$.

(a) Find the values of a and b . [4]

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(b) State a second complex root of this equation. [1]

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(c) Find the real factors of $p(x)$. [2]

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(d) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leq \sqrt{5}$ and $\arg z \leq \frac{1}{4}\pi$. [4]

(ii) Find the least value of $\text{Im } z$ for points in the shaded region. Give your answer in an exact form. [1]

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