Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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2952888595

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

[Turn over

1 Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90° < \theta < 90°$. [4]

2 The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = \mathrm{f}(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

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The point $P\,(5,\ 6)$ lies on the transformed curve $y = \mathrm{f}(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = \mathrm{f}(x)$. [2]

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3 The function f is defined as follows:

$$\mathrm{f}(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of $\mathrm{ff}(5)$. [2]

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(b) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

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4 A curve is such that $\frac{dy}{dx} = \frac{8}{(3x+2)^2}$. The curve passes through the point $\left(2, 5\frac{2}{3}\right)$.

Find the equation of the curve. [4]

5 The first, third and fifth terms of an arithmetic progression are $2\cos x$, $-6\sqrt{3}\sin x$ and $10\cos x$ respectively, where $\frac{1}{2}\pi < x < \pi$.

(a) Find the exact value of x. [3]

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(b) Hence find the exact sum of the first 25 terms of the progression. [3]

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6 The second term of a geometric progression is 54 and the sum to infinity of the progression is 243. The common ratio is greater than $\frac{1}{2}$.

Find the tenth term, giving your answer in exact form. [5]

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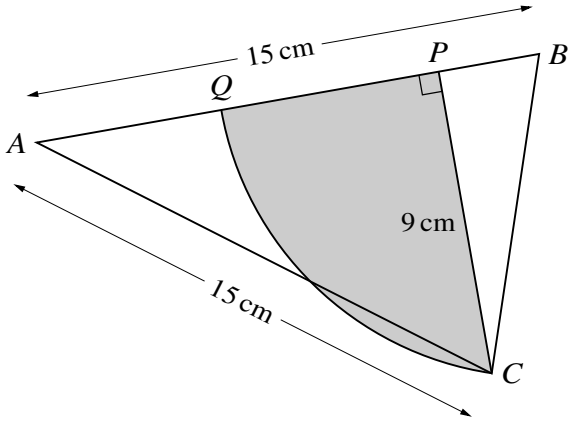

In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length CP = 9 cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle ABC = 1.25 radians, correct to 3 significant figures. [2]

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(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

8 **(a)** It is given that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is -15.

Find the possible values of a. [4]

(b) It is given instead that in the expansion of $(4 + 2x)(2 - ax)^5$, the coefficient of x^2 is k. It is also given that there is only one value of a which leads to this value of k.

Find the values of k and a. [4]

9 The volume $V\,\text{m}^3$ of a large circular mound of iron ore of radius $r\,\text{m}$ is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of $1.5\,\text{m}^3$ per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

10 The function f is defined by $\mathrm{f}(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = \mathrm{f}(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

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(c) Given that this is the only stationary point of the curve, find the range of f. [2]

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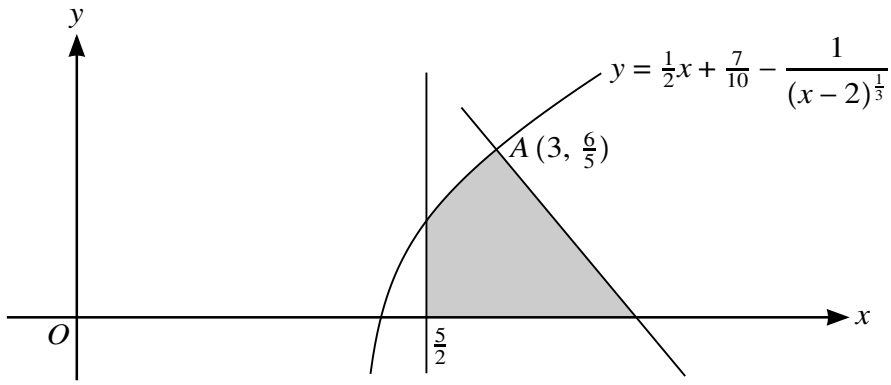
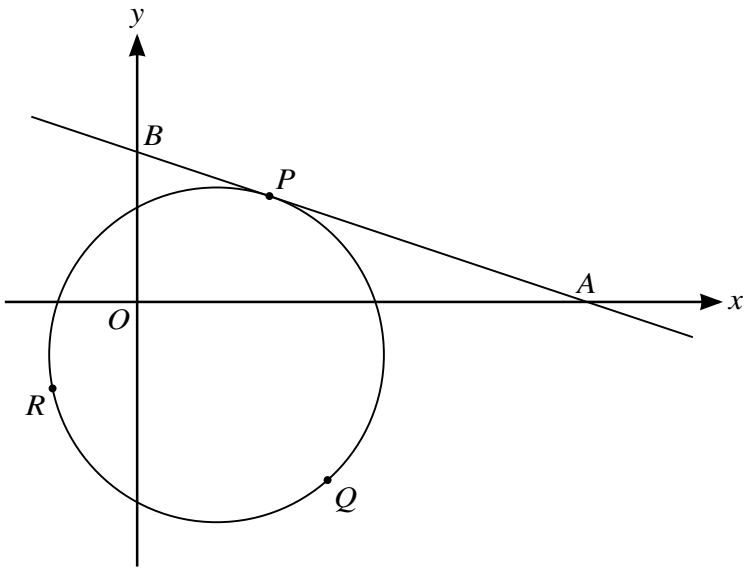


The diagram shows the line $x = \frac{5}{2}$, part of the curve $y = \frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}$ and the normal to the curve at the point $A\left(3, \frac{6}{5}\right)$.

(a) Find the x-coordinate of the point where the normal to the curve meets the x-axis. [5]

(b) Find the area of the shaded region, giving your answer correct to 2 decimal places. [6]

12



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

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(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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