Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2021**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

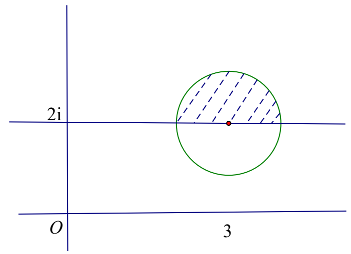
AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product, a quotient or power	*M1	e.g. $\ln(7^x) = x\ln 7$
	Obtain a correct linear equation in any form	A1	e.g. $\ln 3 + (1-x)\ln 2 = x\ln 7$
	Solve a linear equation for x	DM1	
	Obtain answer $x = \dfrac{\ln 6}{\ln 14}$	A1	Maximum 3 out of 4 available if final answer not in required form e.g. 0.67… ISW once correct answer seen.
	Alternative method for Question 1		
	$2^{1-x} = 2\times 2^{-x}$	*M1	OE
	$6 = 2^x 7^x \left[= 14^x\right]$	A1	
	Use law of the logarithm of a power to solve for x	DM1	Must be a linear power. Allow $x = \ln_{14}(6)$.
	Obtain answer $x = \dfrac{\ln 6}{\ln 14}$	A1	ISW once correct answer seen.
		4	

Question	Answer	Marks	Guidance
2	State or imply non-modular inequality $(3x-a)^2 > 2^2(x+2a)^2$, or corresponding quadratic equation, or pair of linear equations or linear inequalities	**B1**	Need 2^2 seen or implied.
	Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for x in terms of a	**M1**	$(5x^2 - 22ax - 15a^2 = 0)$
	Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others	**A1**	OE Accept incorrect inequalities with correct critical values. Must state 2 values i.e. $\frac{a \pm b}{c}$ is not sufficient.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**A1**	Do not condone $\geqslant$ for > or $\leqslant$ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **A0**, 'and' is **A0**.
	Alternative method for Question 2		
	Obtain critical value $x = 5a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}a$ similarly	**B2**	Maximum 2 marks if more than 2 critical values.
	State final answer $x > 5a$, $x < -\frac{3}{5}a$	**B1**	Do not condone $\geq$ for > or $\leq$ for < in the final answer. $5a < x < -\frac{3}{5}a$ is **B0**, 'and' is **B0**.
		4	

Question	Answer	Marks	Guidance
3(a)	Substitute for u and w and state correct conjugate of one side	B1	
	Express the other side without conjugates and confirm $(u+w)^* = u^* + w^*$	B1	**Given answer.** Needs explicit reference to conjugate of both sides.
		2	
3(b)	Substitute and remove conjugates to obtain a correct equation in x and y	B1	e.g. $x+2-(y+1)\mathrm{i}+(2+\mathrm{i})(x+\mathrm{i}y)=0$
	Use $\mathrm{i}^2=-1$ and equate real and imaginary parts to zero	M1	
	Obtain two correct equations in x and y	A1	e.g. $3x-y+2=0$ and $x+y-1=0$. Allow $x\mathrm{i}+y\mathrm{i}-\mathrm{i}=0$.
	Solve and obtain answer $z=-\frac{1}{4}+\frac{5}{4}\mathrm{i}$	A1	Allow for real and imaginary parts stated separately.
		4	

Question	Answer	Marks	Guidance
4	State or imply the form $A + \frac{B}{2x-1} + \frac{C}{x-3}$	**B1**	$\frac{Dx+E}{2x-1} + \frac{F}{x-3}$ and $\frac{P}{2x-1} + \frac{Qx+R}{x-3}$ are also valid.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -3$ and $C = 2$	**A1**	Allow maximum **M1A1** for one or more 'correct' values after **B0**.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
	Alternative method for Question 4		
	Divide numerator by denominator	**M1**	
	Obtain $2\left[+\frac{Px+Q}{(2x-1)(x-3)}\right]$	**A1**	$\left[2+\frac{x+7}{(2x-1)(x-3)}\right]$
	State or imply the form $\frac{D}{2x-1} + \frac{E}{x-3}$	**B1**	
	Obtain one of $D = -3$ and $E = 2$	**A1**	
	Obtain a second value	**A1**	
		5	

Question	Answer	Marks	Guidance
5(a)	Show circle with centre 3 + 2i	**B1**	
	Show circle with radius 1. Must match *their* scales: if scales not identical should have an ellipse.	**B1**	
	Show line $y = 2$ in at least the diameter of a circle in the first quadrant	**B1**	
	Shade the correct region in a correct diagram	**B1**	
		4	
5(b)	Identify the correct point	**B1**	
	Carry out a correct method for finding the argument	**M1**	e.g. $\arg x = \tan^{-1}\frac{2}{3} + \sin^{-1}\frac{1}{\sqrt{13}}$ Exact working required.
	Obtain answer $49.8°$	**A1**	Or better. 0.869 radians scores **B1M1A0**.
		3	**Special Case 1: B1M0** for $45°$ if they have shaded the wrong half of the circle. **Special Case 2:** 3 out of 3 available if they identify the correct point on the correct circle and it is consistent with *their* shading.

Question	Answer	Marks	Guidance
6(a)	State correct expansion of $\sin(3x+2x)$ or $\sin(3x-2x)$	**B1**	
	Substitute expansions in $\frac{1}{2}(\sin 5x+\sin x)$, or equivalent	**M1**	
	Simplify and obtain $\frac{1}{2}(\sin 5x+\sin x)=\sin 3x\cos 2x$	**A1**	Obtain the **given identity** correctly.
		3	
6(b)	Obtain integral $-\frac{1}{10}\cos 5x-\frac{1}{2}\cos x$, or equivalent	**B1**	
	Substitute limits correctly in an expression of the form $p\cos 5x+q\cos x$	**M1**	Correct limits and subtracted the right way around. Do not need values of trig functions for M1. Maximum one slip.
	Obtain $\frac{1}{5}(3-\sqrt{2})$	**A1**	Substitute values and obtain the **given answer** following full, correct and exact working.
		3	

Question	Answer	Marks	Guidance
7	Separate variables correctly	B1	$\int \frac{1}{y^2}\,dy = \int 4xe^{-2x}\,dx$
	$\int \frac{1}{y^2}\,dy = -\frac{1}{y}$	B1	OE
	Commence the other integration and reach $axe^{-2x} + b\int e^{-2x}\,dx$	M1	
	Obtain $-2xe^{-2x} + 2\int e^{-2x}\,dx$ or $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$	A1	SOI (might have taken out factor of 4)
	Complete integration and obtain $-2xe^{-2x} - e^{-2x}$	A1	
	Evaluate a constant or use $x = 0$ and $y = 1$ as limits in a solution containing terms of the form $\frac{p}{y}$, qxe^{-2x}, re^{-2x}, or equivalent.	M1	
	Obtain $y = \frac{e^{2x}}{2x+1}$, or equivalent expression for y	A1	ISW
		7	

Question	Answer	Marks	Guidance
8(a)	Expand the square and equate to 1	**B1**	
	Use correct double angle formula	**M1**	Need to see $\frac{4}{2}$ or $\sin 2\theta = 2\sin\theta\cos\theta$ stated.
	Obtain $\cos^4\theta + \sin^4\theta = 1 - \frac{1}{2}\sin^2 2\theta$	**A1**	Obtain the **given result** correctly.
		3	
8(b)	Use the identity and carry out a method for finding a root	**M1**	$\left(1 - \frac{1}{2}\sin^2 2\theta = \frac{5}{9}\right)$
	Obtain answer $35.3°$	**A1**	Must be correct if overspecified: 35.264...
	Obtain a second answer, e.g. $54.7°$	**A1 FT**	$[\text{e.g } 90° - \textit{their } 35.3°]$ Do not FT if mixing degrees and radians.
	Obtain the remaining answers, e.g. $144.7°$ and $125.3°$ and no others in the given interval	**A1 FT**	$[\text{e.g.} 180° - .. \text{ and } 180° - ..]$ Ignore answers outside the given interval. Treat answers in radians as a misread. $(0.615, 0.955, 2.19, 2.53)$ Do not FT if mixing degrees and radians.
		4	

Question	Answer	Marks	Guidance
9(a)	State correct derivative of ye^{2x} with respect to x	B1	$2ye^{2x} + e^{2x}\dfrac{dy}{dx}$
	State correct derivative of y^2e^x with respect to x	B1	$2ye^x\dfrac{dy}{dx} + y^2e^x$
	Equate attempted derivative of the LHS to zero and solve for $\dfrac{dy}{dx}$	M1	
	Obtain $\dfrac{dy}{dx} = \dfrac{2ye^x - y^2}{2y - e^x}$	A1	Obtain the **given answer** correctly. Condone multiplication by $\dfrac{-1}{-1}$ and cancelling of e^x without comment.
	Alternative method for Question 9(a)		
	Rearrange as $y = \dfrac{2}{e^{2x} - ye^x}$ $\Rightarrow \dfrac{d}{dx}\left(e^{2x} - ye^x\right) = 2e^{2x} - ye^x - e^x\dfrac{dy}{dx}$	B1	Other rearrangements are possible e.g. $y = 2e^{-2x} + y^2e^{-x}$ $\dfrac{d}{dx}\left(y^2e^{-x}\right) = 2ye^{-x}\dfrac{dy}{dx} - y^2e^{-x}$
	$\dfrac{dy}{dx} = -\dfrac{2}{\left(e^{2x} - ye^x\right)^2} \times \left(2e^{2x} - ye^x - e^x\dfrac{dy}{dx}\right)$	B1	$\Rightarrow \dfrac{dy}{dx} = -4e^{-x} + 2ye^{-x}\dfrac{dy}{dx} - y^2e^{-x}$
	Solve for $\dfrac{dy}{dx}$	M1	
	Obtain $\dfrac{dy}{dx} = \dfrac{2ye^x - y^2}{2y - e^x}$	A1	Obtain the **given answer** correctly.
		4	

Question	Answer	Marks	Guidance
9(b)	Equate denominator to zero and substitute for y or for e^x in the equation of the curve	*M1	
	Obtain equation of the form $ae^{3x} = b$ or $cy^3 = d$	DM1	$(e^{3x} = 8, \quad y^3 = 1)$ SOI
	Obtain $x = \ln 2$	A1	Accept $\frac{1}{3}\ln 8$ ISW
	Obtain $y = 1$	A1	
		4	

Question	Answer	Marks	Guidance
10(a)	Obtain direction vector $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, or equivalent	B1	Accept answers as column vectors throughout.
	Use a correct method to form a vector equation	M1	
	State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$, or equivalent correct form	A1	e.g. $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ Allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for $\mathbf{r}$.
		3	
10(b)	Use a correct method to find the position vector of C	M1	e.g. $\mathbf{OC} = \mathbf{OA} + \mathbf{AC} = \begin{pmatrix} 1-3 \\ 2+3 \\ -1+6 \end{pmatrix}$
	Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		2	

Question	Answer	Marks	Guidance
10(c)	State $\overrightarrow{OP}$ in component form	B1 FT	
	Form an equation in λ by equating the modulus of OP to $\sqrt{14}$, or equivalent	M1	
	Simplify and obtain $3\lambda^2 - \lambda - 4 = 0$, or equivalent	A1	$3\lambda^2 + \lambda - 4 = 0$ if using $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ in **(a)**. $3\mu^2 + 5\mu - 2 = 0$ if using $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ in **(a)** and *OB*.
	Solve a 3-term quadratic and find a position vector	M1	$\left(\lambda = -1, \frac{4}{3} \text{ or } \lambda = 1, -\frac{4}{3} \text{ or } \mu = \frac{1}{3}, -2 \text{ or } \mu = -\frac{1}{3}, 2\right)$
	Obtain answers $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i} + \frac{10}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$, or equivalent	A1	Accept as coordinates.
		5	

Question	Answer	Marks	Guidance
11(a)	Use chain rule	M1	Allow if not starting with the correct index.
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$
	Use correct Pythagoras to obtain correct derivative in terms of tan x	A1	e.g. $\frac{dy}{dx} = \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$
	Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$	B1	Confirm the **given statement** from correct work. Should see at least $\frac{2}{2} = 1$.
		4	

Question	Answer	Marks	Guidance
11(b)	Equate answer to part (**a**) to 1 and obtain a quartic equation in t or tan x	***M1**	At least as far as $(1+\tan^2 x)^2 = 4\tan x$.
	Obtain correct answer, i.e. $t^4+2t^2-4t+1=0$	**A1**	Or equivalent horizontal form.
	Commence division by $t-1$	**DM1**	As far as $t^3+t^2+\ldots$by long division or inspection. Allow verification by multiplying given answer by $t-1$.
	Obtain the given answer	**A1**	
		4	
11(c)	Use the iterative process correctly with the given formula at least once	**M1**	Obtain one value and use that to obtain the next. Must be working in radians.
	Obtain final answer a = 0.29	**A1**	
	Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295)	**A1**	e.g. 0.3, 0.2854, 0.2894, 0.2883, ... 0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ... 0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ...
		3	