Cambridge Assessment International Education

# Cambridge International AS & A Level

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| CANDIDATE NAME | | | |
| CENTRE NUMBER | | CANDIDATE NUMBER | |

*8279021580*

**MATHEMATICS** **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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**1** The coefficient of $x^3$ in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant $k$. [4]

**2** The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where $p$ is positive.

Find the sum to infinity of the progression. [5]

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where $m$ is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of $m$, the line intersects the curve at two distinct points. [5]

**4** The sum, $S_n$, of the first $n$ terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The $k$th term in the progression is greater than 200.

Find the smallest possible value of $k$. [5]

**5** Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$

$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

**(a)** Find the value of $fg(7)$. [1]

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**(b)** Find the values of $x$ for which $f^{-1}(x) = g^{-1}(x)$. [5]

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**6** **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

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**(b)** Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

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7 The point (4, 7) lies on the curve $y = f(x)$ and it is given that $f'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the $x$-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the $y$-coordinate when $x = 4$. [3]

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(b) Find the equation of the curve. [4]

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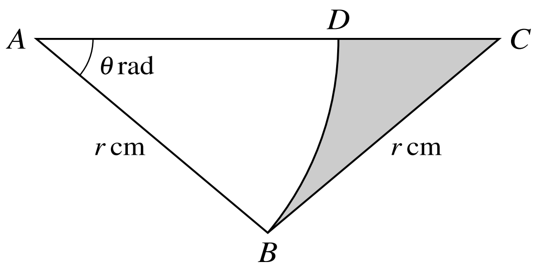


In the diagram, $ABC$ is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point $D$ lies on $AC$ and $ABD$ is a sector of a circle with centre $A$.

**(a)** Express the area of the shaded region in terms of $r$ and $\theta$. [3]

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**(b)** In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

**(a)** Find the equation of the circle. [3]

Point $C$ is such that $AC$ is a diameter of the circle. Point $D$ has coordinates $(5, 16)$.

**(b)** Show that $DC$ is a tangent to the circle. [4]

The other tangent from $D$ to the circle touches the circle at $E$.

**(c)** Find the coordinates of $E$. [2]

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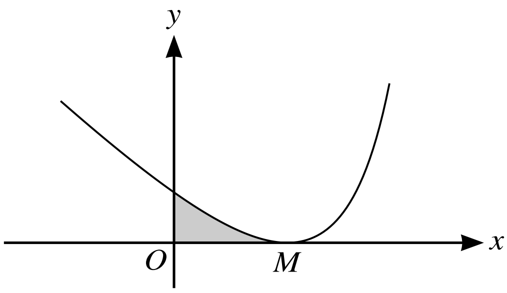


The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point $M$, which lies on the $x$-axis.

**(a)** Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\displaystyle\int y\,dx$. [6]

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**(b)** Find, by calculation, the $x$-coordinate of $M$. [2]

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**(c)** Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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**11** A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

**(a)** State the greatest and least values of $y$. [2]

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**(b)** Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

**(c)** By considering the straight line $y = kx$, where $k$ is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

**(i)** $k = -3$ [1]

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**(ii)** $k = 1$ [1]

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**(iii)** $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

**(d)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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**(e)** Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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**Additional Page**

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