Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/43**

Paper 4 Mechanics **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	For attempt at use of conservation of momentum in one case	**M1**	$0.1\times 4+0=0.4v+0.1v$ or $0.1\times 4+0=0.4v+0.1(-v)$ OE. Must have correct number of terms. Allow sign errors.
	Speed = $0.8\,[\text{m s}^{-1}]$ or $\frac{4}{5}$	**A1**	Must be positive. Allow Max M1A1A0 if *g* included with the masses.
	Speed = $\frac{4}{3}$ $[\text{m s}^{-1}]$ Allow 1.33	**A1**	Must be positive.
		3	

Question	Answer	Marks	Guidance
2	Attempt to use Newton's second law	**M1**	Must have correct number of terms. Allow sign errors. Must use 300 and 1.25, not *T* and *a.*
	Trailer $300-200=m\times 1.25$ or Car $3200-F-300=1500\times 1.25$ System $3200-F-200=(1500+m)\times 1.25$	**A1**	Any 2 equations. Third equation could be with *their m* substituted if found already.
	Solve for *m* or *F*	**M1**	Must get to '*m* =' or '*F* ='. Must have correct number of terms. Allow sign errors. Can be implied by correct answers.
	m = 80 and *F* = 1025	**A1**	
		4	

Question	Answer	Marks	Guidance
3	For attempt to resolve in one direction	**M1**	Must use 0.2 substituted for m if just awarding M1 for vertical equation. Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Allow g missing.
	$X\sin 60 + T\sin 30 - 0.2g = 0$	**A1**	OE. Correct vertical.
	$X\cos 60 - T - T\cos 30 = 0$	**A1**	OE. Correct horizontal. If the two Ts are different, they can get max M1A1A0M0A0, unless they subsequently state that the two Ts are equal.
	For attempt to solve for tension or X	**M1**	Must have correct number of relevant terms in both equations. Must get to '$T =$' or '$X =$'. Allow g missing. Can be implied by correct answers. If no working shown their values must follow from their equations.
	$X = 2$, tension in string = 0.536 [N]	**A1**	Allow exact value of tension = $4 - 2\sqrt{3}$. Allow awrt 2.00 for X.

Question	Answer	Marks	Guidance
3	**Alternative method for Question 3: Resolving parallel and perpendicular to the XN force**		
	For attempt to resolve in one direction, with 0.2 substituted for m	**M1**	Must have correct number of relevant terms (forces must have components as required). Allow sin/cos mix. Allow sign errors. Allow g missing.
	$X - 0.2g\cos 30 - T\cos 60 = 0$	**A1**	OE. Correct parallel to X.
	$T + T\cos 30 - 0.2g\cos 60 = 0$	**A1**	OE. Correct perp to X. If the two Ts are different, they can get max M1A1A0M0A0 unless they subsequently state that the two Ts are equal.
	For attempt to solve for the tension or for X	**M1**	Must have correct number of relevant terms in both equations. Must get to '$T =$' or '$X =$'. Allow g missing. Can be implied by correct answers. If no working shown their values must follow from their equations.
	$X = 2$, Tension in string = 0.536[N] [0.53589…]	**A1**	Allow exact value of tension = $4 - 2\sqrt{3}$. Allow awrt 2.00 for X.
		5	

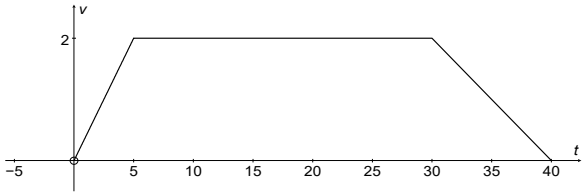
Question	Answer	Marks	Guidance
4(a)	For use of $P = Fv$	**B1**	$P = 20F$ or $P = 25F$ OE (e.g. $F = \frac{P}{20}$ or $F = \frac{P}{25}$). But not with wrong F substituted (e.g. 6000).
	Attempt to use Newton's second law in at least one case	**M1**	Must have 3 terms. Allow sign errors. Allow F.
	$\frac{P}{20} - 6000 = 15000a$ and $\frac{P}{25} - 6000 = 15000\left(\frac{1}{2}a\right)$	**A1**	OE for both. Allow $2a$' and a'. Must be the same P for both.
	For solving simultaneously	**M1**	Dependent on 2 equations of the correct form with the correct number of relevant terms. Must get to '$P =$' or '$a =$', but $P =$ 200 kW or 200 000 W with no attempt at a gets M0. Must be the same P for both.
	Power [= 200 000W] = 200 kW, $a = \frac{4}{15}$ [ms^{-2}]	**A1**	AG. OE awrt 0.267. Do not allow 200 000 [W] as final answer. Must show some working when they find P.
	Alternative Method for Question 4(a): Using two expressions for *P*		
	For use of $P = Fv$	**B1**	$P = 20F$ or $P = 25F$ OE (e.g. $F = \frac{P}{20}$ or $F = \frac{P}{25}$). But not with wrong F substituted (e.g. 6000).
	For one expression for P in terms of a only	**M1**	Allow sign errors. Need 2 term expression.
	$(15000a + 6000) \times 20 = (15000 \times 0.5a + 6000) \times 25$	**A1**	Correct equation.
	For solving for a	**M1**	Must get to '$a =$'.
	Power [= 200 000W] = 200 kW, $a = \frac{4}{15}$ [ms^{-2}]	**A1**	AG. OE awrt 0.267. Do not allow 200 000 [W] as final answer. Must show some working when they find P.

Question	Answer	Marks	Guidance
4(a)	**Alternative Method for Question 4(a): Using the given value of $P = 200$ kW**		
	For use of $P = Fv$	**B1**	e.g. 200 000 = 20F or 200 000 = 25F OE. e.g. $F = \frac{200000}{20}\left[=10000\right]$ or $F = \frac{200000}{25}\left[=8000\right]$.
	Attempt to use Newton's second law in at least one case	**M1**	Must have 3 terms. Allow sign errors. Allow with F. Allow 200 in place of 200 000.
	$\frac{200000}{20} - 6000 = 15000a$ and $\frac{200000}{25} - 6000 = 15000\left(\frac{1}{2}a\right)$	**A1**	For both. Allow 2a' and a' here.
	For solving for a in both cases.	**M1**	
	For showing that both equations lead to $a = \frac{4}{15}$ [m s^{-2}]	**A1**	awrt 0.267.
		5	
4(b)	For attempt at resolving up hill $\frac{200000}{v} - 6000 - 15000g\sin 1 = 0$	**M1**	Or $\frac{200000}{v} - 6000 - 2618 = 0$. May see $\frac{200000}{8618}$. Must have correct number of terms. Allow sin/cos mix. Allow sign errors. Allow g missing, but not a different acceleration. Do not allow F.
	Steady speed = 23.2 [m s^{-1}]	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	For attempt at integration	**M1**	The power of t must increase by 1 with a change of coefficient. Do not penalise missing c. Use of $v = at$ scores M0.
	$v = \frac{2}{3}kt^{\frac{3}{2}}[+c]$	**A1**	Allow unsimplified e.g. $v = \frac{1}{1.5}kt^{\frac{1}{2}+1}[+c]$.
	$1.8 = \frac{2}{3}k \times 9^{\frac{3}{2}} \Rightarrow k = \left[\frac{3}{2} \times 1.8 \div 27 =\right] 0.1$	**B1**	AG. Must show values substituted OE (e.g. $1.8 = 18k$).
		3	

Question	Answer	Marks	Guidance
5(b)	For attempt at integration of either $\int\left(\frac{2}{3}kt^{\frac{3}{2}}\right)\mathrm{d}t$ or $\int\left(0.2(t-9)^2+1.8\right)\mathrm{d}t$ or $\int\left(0.2t^2-3.6t+18\right)\mathrm{d}t$	**M1**	The power of t or $(t-9)$ must increase by 1 with a change of coefficient in at least one term. Use of $s=vt$ is M0.
	$\left[\frac{4}{150}t^{\frac{5}{2}}\right]_0^9$ and $\left[\left(\frac{0.2}{3}(t-9)^3+1.8t\right)\right]_9^{18}$ or $\left[\left(\frac{0.2}{3}t^3-\frac{3.6}{2}t^2+18t\right)\right]_9^{18}$	**A1**	Allow unsimplified. No need for limits. Could include '+c' with either or both.
	$=\frac{4}{150}\times 9^{\frac{5}{2}}\ \left[=6.48\text{ or }\frac{162}{25}\right]$ or $=\frac{0.2}{3}(18-9)^3+1.8\times 18-1.8\times 9=\left[64.8\text{ or }\frac{324}{5}\right]$ or $\left(\frac{0.2}{3}\times 18^3-\frac{3.6}{2}\times 18^2+18\times 18\right)-\left(\frac{0.2}{3}\times 9^3-\frac{3.6}{2}\times 9^2+18\times 9\right)$ or $\left[=\frac{648}{5}-\frac{324}{5}=\frac{324}{5}\right]$	**M1**	Correct use of limits 0, 9 or limits 9, 18. Can be implied by either answer following integration "+c" method is as follows: $s=\frac{0.2}{3}(t-9)^3+1.8t-\frac{243}{25}$, or $s=\frac{0.2}{3}t^3-\frac{3.6}{2}t^2+18t-\frac{1458}{25}$, then substitute t = 18 to get $\frac{1782}{25}$, subtract $\frac{162}{25}$, so distance = $\frac{324}{5}$.
	$6.48=\frac{1}{10}\times 64.8$ or $64.8=10\times 6.48$ or $\frac{324}{5}=10\times\frac{162}{25}$	**A1**	AG OE**.** Check working as can get answer from wrong working NFWW (not from wrong working).

Question	Answer	Marks	Guidance
5(b)	**Alternative Method for Question 5(b): Special Case for those who use a calculator to integrate. Award max 2/4**		
	Either $\int_0^9 \left(\frac{2}{3}kt^{\frac{3}{2}}\right)\mathrm{d}t = 6.48$ or $\frac{162}{25}$ Or $\int_9^{18} \left(0.2(t-9)^2 + 1.8\right)\mathrm{d}t = 64.8$ or $\frac{324}{5}$	**SC B1**	
	$6.48 = \frac{1}{10} \times 64.8$ or $64.8 = 10 \times 6.48$ or $\frac{324}{5} = 10 \times \frac{162}{25}$	**SC B1**	OE.
		4	
5(c)	For differentiation Should get $a = 0.4(t-9)$ or $a = 0.4t - 3.6$	**M1**	The power of t or $(t-9)$ must decrease by 1 with a change of coefficient. M0 for $a = \frac{v}{t}$.
	$0.4\,[\mathrm{ms}^{-2}]$ [at $t = 10$]	**A1**	**SC B1** for 0.4 with no differentiation seen.
	0.3 seen (from the first phase) and state that 0.4 is final answer	**B1**	No working needed. If M1A0 or M0A0 scored, then **SC B1** for 0.3 without mention of the maximum acceleration.
		3	

Question	Answer	Marks	Guidance
6(a)	Greatest speed = $2\,[\text{m s}^{-1}]$ $[0.4\times 5]$	**B1**	This can be seen on the graph and not stated explicitly.
	Trapezium shape	**B1**	Sitting on t-axis, starting at origin.
		**B1**	All correct including height of 2 and t-values of 5, 30, 40 on the horizontal axis. Labels not needed. Does not need to be to scale.
		3	
6(b)	Distance = $\frac{1}{2}(25+5+25+their\,10)\times their\,2$ or $\frac{1}{2}\times 5\times their\,2+25\times their\,2+\frac{1}{2}\times their 10\times their\,2$	**M1**	Allow M1 for finding total area under their trapezium or appropriate 'suvat' in each phase. If presented as 3 areas, they do not need to be added for M1. Allow one wrong value but must represent all 3 phases of motion.
	Distance = 65 [m]	**A1**	
		2	
6(c)	Attempt at Newton's second law	**M1**	Must have correct number of terms (5). Allow sign errors. Allow g missing. Use of $a = g$ is M0A0A0 but condone use of $a = 0.4$ (from wrong phase).
	$12250-1200g-mg=(1200+m)\times(-0.2)$ Or $1200g+mg-12250=(1200+m)\times 0.2$	**A1**	Correct equation. Note that taking $a = 0.2$ and omitting mg gets M0A0A0.
	$m = 50$	**A1**	
		3	

Question	Answer	Marks	Guidance
6(d)	Realise that this is when accelerating and attempt Newton's second law for the crate only	**M1**	Must have correct number of terms (3). Allow sign errors. Allow g missing. Must use $a = \pm 0.4$, M0A0A0 otherwise.
	$R - 50g = 50 \times 0.4$ or $50g - R = 50 \times (-0.4)$	**A1FT**	Correct equation using their 50.
	Force $R = 520$[N], **upwards**	**A1**	Must include 'upwards' OE.
		3	

Question	Answer	Marks	Guidance
7(a)	PE lost = $mgh = 25g \times 1.8 [= 450]$	**B1**	
	For work energy equation	**M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Must use 25, not m. Candidates who try to use constant acceleration can only score B1.
	$25g \times 1.8 - 50 = \frac{1}{2} \times 25v^2$	**A1**	OE. Must be correct.
	$v = 4\sqrt{2}$ [ms^{-1}] or 5.66 [5.6568…]	**A1**	Allow $\sqrt{32}$.
		4	

Question	Answer	Marks	Guidance
7(b)	PE gained/lost = $\pm 25g \times 2 \times 0.28 [= \pm 140]$ or KE gained/lost = $\pm\frac{1}{2} \times 25\left(their\, 4\sqrt{2}\right)^2$ [KE = ± 400]	**B1FT**	For either. FT from *their v* for KE. Must have α substituted for PE. Allow $25g \times 2\sin 16.26°$ or $25g \times 2\sin 16.3°$.
	For work energy equation	***M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Allow sin/cos mix Do not allow with WD instead of $F \times 2$. Must have substituted α and v.
	$F \times 2 = 25g \times 2 \times 0.28 + \frac{1}{2} \times 25\left(4\sqrt{2}\right)^2$ [$\Rightarrow F = 270$]	**A1FT**	FT *their* v^2 or v.
	$R = 25g \times 0.96$ [= 240]	**B1**	Allow $25g\cos 16.26°$ or $25g\cos 16.3°$.
	Use of $F = \mu R$ to form an equation in μ only	**DM1**	Must be from 3 term F, dimensionally correct and single term R. Allow sin/cos mix but must be different components of weight. F and R must be numerical expressions.
	$\mu = \frac{9}{8}$	**A1**	CAO. Allow $1\frac{1}{8}$, but no other answer.
	Alternative method 1 for first 3 marks: Using energy from the initial position (use existing scheme for final 3 marks).		
	PE lost = $\pm 25g \times (1.8 + 2 \times 0.28) [= \pm 590]$	**B1**	Allow $25g \times (1.8 + 2\sin 16.26°)$ or $25g \times (1.8 + 2\sin 16.3°)$.
	For work energy equation	***M1**	Must have correct number of terms. Allow sign errors. Dimensionally correct. Allow sin/cos mix. Do not allow with WD instead of $F \times 2$. Must have substituted α.
	$F \times 2 = 25g \times (1.8 + 2 \times 0.28) - 50$ [$\Rightarrow F = 270$]	**A1**	

Question	Answer	Marks	Guidance
7(b)	**Special Case: Use of constant acceleration. Award max 4/6**		
	$0=\left(4\sqrt{2}\right)^2+2\left(\pm a\right)\times 2$		Use of $v^2=u^2+2as$.
	$a=\pm 8$	**SC B1FT**	FT *their* v^2 or v. Note: 8.01 or 8.0089 from use of 5.66.
	$R=25g\times 0.96$	**SC B1**	Allow $25g\cos 16.26°$ or $25g\cos 16.3°$.
	Use of $F=\mu R$ and attempt at N2L If correct should get $25g\sin 16.3°-\mu\times 25g\cos 16.3°=25\times\left(-8\right)\ \left[\Rightarrow 70-240\mu=-200\right]$	**SC M1**	To form an equation in μ only. Using *their a.* Allow sign errors. Allow sin/cos mix but must be different components of weight. F and R must be numerical expressions. Must have substituted α.
	$\mu=\frac{9}{8}$	**SC A1**	CAO. Allow $1\frac{1}{8}$, but no other answer.
		6	