Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **16** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {[scale] factor 2} {parallel to *x*-axis}	**B2, 1, 0**	B2 for fully correct, B1 with two elements correct.
		4	Transformations can be in either order.

Question	Answer	Marks	Guidance
2	$x^2-6x+c>2$ leading to $(x-3)^2-9+c>2$	M1 A1	M1 for completion of the square with an equation or in equality with the '2'.
	$c>11-(x-3)^2$ and $(x-3)^2 \geq 0$	M1	SOI
	$c>11$	A1	
	Alternative Method 1		
	$\frac{dy}{dx}=2x-6=0$	M1	M1 for differentiating and setting $\frac{dy}{dx}=0$.
	$x=3$	A1	
	When $x=3$, $y=9-18+c$	M1	
	$[-9+c>2]$ $c>11$	A1	
	Alternative Method 2		
	$x^2-6x+c>2$ leading to $x^2-6x+c-2[>0]$ then use of 'b^2-4ac'	M1	
	$36-4(1)(c-2)<0$	M1 A1	OE Must be correct inequality for M1.
	$c>11$	A1	
		4	

Question	Answer	Marks	Guidance
3(a)	$x^5+10x^3+40x+\frac{80}{x}+\frac{80}{x^3}+\frac{32}{x^5}$ or $x^5+10x^3+40x+80x^{-1}+80x^{-3}+32x^{-5}$	**B2, 1, 0**	B2, all terms correct, B1 5 terms correct. Terms must be simplified. Lists of terms allowed.
		2	
3(b)	*their* $40\times a+\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times b=0$	**M1**	Coefficients of a and b must be non-zero, allow x's so long as they are dealt with correctly.
	$\left(\textit{their}\text{ coefficient of }x^{-1}\right)\times a+\left(\textit{their}\text{ coefficient of }x^{-3}\right)\times b=80$	**M1**	Coefficients of a and b must be non-zero, allow x's as long as they are dealt with correctly.
	$a=2 \quad b=-1$	**A1 A1**	Dependent on both M marks, may be seen without working.
		4	

Question	Answer	Marks	Guidance
4(a)	$3\sin^2 x-3\sin^2 x\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\tan^2 x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$.
	$3\left(1-\cos^2 x\right)-3\left(1-\cos^2 x\right)\cos^2 x-4\cos^2 x\ [=0]$	**M1**	Replace $\sin^2 x$ by $1-\cos^2 x$ twice.
	$3\cos^4 x-10\cos^2 x+3=0$ or $-3\cos^4 x+10\cos^2 x-3=0$	**A1**	Or multiple of these equations.
		3	

Question	Answer	Marks	Guidance
4(b)	$(3\cos^2 x - 1)(\cos^2 x - 3)$ $[=0]$	**M1**	OE, using *their* equation in the given form. Allow unusual notation if meaning is clear.
	$\cos x = [\pm]\dfrac{1}{\sqrt{3}}$	**A1**	SOI Answer only **SC B1**.
	54.7°,	**A1**	
	125.3°	**A1 FT**	Only other answer and must be from correct factorisation for A1. FT for $180° - \textit{their}$ first answer . Answers only **SC B1**, **SC B1 FT**.
		4	

Question	Answer	Marks	Guidance
5(a)	$(x-1)^2 + (x-9+4)^2 = 40$	**M1**	Substitute line into circle.
	$x^2 - 6x - 7$ $[=0]$ leading to $(x+1)(x-7)$ $[=0]$	**M1**	Simplify to 3-term quadratic and factorise OE.
	(–1, –10), (7, –2) or $x = -1$ and 7, $y = -10$ and –2	**A1 A1**	Answers only **SC B1**, **SC B1** but must see a correct quadratic equation.
		4	

Question	Answer	Marks	Guidance
5(b)	[C is mid-point =] $\left(\frac{their\ x_1 + their\ x_2}{2}, \frac{their\ y_1 + their\ y_2}{2}\right)$	M1	Expect (3, –6).
	Radius = $\sqrt{(their\ x - their\ 3)^2 + (their\ y - their\ (-6))^2}$ OR $their\sqrt{\left((7-(-1))^2 + (-2-(-10))^2\right)}/2$	M1	Expect $\sqrt{32}$.
	$(x-3)^2 + (y+6)^2 = 32$	A1	OE
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{½r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate θ *or* r using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	ΔOAB = ½ × *their* 16^2 × sin *their* 0.6	M1	Allow Segment = 76.8 –½ × *their* 16^2 × sin *their* 0.6. Expect 72.27 .
	[Area = 76.8 – 72.27 =] 4.53	A1	AWRT
		5	
6(b)	$AB = 2 \times 16 \times \sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 0.6$	M1	Any valid method with *their* r, θ. Expect *AB* = 9.46.
	Perimeter = 9.6 + 9.46 = 19.1	A1	AWRT
		2	

Question	Answer	Marks	Guidance
7(a)	$[y]<2$ OR $[\mathrm{f}(x)]<2$	**B1**	OE e.g. $f<2,(-\infty,2),-\infty<f[x]<2$. Do not accept $x<2$ or $\mathrm{f}(x)\leqslant 2$.
		1	
7(b)	$y=2-\dfrac{5}{x+2}$ leading to $y(x+2)=2(x+2)-5$ leading to $xy+2y=2x-1$	**M1**	or $\dfrac{5}{x+2}=2-y$ (allow sign errors).
	$2y+1=2x-xy$ leading to $2y+1=x(2-y)$	**DM1**	or $\dfrac{5}{2-y}=x+2$ (allow sign errors).
	$x=\dfrac{2y+1}{2-y}\rightarrow \mathrm{f}^{-1}(x)=\dfrac{2x+1}{2-x}$	**A1**	OE or $y=\dfrac{5}{2-x}-2$.
	Domain is $x<2$	**B1 FT**	FT on the numerical part of *their* range from part **(a),** including $x\neq 2$ not penalized. No FT for $x\in\mathcal{R},x=k,x\neq k$.
		4	
7(c)	$\mathrm{fg}(x)=2-\dfrac{5}{x+3+2}$	**B1**	
	$=\dfrac{2(x+5)-5}{x+5}$ or $\dfrac{2(x+5)}{x+5}-\dfrac{5}{x+5}$	**M1**	Use of *their* common denominator.
	$=\dfrac{2x+5}{x+5}$	**A1**	
		3	

Question	Answer	Marks	Guidance
8(a)	$r = \frac{a}{a+2}$	**B1**	OE SOI
	$\frac{a}{1-\frac{a}{a+2}} = 264$	**M1**	Use of S∞ formula.
	$\frac{a(a+2)}{a+2-a} = 264$ leading to $\frac{a(a+2)}{2} = 264$ leading to $a^2 + 2a - 528\ [=0]$	**M1***	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24)\ [=0]$	**DM1**	Attempt to solve.
	$a = 22$ (only)	**A1**	22 without working **SC DB1** (dep on 2nd M1).
		5	
8(b)	$d = \frac{6^2}{6+2} - 6 = -\frac{3}{2}$	**B1**	
	$\frac{n}{2}\left\{12 + (n-1)\left(\frac{-3}{2}\right)\right\}[<] - 480$	**M1***	Forming an inequation with *their* numerical d. May use an equality.
	$[3]\left(n^2 - 9n - 640\right)\ [>0]$	**A1**	OE May contain terms on LHS and RHS.
	$[n =]\ \frac{9 \pm \sqrt{81+2560}}{2}$	**DM1**	OE. Expect 30.19 . Working for solution must be shown.
	31 only	**A1**	Must come from a correct first inequality (or an equality). 31 no working **SC DB1** (dep on correct quadratic and correct inequality/equality).
		5	

Question	Answer	Marks	Guidance
9(a)	$[y=]\ \{x\}\ \{+(x-1)^{-2}\ \}\ \ [+c]$	B1 B1	May be unsimplified.
	Sub x = 0, y = 3 leading to 3 = 0 + 1 + c	M1	Substitution into an integral, expect c = 2.
	$y=x+(x-1)^{-2}+2$ or $f(x)=x+(x-1)^{-2}+2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] f′(0) = 3	B1	
	Equation of tangent is $y-3=$ *their* gradient at $x=0(x-0)$	M1*	Expect $y=3x+3$, normal gets M0.
	Intersection given by $3x+3=x+(x-1)^{-2}+2$	DM1	FT *their* equation from part **(a)**.
	$2x+1=\frac{1}{(x-1)^2}\ \rightarrow\ (2x+1)(x-1)^2-1=0$ or solve equation before given form reached and show solution ($x=3/2$) satisfies given result	A1	WWW AG
		4	
9(c)	Substitute $x=\frac{3}{2}$ leading to $(2x+1)(x-1)^2-1$ leading to $4\times¼-1=0$. Hence $x=\frac{3}{2}$ If shown in **(b)** must be referenced here (in part **(c)**)	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2-1=0$ is acceptable.
	When $x=\frac{3}{2}$ y = 7½	B1	
		2	

Question	Answer	Marks	Guidance
10(a)	$\left[\dfrac{dy}{dx}=\right]\{9\}+\left\{-\dfrac{3}{2}(2x+1)^{1/2}\times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9-3(2x+1)^{1/2}=0$ leading to $2x+1=9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2-3^2(2x+1)=0$ M0A0. $2x+1=\sqrt{3}$ *or* $2x+1=\pm 9$ get M0.
	Max point = (4, 9)	A1	WWW y = 9 must come from original equation.
		4	
10(b)	When $x = 1½$, shows substitution or $\dfrac{dy}{dx}=3$	M1	Substituting $x = 1½$ into their $\dfrac{dy}{dx}$.
	Gradient of AB is $\dfrac{5½-3½}{1½-7½}\left[=\dfrac{-1}{3}\right]$	M1	Substituting into a correct expression for m_{AB}.
	$-\dfrac{1}{3}\text{x}3=-1$. [Hence AB is the normal]	A1	
	Alternative method for Question 10(b)		
	When $x = 1½$ $\dfrac{dy}{dx}=3$,[perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y=\dfrac{-x}{3}+6$ which contains B(7.5,3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

Question	Answer	Marks	Guidance
10(c)	$\left\{\frac{9x^2}{2}\right\}+\left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2}\times 2}\right\}$	B1 B1	Integrating y with respect to x.
	$\left\{\frac{9}{2}7.5^2-\frac{1}{5}(2\times 7.5+1)^{2.5}\right\}-\left\{\frac{9}{2}1.5^2-\frac{1}{5}(2\times 1.5+1)^{2.5}\right\}$ or $\left(\frac{9}{2}\times\frac{225}{4}-\frac{1024}{5}\right)-\left(\frac{81}{8}-\frac{32}{5}\right)$ or $\frac{1933}{40}-\frac{149}{40}$ or $48.325-3.725$	M1	OE Apply limits 1½ to 7½ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2}\left(5\frac{1}{2}+3\frac{1}{2}\right)\times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}}(\frac{-1}{3}x+6)\mathrm{d}x =$ $\left(\frac{-1}{6}\times\left(\frac{15}{2}\right)^2+6\times\frac{15}{2}\right)-\left(\frac{-1}{6}\times\left(\frac{3}{2}\right)^2+6\times\frac{3}{2}\right)$ or $\frac{285}{8}-\frac{69}{8}$ [= 27]	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = 44.6 – 27 =] 17.6	A1	**SC B1** if no substitution of the limits seen.
		5	

Question	Answer	Marks	Guidance
10(c)	**Alternative method for Question 10(c)**		
	$A = \int_{\frac{3}{2}}^{\frac{15}{2}} ((9x - (2x+1)^{\frac{3}{2}}) - \left(\frac{-1}{3}x + 6\right))\mathrm{d}x = \int_{\frac{3}{2}}^{\frac{15}{2}} ((\frac{28}{3}x - (2x+1)^{\frac{3}{2}} - 6)\mathrm{d}x$	**M1**	Finding the equation of AB and subtracting from the equation of the curve.
	$= \left\{\frac{28}{3 \times 2}x^2 - 6x\right\} + \left\{\frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2} \times 2}\right\}$	**A1 A1**	
	$\frac{127}{10} - \frac{-49}{10}$	**M1**	Apply limits 1½ to 7½ to an integral. Working must be seen.
	17.6	**A1**	**SC B1** if no substitution of limits seen.
		5	