



Cambridge International AS & A Level

CANDIDATE
NAME

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 2 (a) Expand $(2 - x^2)^{-2}$ in ascending powers of x , up to and including the term in x^4 , simplifying the coefficients. [4]

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- (b) State the set of values of x for which the expansion is valid. [1]

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3 Solve the equation $2 \cot 2x + 3 \cot x = 5$, for $0^\circ < x < 180^\circ$. [6]

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4 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = \frac{xy}{1+x^2},$$

and $y = 2$ when $x = 0$.

Solve the differential equation, obtaining a simplified expression for y in terms of x . [7]

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A series of 25 horizontal dotted lines for writing.

5 The polynomial $ax^3 - 10x^2 + bx + 8$, where a and b are constants, is denoted by $p(x)$. It is given that $(x - 2)$ is a factor of both $p(x)$ and $p'(x)$.

(a) Find the values of a and b . [5]

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(b) When a and b have these values, factorise $p(x)$ completely.

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6 Let $I = \int_0^3 \frac{27}{(9+x^2)^2} dx.$

(a) Using the substitution $x = 3 \tan \theta$, show that $I = \int_0^{\frac{1}{4}\pi} \cos^2 \theta d\theta.$ [4]

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7 The complex number u is defined by $u = \frac{\sqrt{2} - a\sqrt{2}i}{1 + 2i}$, where a is a positive integer.

(a) Express u in terms of a , in the form $x + iy$, where x and y are real and exact. [3]

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It is now given that $a = 3$.

- (b) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [2]

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- (c) Using your answer to part (b), find the two square roots of u . Give your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [3]

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8 The equation of a curve is $x^3 + y^3 + 2xy + 8 = 0$.

(a) Express $\frac{dy}{dx}$ in terms of x and y . [4]

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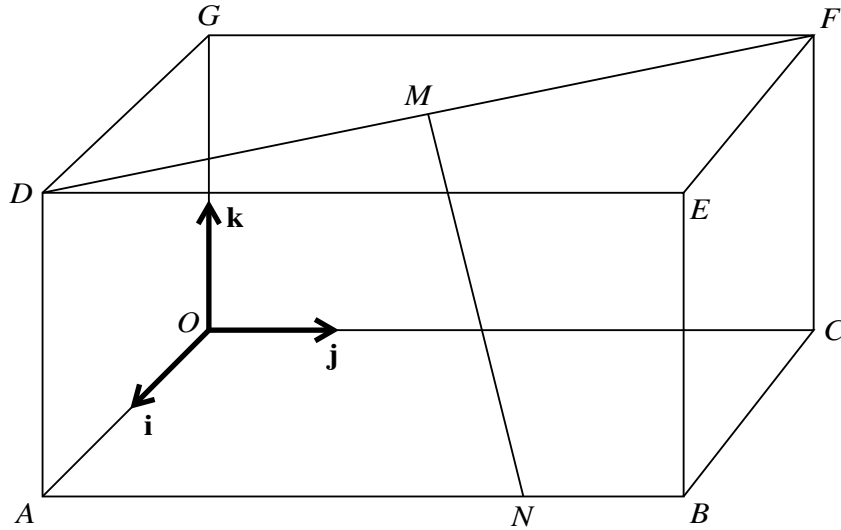
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In the diagram, $OABCDEFG$ is a cuboid in which $OA = 2$ units, $OC = 4$ units and $OG = 2$ units. Unit vectors \mathbf{i} , \mathbf{j} and \mathbf{k} are parallel to OA , OC and OG respectively. The point M is the midpoint of DF . The point N on AB is such that $AN = 3NB$.

- (a) Express the vectors \overrightarrow{OM} and \overrightarrow{MN} in terms of \mathbf{i} , \mathbf{j} and \mathbf{k} . [3]

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- (b) Find a vector equation for the line through M and N . [2]

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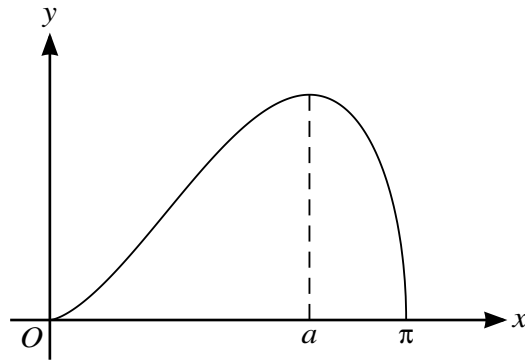
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(c) Show that the length of the perpendicular from O to the line through M and N is $\sqrt{\frac{53}{6}}$. [4]

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The curve $y = x\sqrt{\sin x}$ has one stationary point in the interval $0 < x < \pi$, where $x = a$ (see diagram).

(a) Show that $\tan a = -\frac{1}{2}a$. [4]

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(b) Verify by calculation that a lies between 2 and 2.5. [2]

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(c) Show that if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi - \tan^{-1}(\frac{1}{2}x_n)$ converges, then it converges to a , the root of the equation in part (a). [2]

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(d) Use the iterative formula given in part (c) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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