Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **23** printed pages.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $2^2(3x+a)^2 < (2x+3a)^2$, or corresponding quadratic equation, or pair of linear equations	**B1**	e.g. $(6x+2a)^2 = (2x+3a)^2$ or $32x^2 + 12xa - 5a^2 = 0$ $2(3x + a) = (2x + 3a)$ and $-2(3x + a) = (2x + 3a)$
	Solve 3-term quadratic, or solve two linear equations for x	**M1**	Apply general rules for solving quadratic equation by formula or by factors. Instead of x = {formula}, have {formula} = 0 and try to solve for a then M0
	Obtain critical values $x = \frac{1}{4}a$ and $x = -\frac{5}{8}a$	**A1**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**A1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
	Alternative method for question 1		
	Obtain critical value $x = \frac{1}{4}a$ from a graphical method, or by solving a linear equation or linear inequality	**B1**	
	Obtain critical value $x = -\frac{5}{8}a$ similarly	**B2**	
	State final answer $-\frac{5}{8}a < x < \frac{1}{4}a$ or $-0.625a < x < 0.25a$ or $x > -\frac{5}{8}a$ **and** $x < \frac{1}{4}a$ or $x > -\frac{5}{8}a \cap x < \frac{1}{4}a$	**B1**	Do not condone $\leqslant$ for $<$ in the final answer. Do not ISW. **SC** Set a to value, (say $a = 1$), after initial B1 gained, then $-\frac{5}{8} < x < \frac{1}{4}$ **B1** maximum 2 out of 4.
		4	

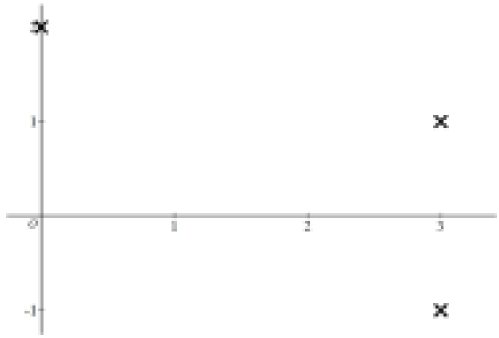
Question	Answer	Marks	Guidance
2	Use correct $\cos(A-B)$ formula to obtain an equation in $\cos\theta$ and $\sin\theta$	B1	$\cos\theta\cos 60 + \sin\theta\sin 60 = 3\sin\theta$
	Use trigonometric formula and substitute values for cos 60 and sin 60 to obtain an equation in $\tan\theta$ (or $\cos\theta$ or $\sin\theta$)	M1	Allow $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ interchanged. $\frac{1}{2} + \frac{\sqrt{3}}{2}\tan\theta = 3\tan\theta$ $\frac{1}{4}\cos^2\theta = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)(1-\cos^2\theta)$ $\frac{1}{4}(1-\sin^2\theta) = \left(3-\frac{\sqrt{3}}{2}\right)\left(3-\frac{\sqrt{3}}{2}\right)\sin^2\theta$
	Obtain $\tan\theta = \frac{1}{6-\sqrt{3}}$ or $\tan\theta = \frac{6+\sqrt{3}}{33}$ or 0.2343, $\cos\theta = \frac{3\frac{\sqrt{3}}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.9736 or $\sin\theta = \frac{\frac{1}{2}}{\sqrt{10-3\sqrt{3}}}$ or 0.2281	A1	OE
	Obtain answer, e.g. $\theta = 13.2°$	A1	May be more accurate, allow value rounding to 13.2°. $\theta = 13.1867°$.
	Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval	A1 FT	May be more accurate. Allow value rounding to 193.2°. FT is on previous value of θ, must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.

Question	Answer	Marks	Guidance
2	**Alternative method for question 2** – using $R\cos(\theta \pm \alpha)$ or equivalent		
	Use correct cos($A - B$) formula to obtain an equation in $\cos\theta$ and $\sin\theta$	**B1**	$\cos\theta \cos 60 + \sin\theta \sin 60 = 3\sin\theta$
	Correct method for finding $\tan\alpha$ from $p\cos\theta + q\sin\theta = 0$	**M1**	$\tan\alpha = \pm\frac{q}{p}$
	Correct value of α	**A1**	76.8° or 1.34 radians (may be more accurate).
	Obtain answer, e.g. $\theta = 13.2°$	**A1**	May be more accurate, allow value rounding to 13.2°. $\theta = 13.1867°$.
	Obtain second answer, e.g. $\theta = 193.2°$ and no others in the given interval	**A1 FT**	May be more accurate. Allow value rounding to 193.2°. FT is on previous value of θ, must have scored M1. Note if θ is negative (e.g. -13.2): $-13.2 + 180 = 166.8$ A0 but $-13.2 + 360 = 346.8$ A1 FT. Ignore answers outside the given interval. Treat answers in radians as a misread. 0.23015, 3.3717.
		5	

Question	Answer	Marks	Guidance
3(a)	Use law of logarithm of a power	M1	$\log_3(2x+1) = 1 + \log_3(x-1)^2$
	Use $\log_3 3 = 1$	B1	$\log_3(2x+1) = \log_3 3 + 2\log_3(x-1)$ $\left[\log 3\left(\dfrac{2x+1}{(x-1)^2}\right) = \log_3 3 \quad \text{or} \quad \left(\dfrac{2x+1}{(x-1)^2}\right) = 3\right]$ **SC** For candidates scoring M0 B0 due to combining logs before dealing with coefficient 2, and confusing coefficients, allow $\log_3(\ldots) = c$ leading to $(...) = 3^c$ **B1**.
	Obtain $3x^2 - 8x + 2 = 0$ or $1.5x^2 - 4x + 1 = 0$	A1	OE 3 terms only and = 0 required.
		3	
3(b)	Solve 3-term quadratic equation from part **3(a)** or restart to find y	M1	$y = \dfrac{4 \pm \sqrt{10}}{6}$ or $y = 1.1937\ldots$ or $y = 0.1396\ldots$ ($x = 2.3874$ or $x = 0.2792$) May solve for x but must find $y = \dfrac{x}{2}$ to gain M1.
	Obtain answer 1.19	A1	CAO. 2 dp required.
		2	

Question	Answer	Marks	Guidance
4(a)	Use correct product rule or quotient rule, and attempt at chain rule	**M1**	$k\mathrm{e}^{-4x}\tan x + \mathrm{e}^{-4x}\sec^2 x$ or $\dfrac{\mathrm{e}^{4x}\sec^2 x - \tan x(k\mathrm{e}^{4x})}{(\mathrm{e}^{4x})^2}$ Need to see $\mathrm{d}(\tan x)/\mathrm{d}x = \sec^2 x$ (formula sheet) and attempt at $k\mathrm{e}^{-4x}$, where $k \neq 1$.
	Obtain correct derivative in any form	**A1**	$-4\mathrm{e}^{-4x}\tan x + \mathrm{e}^{-4x}\sec^2 x$ or $\dfrac{\mathrm{e}^{4x}\sec^2 x - \tan x(4\mathrm{e}^{4x})}{(\mathrm{e}^{4x})^2}$
	Use trigonometric formulae to express derivative in the form $k\mathrm{e}^{-4x}\sin x\cos x\sec^2 x + a\mathrm{e}^{-4x}\sec^2 x$ or $k\mathrm{e}^{-4x}\dfrac{\sin x\cos x}{\cos x\cos x} + a\mathrm{e}^{-4x}\sec^2 x$ or $\sec^2 x(k\mathrm{e}^{-4x}\sin x\cos x + a\mathrm{e}^{-4x})$ Allow $\dfrac{1}{\cos^2 x}$ instead of $\sec^2 x$	**M1**	Need to use $\dfrac{\tan x}{\sec^2 x} = \sin x\cos x$ or $\tan x = \dfrac{\sin x}{\cos x}.\dfrac{\cos x}{\cos x}$ OE. M1 is independent of previous M1, but expression must be of appropriate form.
	Obtain correct answer with $a = 1$ and $b = -2$	**A1**	At least one line of trigonometric working is required from $-4\mathrm{e}^{-4x}\tan x + \mathrm{e}^{-4x}\sec^2 x$ to given answer $\sec^2 x(1 - 2\sin 2x)\,\mathrm{e}^{-4x}$ with elements in any order. If only error: $4\sin x\cos x = 4\sin 2x$ M1 A1 M1 A0.
		4	

Question	Answer	Marks	Guidance
4(b)	Equate derivative to zero and use correct method to solve for x	**M1**	$\sin 2x = \frac{1}{2}$, hence $x = \frac{1}{2}\sin^{-1}\frac{1}{2}$ or $x = \tan^{-1}(2 \pm \sqrt{3})$ Allow M1 for correct method for non-exact value.
	Obtain answer, e.g. $x = \frac{1}{12}\pi$	**A1**	[0.262 M1 A0]
	Obtain second answer, e.g. $\frac{5}{12}\pi$ and no other in the given interval	**A1 FT**	FT $\frac{\pi - their\ 2x}{2}$ if exact values; x must be $< \frac{\pi}{2}$. Ignore answers outside the given interval. Treat answers in degrees as a misread. 15°, 75°. **SC** No values found for a and b in **4(a)** but chooses values in **4(b)**: max **M1** for x.
		3	

Question	Answer	Marks	Guidance
5(a)	Show u and u^* in relatively correct positions. Must have sense of scale on axes	B1	$u = 3 - \mathrm{i}$, $u^* = 3 + \mathrm{i}$ Ignore labels.
	Show $u^* - u$ in a relatively correct position. Must have sense of scale on axes	B1	2i. Scale only on Imaginary axis is sufficient for this mark.
	State that *OABC* is a parallelogram [independent of previous marks]	B1	Ignore 'quadrilateral'. Allow 'trapezium' from correct work.
		3	

Question	Answer	Marks	Guidance
5(b)	Multiply *their* numerator and the given denominator by 3 + i and attempt to evaluate either	M1	Can have missing term and arithmetic errors but need $i^2 = -1$ once, seen or implied.
	Obtain numerator 8 + 6i or denominator 10	A1	
	State final answer $\frac{4}{5}+\frac{3}{5}\mathrm{i}$ or $\frac{8}{10}+\frac{6}{10}\mathrm{i}$ or 0.8 + 0.6i	A1	Correct answer with no working scores 0/3.
	Alternative method for question 5(b)		
	Obtain two equations in x and y, and attempt to solve for x or for y	M1	$3 = 3x + y$ and $1 = -x + 3y$
	Obtain $x = \frac{4}{5}$ or $\frac{8}{10}$ or 0.8 $y = \frac{3}{5}$ or $\frac{6}{10}$ or 0.6	A1	
	State final answer $\frac{4}{5}+\frac{3}{5}\mathrm{i}$ or $\frac{8}{10}+\frac{6}{10}\mathrm{i}$ or 0.8 + 0.6i	A1	Correct answer with no working scores 0/3.
		3	

Question	Answer	Marks	Guidance
5(c)	State or imply $\arg \frac{u^*}{u} = \arg u^* - \arg u$ or $2\arg u^*$	M1	
	Justify the given statement correctly	A1	AG $\arg \frac{u^*}{u} = \tan^{-1}\frac{3}{4}$, $\arg u^* = \tan^{-1}\frac{1}{3}$ and $\arg u = \tan^{-1} -\frac{1}{3}$ (or $\arg u = -\tan^{-1}\frac{1}{3}$), needed if use first expression in M1; or $\arg \frac{u^*}{u} = \tan^{-1}\frac{3}{4}$ and $\arg u^* = \tan^{-1}\frac{1}{3}$, needed if use second expression in M1.
	Alternative method for question 5(c)		
	Use $\tan 2A$ formula with $\tan A = \frac{1}{3}$	M1	$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$, $\tan A = \frac{1}{3}$, hence $\tan 2A = \frac{3}{4}$.
	Justify the given statement correctly	A1	AG So $2A = \tan^{-1}\frac{3}{4} = \arg \frac{u^*}{u}$ and $A = \tan^{-1}\frac{1}{3} = \arg u^*$ hence $\arg \frac{u^*}{u} = 2\arg u^*$.
		2	

Question	Answer	Marks	Guidance
6(a)	Use chain rule at least once	M1	Needs $\frac{dy}{dt}=\frac{1}{\tan t}\frac{d}{dt}(\tan t)$ or $\frac{dx}{dt}=(-1)(\cos^{-2}t)\frac{d}{dt}(\cos t)$. BOD if + and (– 1) (– 1) not seen. $\frac{dx}{dt}$ = sec t tan t (from List of Formulae MF19) M1 A1. If $\frac{dx}{dt}$ = – sec t tan t M1 A0.
	Obtain $\frac{dx}{dt}=\sec t\tan t$	A1	OE e.g. sin t (cos t)$^{-2}$. If e.g. $\frac{dx}{dt}=\sec x\tan x$ or $\sec\theta\tan\theta$ or $\sec t\tan x$, condone recovery on next line.
	Obtain $\frac{dy}{dt}=\frac{\sec^2 t}{\tan t}$	A1	OE e.g. $\frac{1}{\sin t\cos t}$. If e.g. $\frac{dy}{dt}=\frac{\sec^2 x}{\tan x}$ or $\frac{\sec^2\theta}{\tan\theta}$, condone recovery on next line. Only penalise notation errors **once** in $\frac{dx}{dt}$ and $\frac{dy}{dt}$ if no recovery.
	Use $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$	M1	Allow even if previous M0 scored, but must be using derivatives.
	Obtain given answer $\frac{\cos t}{\sin^2 t}$	A1	AG After $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$ used, any notation error A0. Must cancel cos t correctly.
		5	

Question	Answer	Marks	Guidance
6(b)	State or imply $t = \frac{1}{4}\pi$ when $y = 0$	**B1**	
	Form the equation of the tangent at $y = 0$ or find c	**M1**	$x = \sqrt{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{2}$ and $y = 0$, *their* coordinates and gradient used in $y = mx + c$.
	Obtain answer $y = \sqrt{2}x - 2$	**A1**	OE e.g. $y = \sqrt{2}\,(x - \sqrt{2})$ ISW. Allow $y = 1.41x - 2[.00]$ or $1.41(x - 1.41)$.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$	**B1**	If $1 - \frac{A}{x-2} + \frac{Bx+C}{2x^2+3}$ or $\frac{A}{x-2} + \frac{C}{2x^2+3}$ B0 then M1 A1 (for $A = 3$) still possible.
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 3$, $B = -1$ and $C = 6$	**A1**	Allow all A marks obtained even if method would give errors if equations solved in a different order.
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	

Question	Answer	Marks	Guidance
7(b)	Use correct method to find the first two terms of the expansion of $(x-2)^{-1}$, $\left(1-\frac{1}{2}x\right)^{-1}$, $(2x^2+3)^{-1}$ or $\left(1+\frac{2}{3}x^2\right)^{-1}$	**M1**	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions, up to the term in x^2, of each partial fraction	**A1 FT** **A1 FT**	The FT is on A, B and C. $-\frac{A}{2}\left[1-\left(-\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(-\frac{x}{2}\right)^2+\ldots\right]$ $\frac{Bx+C}{3}\left[1-\frac{2x^2}{3}+\ldots\right]$
	Extract the coefficient 3 correctly from $(2x^2+3)^{-1}$ with expansion to $1\pm\frac{2}{3}x^2$ then multiply by $Bx+C$ up to the terms in x^2, where $BC\neq 0$	**M1**	$\frac{C}{3}+\frac{Bx}{3}\pm\frac{C}{3}\left(\frac{2}{3}\right)x^2$ or $\frac{1}{3}\left(C+Bx\pm C\left(\frac{2}{3}\right)x^2\right)$ Allow a slip in multiplication for M1. Allow miscopies in B and C from **7(a)**.
	Obtain final answer $\frac{1}{2}-\frac{13}{12}x-\frac{41}{24}x^2$	**A1**	Do not ISW.
		5	

Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	**B1**	$\frac{\mathrm{d}N}{N^{\frac{3}{2}}} = (k\cos 0.02t)\,\mathrm{d}t$ Allow without integral signs.
	Obtain term $-\frac{2}{\sqrt{N}}$	**B1**	OE Ignore position of k.
	Obtain term $50\sin 0.02t$	**B1**	OE Ignore position of k.
	Use $t = 0$, $N = 100$ to evaluate a constant, or as limits, in a solution containing terms $\frac{a}{\sqrt{N}}$ and $b\sin 0.02t$, where $ab \neq 0$	**M1**	$\left[\text{e.g. } c = -0.2 \quad \text{or} \quad c = \frac{-0.2}{k}\right]$
	Obtain correct solution in any form, e.g. $-\frac{2}{\sqrt{N}} = 50k\sin 0.02t - 0.2$	**A1**	OE ISW e.g. $N = \frac{1}{(25k\sin 0.02t - 0.1)^2}$ $\quad -2N^{-\frac{1}{2}} = \frac{k}{0.02}\sin 0.02t - \frac{1}{5}$ $50k\sin 0.02t = -\frac{2}{\sqrt{N}} + \frac{1}{5}$ $\quad \frac{1}{\sqrt{N}} = -\frac{1}{2}k(50\sin 0.02t) + \frac{1}{10}$ $50\sin\left(\frac{1}{50}t\right) = -\frac{2\sqrt{N}}{kN} + \frac{20}{100k}$
		5	
8(b)	Use the substitution $N = 625$ and $t = 50$ in expression of appropriate form to evaluate k	**M1**	Expression must contain $a + bk\sin 0.02t$, $(\sqrt{N})^{\pm n}$, where $n = -1, 1, 3$ or 5 and a and b are constants $ab \neq 0$ or $(a + bk\sin 0.02t)^{\pm 2}$ and $(N)^{\pm n}$. Allow with k replaced by $\frac{1}{k}$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. If invert term by term when 3 terms shown then M0.
	Obtain $k = 0.00285[2148]$	**A1**	Must evaluate sin1. Degrees $k = 0.138$ M1 A0.
		2	

Question	Answer	Marks	Guidance
8(c)	Rearrange and obtain $N = 4\left(0.2 - 0.142\left(607\right)\sin 0.02t\right)^{-2}$ Substitution for *k* required	M1	Anything of the form $N = c(d - ek\sin 0.02t)^{-2}$, where *c*, *d* and *e* are constants $cde \neq 0$ and value of *k* substituted. Allow with *k* replaced by $1/k$, error due to $k(N^{-3/2})$ when separating variables in **8(a)**. OE ISW e.g. $N = \left(-\dfrac{10}{0.7125\sin 0.02t - 1}\right)^2$ $N = \dfrac{1}{\left(-0.0713\sin 0.02t + 0.1\right)^2}$ $N = \dfrac{100}{\left(\left(\dfrac{0.6}{\sin 1}\right)\sin 0.02t - 1\right)^2}$ $N = \dfrac{1}{\left(\dfrac{3}{-50\sin 1}\times\sin 0.02t + \dfrac{1}{10}\right)^2}$ $N = \left(-\dfrac{0.06}{\sin 1}\sin 0.02t + 0.1\right)^{-2}$ $N = \left(\dfrac{800}{80 - 57\sin 0.02t}\right)^2$ Do not need to substitute for $\sin(0.02t) = 1$, but must substitute for *k*.
	Accept answers between 1209 and 1215	A1	ISW Substitute $\sin 0.02t = 1$ or $t = 50\sin^{-1} 1$ or 78.5 or 25π. Answer with no working (rubric) 0/2. **SC** $N = \ldots$ not seen but correct numerical answer **B1** 1/2.
		2	

Question	Answer	Marks	Guidance
9(a)	Using the correct process find the scalar product of direction vectors of l and **OA**	**M1**	(1, 5, 6)**.**(– 1, 2, 3) = –1.1 + 5.2 + 6.3 = – 1 + 10 + 18
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and find the inverse cosine of the result	**M1**	*Their* scalar product ÷ $[\sqrt{(1^2+5^2+6^2)}\sqrt{((-1)^2+2^2+3^2)}]$. Angle = $\cos^{-1}\dfrac{27}{\sqrt{62}\sqrt{14}}$.
	Obtain answer 23.6°.	**A1**	AWRT 23.6°. 23.5889°. Radians 0.412 scores A0 (0.4117…).
		3	
9(b)	Taking a general point P on l, state **AP** (or **PA**) in component form, e.g. $(3-\lambda, -5+2\lambda, -5+3\lambda)$	**B1**	Note: (4, 1, 0) or (4, 1, 1), for 4**i** + **k** is not MR, but M1 possible.
	Either equate scalar product of **AP** and direction vector of l to zero and solve for λ *or* use Pythagoras in a relevant triangle and solve for λ	**M1**	$(3-\lambda, -5+2\lambda, -5+3\lambda)$**.**$(-1, 2, 3) = 0$ $-3\ -10\ -15+\lambda+4\lambda+9\lambda=0$ or let **OQ** = (4, 0, 1) so **AQ** = (3, – 5, – 5), **QP** = $(-\lambda, 2\lambda, 3\lambda)$, **AP** = $(3-\lambda, -5+2\lambda, -5+3\lambda)$ hence $3^2+(-5)^2+(-5)^2=$ $(3-\lambda)^2+(-5+2\lambda)^2+(-5+3\lambda)^2+(-\lambda)^2+(2\lambda)^2+(3\lambda)^2$ Other alternative approaches are possible, e.g. minimise AP or AP^2, either by completing the square or by differentiating.
	Obtain $\lambda = 2$	**A1**	$\lambda = 2$
	State that the position vector **OP*** of the foot is 2**i** + 4**j** + 7**k**	**A1**	OE Condone coordinates.
		4	

Question	Answer	Marks	Guidance
9(c)	Set up a correct method for finding the position vector of the reflection of A in l	**M1**	For all methods, allow a sign error in one component only: **OA′** = **OP*** + (**OP*** – **OA**) $\textit{their}(2,4,7)+(\textit{their}\,2,4,7-1,5,6)$ or **OA′** = **OP*** – (**OA** – **OP***) $\textit{their}(2,4,7)-(1,5,6-\textit{their}\,2,4,7)$ or **OA′** = **OA** + 2(**OP*** – **OA**) $\begin{pmatrix} 1+2(\textit{their}\,2-1) \\ 5+2(\textit{their}\,4-5) \\ 6+2(\textit{their}\,7-6) \end{pmatrix}$ or midpoint **OP*** = (**OA** + **OA′**)/2 with *their* λ value substituted. $\frac{1+x}{2}=\textit{their}\,2 \quad \frac{5+y}{2}=\textit{their}\,4 \quad \frac{6+z}{2}=\textit{their}\,7$
	Obtain answer $3\mathbf{i}+3\mathbf{j}+8\mathbf{k}$ or $3\left(\mathbf{i}+\mathbf{j}+\frac{8}{3}\right)$	**A1**	OE Condone coordinates $x=3, y=3, z=8$ A1. No method shown and correct answer 2/2.
		2	

Question	Answer	Marks	Guidance
10(a)	Commence integration and reach $ax^3 \ln x + b\int x^3 \cdot \frac{1}{x}\, dx$	*M1	OE Allow omission of dx.
	Obtain $\frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^3 \cdot \frac{1}{x}\, dx$	A1	OE Allow omission of dx.
	Complete integration and obtain $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3$	A1	Allow $-\frac{1}{3}\left(\frac{1}{3}x^3\right)$.
	Use limits correctly and equate to 4, having integrated twice	DM1	$\frac{1}{3}a^3 \ln a - \frac{1}{9}a^3 - (0 - \frac{1}{9}) = 4$ allow one sign error OR one numerical error, but 0 may be absent or expressed as $\frac{a^3}{3}\ln 1$. Allow $-\frac{1}{3}\left(\frac{1}{3}ax^3\right)$ and $-\frac{1}{3}\left(\frac{1}{3}\right)$.
	Obtain given result correctly	A1	$a = \left(\frac{35}{3\ln a - 1}\right)^{\frac{1}{3}}$ AG After substitution, any errors even if corrected A0. Need to see at least one line of working between substitution and the given answer.
		5	
10(b)	Calculate the values of a relevant expression or pair of expressions at $a = 2.4$ and $a = 2.8$ All values must be correct for M1 (numerical question)	M1	
	Justify the given statement with correct calculated values	A1	$2.4 < 2.7(8)$ and $2.8 > 2.5(6)$ sign change here insufficient OR $-0.3(8)$ and $0.2(4)$ $< 0, > 0$ or change of sign.
		2	

Question	Answer	Marks	Guidance
10(c)	Use the iterative process $a_{n+1} = \left(\frac{35}{3\ln a_n - 1}\right)^{\frac{1}{3}}$ correctly at least twice	**M1**	
	Obtain final answer $a = 2.64$	**A1**	Must be 2 dp.
	Show sufficient iterations to 4 dp to justify 2.64 to 2 dp, or show there is a sign change in (2.635, 2.645)	**A1**	2.635 $(35/(3\ln a - 1))^{1/3} - a = 0.0029(4) > 0$ 2.645 $(35/(3\ln a - 1))^{1/3} - a = -0.012 < 0$
		3	