Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **May/June 2022**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Use law of the logarithm of a product or a quotient or a power	*M1	
	Obtain a correct linear equation in any form	A1	e.g. $\ln 2 + (2x-1)\ln 3 = (x+1)\ln 4$ or $\log_2 2 + (2x-1)\log_2 3 = (2x+2)\log_2 2$
	Solve for x	DM1	Allow for unsimplified expression $x = \ldots$ Allow M1 M1 for $x = 1.45$ from $6^{2x-1} = 4^{x+1}$.
	Obtain answer $x = 2.21$	A1	The question asks for 2 dp.
	Alternative method for question 1		
	Correct use of indices to obtain $2.25^x = 6$ or $1.5^{2x} = 6$	M1 A1	
	Correct use of logarithms to solve for x	M1	Allow solution of $2.25^x = 6$ by trial and improvement as far as 2.2…
	Obtain answer $x = 2.21$	A1	Need to see an intermediate step / sequence of iterations.
		4	

Question	Answer	Marks	Guidance
2(a)	State a correct unsimplified version of the x^2 or the x^4 term of the expansion of $(2-x^2)^{-2}$ or $\left(1-\frac{1}{2}x^2\right)^{-2}$	**M1**	$\frac{1}{4}\left(1+2\frac{x^2}{2}+\frac{-2.-3}{2}\left(\frac{x^2}{2}\right)^2 ...\right)$ Symbolic binomial coefficients are not sufficient for the M1.
	State correct first term $\frac{1}{4}$	**B1**	Accept 2^{-2}.
	Obtain the next two terms $\frac{1}{4}x^2+\frac{3}{16}x^4$	**A1 A1**	A1 for each one correct ISW. Full marks for $\frac{1}{4}\left(1+x^2+\frac{3}{4}x^4\right)$ ISW.
			SC allow **M1 A1 A1** for $\frac{1}{4}$ and $1+x^2+\frac{3}{4}x^4$ SOI. **SC** allow **M1 A1** for $1+x^2+\frac{3}{4}x^4$
		4	
2(b)	State answer $\lvert x\rvert<\sqrt{2}$	**B1**	Or $-\sqrt{2}<x<\sqrt{2}$.
		1	

Question	Answer	Marks	Guidance
3	Use correct trigonometric formulae to form an equation in $\tan x$	*M1	e.g. $\frac{1-\tan^2 x}{\tan x}+\frac{3}{\tan x}=5$
	Obtain a correct linear equation in any form	A1	$1-\tan^2 x+3=5\tan x$
	Reduce equation to a 3-term quadratic	A1	$\tan^2 x+5\tan x-4=0$, or 3-term equivalent
	Solve a 3-term quadratic in tan x and obtain a value of x	DM1	
	Obtain answer, e.g. $x = 35.1°$	A1	
	Obtain second answer, e.g. $x = 99.9°$, and no other in $(0°, 180°)$	A1	Ignore answers outside $(0°, 180°)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
	Alternative method for question 3		
	Use correct formulae for $\sin 2x$ and $\cos 2x$ to form an equation in $\sin x$ and $\cos x$	*M1	
	Obtain $4\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}=5$	A1	
	Reduce equation to a 3-term quadratic	A1	$\tan^2 x+5\tan x-4=0$, or 3-term equivalent
	Solve a 3-term quadratic in tan x and obtain a value of x	DM1	
	Obtain answer, e.g. $x = 35.1°$	A1	
	Obtain second answer, e.g. $x = 99.9°$, and no other in $(0°, 180°)$	A1	Ignore answers outside $(0°, 180°)$. Treat answers in radians $(0.612, 1.74)$ as a misread.
		6	

Question	Answer	Marks	Guidance
4	Separate variables correctly	**B1**	$\int \frac{x}{1+x^2}\,dx = \int \frac{1}{y}\,dy$ Accept without integral signs.
	Obtain term $\ln y$	**B1**	
	State term of the form $k\ln\left(1+x^2\right)$	**M1**	
	State correct term $\frac{1}{2}\ln\left(1+x^2\right)$	**A1**	OE
	Evaluate a constant, or use limits $x = 0$, $y = 2$ in a solution containing terms $a\ \ln y$ and $b\ln\left(1+x^2\right)$ where $ab \neq 0$	**M1**	If they remove logs first the constant must be of the correct form.
	Obtain correct solution in any form	**A1**	e.g $\ln y + \ln\frac{1}{2} = \frac{1}{2}\ln\left(1+x^2\right)$
	Simplify and obtain $y = 2\sqrt{1+x^2}$	**A1**	OE The question asks for simplification, so need to deal with $\exp(\ln(...))$.
		7	

Question	Answer	Marks	Guidance
5(a)	Substitute $x = 2$, equate to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain a correct equation, e.g. $8a - 40 + 2b + 8 = 0$	**A1**	Seen or implied in subsequent work.
	Differentiate p(x), substitute $x = 2$ and equate result to zero	**M1**	Or divide by $x - 2$ and equate constant remainder to zero.
	Obtain $12a - 40 + b = 0$, or equivalent	**A1**	SOI in subsequent work.
	Obtain $a = 3$ and $b = 4$	**A1**	
	Alternative method for question 5(a)		
	State or imply $(x-2)^2$ is a factor	**M1**	
	$\mathrm{p}(x) = (x-2)^2(ax+2)$	**A1**	
	Obtain an equation in b	**M1**	
	e.g. by comparing coefficients of x: $b = 4a - 8$	**A1**	
	Obtain $a = 3$ and $b = 4$	**A1**	
			SC If uses $x = -2$ in both equations allow **M1** and allow **A1** for $a = -3$, $b = -4$.
		5	

Question	Answer	Marks	Guidance
5(b)	Attempt division by $(x-2)$	M1	The M1 is earned if division reaches a partial quotient of $ax^2 + kx$, or if inspection has an unknown factor $ax^2 + ex + f$ and an equation in e and/or f. Where a has the value found in part **5(a)**.
	Obtain quadratic factor $3x^2 - 4x - 4$	A1	
	Obtain factorisation $(3x+2)(x-2)(x-2)$	A1	
	Alternative method for question 5(b)		
	State or imply $(x-2)^2$ is a factor	B1	
	Attempt division by $(x-2)^2$, reaching a quotient $ax + k$ or use inspection with unknown factor $cx + d$ reaching a value for c or for d	M1	
	Obtain factorisation $(3x+2)(x-2)^2$	A1	Accept $3\left(x+\frac{2}{3}\right)(x-2)^2$.
		3	

Question	Answer	Marks	Guidance
6(a)	State or imply $dx = 3\sec^2\theta\, d\theta$	B1	
	Substitute throughout for x and dx	M1	
	Obtain any correct form in terms of θ	A1	e.g. $\int \frac{81\sec^2\theta}{(9+9\tan^2\theta)^2}d\theta$
	Justify change of limits and obtain $\int_0^{\frac{\pi}{4}} \cos^2\theta\, d\theta$ correctly	A1	AG
		4	
6(b)	Obtain indefinite integral of the form $\int a + b\cos 2\theta\, d\theta$, where $ab \neq 0$	*M1	
	Obtain $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$	A1	
	Use correct limits correctly in an expression containing $p\theta$ and $q\sin 2\theta$ where $pq \neq 0$	DM1	$\frac{\pi}{8}+\frac{1}{4}(-0)$
	Obtain answer $\frac{1}{8}(\pi+2)$	A1	Or exact equivalent e.g. $\frac{1}{8}\pi + \frac{1}{4}$.
		4	

Question	Answer	Marks	Guidance
7(a)	Multiply numerator **and** denominator by 1 – 2i, or equivalent	**M1**	At least one multiplication completed.
	Obtain correct numerator $(1-2a)\sqrt{2}-(2+a)\sqrt{2}\text{i}$	**A1**	OE
	Obtain final answer $\frac{1-2a}{5}\sqrt{2}-\frac{2+a}{5}\sqrt{2}\text{i}$	**A1**	OE
	Alternative method for question 7(a)		
	Multiply $x+\text{i}y$ by $1+2\text{i}$ and compare real and imaginary parts	**M1**	
	Obtain $x-2y=\sqrt{2}$ and $2x+y=a\sqrt{2}$	**A1**	
	Obtain final answer $\frac{1-2a}{5}\sqrt{2}-\frac{2+a}{5}\sqrt{2}\text{i}$	**A1**	OE
		3	
7(b)	Obtain $r=2$	**B1 FT**	
	Obtain $\theta=-\frac{3}{4}\pi$	**B1**	
		2	
7(c)	Use correct method to find r or θ	**M1**	
	State answer $\sqrt{2}\text{e}^{-\frac{3}{8}\pi\text{i}}$	**A1 FT**	
	State answer $\sqrt{2}\text{e}^{\frac{5}{8}\pi\text{i}}$	**A1 FT**	
		3	

Question	Answer	Marks	Guidance
8(a)	State or imply $3y^2\frac{dy}{dx}$ as derivative of y^3	B1	
	State or imply $2y+2x\frac{dy}{dx}$ as derivative of $2xy$	B1	
	Complete differentiation and equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $-\frac{3x^2+2y}{3y^2+2x}$	A1	
		4	
8(b)	Find gradient at either (0, – 2) or (– 2, 0)	M1	
	Obtain answers $\frac{1}{3}$ and 3	A1 A1	
	Use $\tan(A \pm B)$ formula to find $\tan\alpha$	M1	
	Obtain answer $\tan\alpha = \frac{4}{3}$	A1	
		5	

Question	Answer	Marks	Guidance
9(a)	Obtain $\overrightarrow{OM} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{MN}$	M1	e.g. $\overrightarrow{MO} + \overrightarrow{OA} + \overrightarrow{AN}$ or $\overrightarrow{MO} + \overrightarrow{ON}$
	Obtain $\overrightarrow{MN} = \mathbf{i} + \mathbf{j} - 2\mathbf{k}$	A1	Accept any notation.
		3	
9(b)	Use a correct method to form an equation for *MN*	M1	Allow without $\mathbf{r} = \ldots$
	Obtain $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + \lambda(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$	A1 FT	OE e.g. $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ Must have $\mathbf{r} = \ldots$ Follow *their* answers to part **9(a)**.
		2	
9(c)	State $\overrightarrow{OP}$ for a general point *P* on *MN* in component form, e.g. $(2+\lambda, 3+\lambda, -2\lambda)$	B1	
	Equate scalar product of $\overrightarrow{OP}$ and a direction vector for *MN* to zero and solve for λ	M1	
	Obtain $\lambda = -\dfrac{5}{6}$	A1	OE e.g. $\mu = \frac{1}{6}$
	Obtain $\sqrt{\frac{53}{6}}$ correctly	A1	AG e.g. from $\sqrt{\left(\frac{7}{6}\right)^2 + \left(\frac{13}{6}\right)^2 + \left(\frac{5}{3}\right)^2}$
		4	

Question	Answer	Marks	Guidance
10(a)	Use correct product rule	**M1**	Condone incorrect / missing chain rule
	Obtain correct derivative in any form	**A1**	e.g. $\frac{dy}{dx} = \sqrt{\sin x} + \frac{x \cos x}{2\sqrt{\sin x}}$ or $2y\frac{dy}{dx} = 2x\sin x + x^2 \cos x$
	Equate derivative to zero and obtain an equation in tan x or tan a	**M1**	
	Obtain $\tan a = -\frac{1}{2}a$ correctly	**A1**	AG
		4	
10(b)	Calculate the value of a relevant expression or pair of expressions at $a = 2$ and $a = 2.5$	**M1**	Must be working in radians At least one correct
	Complete the argument correctly with correct calculated values	**A1**	e.g. $-1 > -2.18$ and $-1.25 < -0.747$
		2	
10(c)	State a suitable equation, e.g. $x = \pi - \tan^{-1}\left(\frac{1}{2}x\right)$	**B1**	A correct equation without subscripts or quote $\tan\theta = -\tan(\pi - \theta)$
	Using $\tan(A \pm B)$ formula, or otherwise, rearrange this as $\tan x = -\frac{1}{2}x$	**B1**	Complete argument correctly
		2	

Question	Answer	Marks	Guidance
10(d)	Use the iterative process correctly at least once	**M1**	Must be working in radians
	Obtain answer $a = 2.29$	**A1**	
	Show sufficient iterations to 4 dp to justify 2.29 to 2 dp or show there is a sign change in the interval (2.285, 2.295)	**A1**	e.g. 2.25, 2.2974, 2.2871, 2.2893, 2.2888, …
		3	