Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/12**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
SOI	Seen or implied
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks	Guidance
1	For $\left(\frac{2}{\boldsymbol{x}}-3x\right)^5$ term in x is 10 or $5C_3$ or 5C2 × $\left(\frac{2}{x}\right)^2$ × $(-3x)^3$ or $\left(\frac{2}{x}\right)^5\frac{5.4.3}{3!}\left(-\frac{3}{2}x^2\right)^3$ or $(-3x)^5\frac{5.4}{2!}\left(\frac{2}{3x^2}\right)^2$	**B2,1**	3 elements required. –1 for each error with or without x's. Can be seen in an expansion.
	−1080 identified	**B1**	Allow $-1080x$ Allow if expansion stops at this term. Allow from expanding brackets.
		3	

Question	Answer	Marks	Guidance
2	Midpoint of AB is (5, 1)	**B1**	Can be seen in working, accept $\left(\frac{10}{2},\frac{2}{2}\right)$.
	$m_{AB} = -½$ oe	**B1**	
	C to (5, 1) has gradient 2	***M1**	Use of $m_1 \times m_2 = -1$.
	Forming equation of line ($y = 2x - 9$)	**DM1**	Using their perpendicular gradient and their midpoint to form the equation.
	C (0, −9) **or** $y = -9$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} = 7 \times -0.05$	M1	Multiply numerical gradient at $x = 2$ by ± 0.05.
	−0.35 (units/s) **or** Decreasing at a rate of (+) 0.35	A1	Ignore notation and omission of units
		2	
3(ii)	$(y) = \frac{x^4}{4} + \frac{4}{x}$ $(+c)$ oe	B1	Accept unsimplified
	Uses (2, 9) in an integral to find c.	M1	The power of at least one term increase by 1.
	$c = 3$ **or** $(y =) \frac{x^4}{4} + \frac{4}{x} + 3$ oe	A1	A0 if candidate continues to a final equation that is a straight line.
		3	

Question	Answer	Marks	Guidance
4(i)	$a^2+2ab+b^2, a^2-2ab+b^2$	B1	Correct expansions.
	$\sin^2 x + \cos^2 x = 1$ used $\rightarrow (a+b)^2+(a-b)^2=1$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$ with $(a+b)^2$ and $(a-b)^2$
	$a^2 + b^2 = ½$	A1	No evidence of $\pm 2ab$, scores 2/3
	Alternative method for question 4(i)		
	$2a = (s+c)$ & $2b = (s-c)$ or $a = ½(s+c)$ & $b = ½(s-c)$	B1	
	$a^2+b^2 = \frac{1}{4}(s+c)^2+\frac{1}{4}(s-c)^2 = ½(s^2+c^2)$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$
	$a^2 + b^2 = ½$	A1	Method using only $(\sin x - b)^2$ and $(a - \cos x)^2$ scores 0/3.
		3	SC B1 for assuming θ is acute giving $a = \frac{1}{\sqrt{5}} + b$ or $2\sqrt{5} - b$

Question	Answer	Marks	Guidance
4(ii)	$\tan x = \frac{\sin x}{\cos x} \rightarrow \frac{a+b}{a-b} = 2$	M1	Use of $\tan x = \frac{\sin x}{\cos x}$ to form an equation in a and b only
	$a = 3b$	A1	
		2	

Question	Answer	Marks	Guidance
5	Perimeter of $AOC = 2r + r\theta$	B1	
	Angle $COB = \pi - \theta$	B1	Could be on the diagram. Condone $180 - \theta$.
	Perimeter of $BOC = 2r + r(\pi - \theta)$	B1	**FT** on angle COB if of form $(k\pi - \theta)$, $k > 0$.
	$(2r +)\ \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \rightarrow \theta = \frac{\pi - 2}{3}$)	M1	Sets up equation using $r(k\pi - \theta)$ and ×2 on correct side. Condone any omissions of OA, OB and/or OC.
	$\theta = 0.38$	A1	Equivalent answer in degrees scores A0.
		5	

Question	Answer	Marks	Guidance
6(i)	3, –3	**B1**	Accept ± **3**
	–½	**B1**	
	2½	**B1**	
		3	Condone misuse of inequality signs.
6(ii)			Only mark the curve from 0 → 2π. If the *x* axis is not labelled assume that 0 → 2π is the range shown. Labels on axes are not required.
	2 complete oscillations of a cosine curve starting with a maximum at (0,a), a>0	**B1**	
	Fully correct curve which must appear to level off at 0 and/or 2π.	**B1**	
	Line starting on positive *y* axis and finishing below the *x* axis at 2π. Must be straight.	**B1**	
		3	
6(iii)	4	**B1**	
		1	

Question	Answer	Marks	Guidance
7(i)	$(\mathrm{f}^{-1}(x)) = \dfrac{x+2}{3}$ oe	B1	
	$y = \dfrac{2x+3}{x-1} \rightarrow (x-1)y = 2x+3 \rightarrow x(y-2) = y+3$	M1	Correct method to obtain $x =$, (or $y =$, if interchanged) but condone $+/-$ sign errors
	$(\mathrm{g}^{-1}(x)$ or $y) = \dfrac{x+3}{x-2}$ oe $\left(eg \dfrac{5}{x-2}+1\right)$	A1	Must be in terms of x
	$x \neq 2$ only	B1	**FT** for value of x from their denominator = 0
		4	
7(ii)	$\left(fg(x) =\right) \dfrac{3(2x+3)}{x-1} - 2\ (= \dfrac{7}{3})$	B1	
	$18x + 27 = 13x - 13$ or $3(4x + 11) = 7(x - 1)$ $(5x = -40)$	M1	Correct method from their $fg = \dfrac{7}{3}$ leading to a linear equation and collect like terms. Condone omission of $2(x-1)$.
	Alternative method for question 7(ii)		
	$(\mathrm{f}^{-1}(\dfrac{7}{3})) = \dfrac{13}{9}$	B1	
	$\dfrac{2x+3}{x-1} = \dfrac{13}{9} \rightarrow 9(2x+3) = 13(x-1)\ (\rightarrow 5x = -40)$	M1	Correct method from $g(x) =$ their $\dfrac{13}{9}$ leading to a linear equation and collect like terms.
	$x = -8$	A1	
		3	

Question	Answer	Marks	Guidance
8(i)	6×3+-2×k+-6×-3 = 0 (18 – 2k + 18 = 0)	M1	Use of scalar product = 0. Could be $\overrightarrow{AO}.\overrightarrow{OB}$, $\overrightarrow{AO}.\overrightarrow{BO}$ or $\overrightarrow{OA}.\overrightarrow{BO}$
	k = 18	A1	
	Alternative method for question 8(i)		
	$76 + 18 + k^2 = 18 + (k + 2)^2$	M1	Use of Pythagoras with appropriate lengths.
	k = 18	A1	
		2	
8(ii)	$36 + 4 + 36 = 9 + k^2 + 9$	M1	Use of modulus leading to an equation and solve to $k=$ or $k^2 =$
	$k = \pm\sqrt{58}$ or ±7.62	A1	Accept exact or decimal answers. Allow decimals to greater accuracy.
		2	

Question	Answer	Marks	Guidance
8(iii)	$\overrightarrow{AB} = \begin{pmatrix} -3 \\ 6 \\ 3 \end{pmatrix} \rightarrow \overrightarrow{AC} = \begin{pmatrix} -2 \\ 4 \\ 2 \end{pmatrix}$ then $\overrightarrow{OA} + \overrightarrow{AC}$	**M1**	Complete method using $\overrightarrow{AC} = \pm \frac{2}{3} \overrightarrow{AB}$ And then $\overrightarrow{OA} + their\, \overrightarrow{AC}$
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$	**A1**	
	$\div \sqrt{(their\, 4)^2 + (their\, 2)^2 + (their\, -4)^2}$	**M1**	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\ (4i + 2j - 4k)$	**A1**	
	Alternative method for question 8(iii)		
	Let $\overrightarrow{OC} = \begin{pmatrix} p \\ q \\ r \end{pmatrix} \rightarrow \overrightarrow{AC} = \begin{pmatrix} p-6 \\ q+2 \\ r+6 \end{pmatrix} \& \overrightarrow{CB} = \begin{pmatrix} 3-p \\ 4-q \\ -3-r \end{pmatrix}$	**M1**	Correct method. Equates coefficients leading to values for p, q, r
	$p - 6 = 2(3 - p)$; $q+2 = 2(4 - q)$; $r + 6 = 2\ (-3 - r)$ $\rightarrow p=4,\ q=2\ \&\ r= -4$	**A1**	
	$\div \sqrt{(their\, 4)^2 + (their\, 2)^2 + (their\, -4)^2}$	**M1**	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\ (4i + 2j - 4k)$	**A1**	

Question	Answer	Marks	Guidance
8(iii)	**Alternative method for question 8(iii)**		
	$\overrightarrow{CB} = \overrightarrow{OB} - \overrightarrow{OC} \;\therefore 2\left(\overrightarrow{OB} - \overrightarrow{OC}\right) = \overrightarrow{OC} - \overrightarrow{OA}$ $\rightarrow 2\overrightarrow{OB} + \overrightarrow{OA} = 3\overrightarrow{OC} \therefore\; 3\overrightarrow{OC} = \begin{pmatrix} 12 \\ 6 \\ -12 \end{pmatrix}$	**M1**	Correct method. Gets to a numerical expression for k$\overrightarrow{OC}$ from $\overrightarrow{OA}$ & $\overrightarrow{OB}$.
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$	**A1**	
	$\div \sqrt{(their\,4)^2 + (their\,2)^2 + (their\,-4)^2}$	**M1**	Divides by modulus of their $\overrightarrow{OC}$
	$= \frac{1}{6}\begin{pmatrix} 4 \\ 2 \\ -4 \end{pmatrix}$ **or** $\frac{1}{6}\,(4i + 2j - 4k)$	**A1**	
		4	

Question	Answer	Marks	Guidance
9	For C_1: $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y – '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	$2x - 2 = \sqrt{4x + k}$ $(\rightarrow 4x^2 - 12x + 4 - k = 0)$	*M1	Forms an equation in one variable using tangent & C_2
	Use of $b^2 - 4ac = 0$ on a 3 term quadratic set to 0.	*DM1	Uses 'discriminant = 0'
	$144 = 16(4 - k) \rightarrow k = -5$	A1	
	$4x^2 - 12x + 4 - k = 0 \rightarrow 4x^2 - 12x + 9 = 0$	DM1	Uses k to form a 3 term quadratic in x
	$x = \frac{3}{2}\left(or \frac{1}{2}\right), y = 1(\text{or} - 1)$.	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y – '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C_2: $\frac{dy}{dx} = A(4x + k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x + k)^{-\frac{1}{2}}$

Question	Answer	Marks	Guidance
9	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" → ($x=\frac{1-k}{4}$ or $4x+k=1$)	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x + k$ and a constant.
	$(2x-2)^2 = 4x+k \to (2x-2)^2 = 1 \to (4x^2-8x+3=0)$	DM1	Using *their* $y = 2x - 2$, $y^2 = 4x + k$ and *their* $4x+k=1$ (but not =0) to form a 3 term quadratic in x.
	$x=\frac{3}{2}\left(or\,\frac{1}{2}\right)$ and from $k=-5(or\,-1)$	A1	Needs correct values for x and k.
	from $y^2 = 4x + k$, $y = 1$(or – 1).	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\frac{dy}{dx} = 2x - 4 \to m = 2$	B1	
	y – '*their 4*' = '*their* m' (x – 3) or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C_2: $\frac{dy}{dx} = A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$
	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" → ($x=\frac{1-k}{4}$ or $4x+k=1$)	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x + k$ and a constant.
	From $4x + k = 1$ and $y^2 = 4x + k \to y^2 = 1$	DM1	Using *their* $4x + k = 1$ (but not =0) and C_2 to form y^2 = a constant

Question	Answer	Marks	Guidance
9	$y = 1(\text{or} -1)$ and $x = \frac{3}{2}\left(or \frac{1}{2}\right)$	A1	Needs correct values for y and x.
	From $4x + k = 1$, $k = -5$ (or – 1)	A1	Condone 'correct' extra solution
		8	

Question	Answer	Marks	Guidance
10(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11 \text{ to } 15)}$	M1	Either statement seen or implied.
	$5(2a + 9d)$ oe	B1	
	$7.5(2a + 14d) - 5(2a + 9d)$ or $\frac{5}{2}[(a + 10d) + (a+14d)]$ oe	A1	
	$d = \frac{a}{3}$ **AG**	A1	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
10(a)(ii)	$(a+9d) = 36 + (a+3d)$	M1	Correct use of $a+(n-1)d$ twice and addition of ±36
	$a = 18$	A1	
		2	Correct answer www scores 2/2

Question	Answer	Marks	Guidance
10(b)	$S_\infty = 9 \times S_4$; $\frac{a}{1-r} = 9\frac{a\left(1-r^4\right)}{1-r}$ or $9(a + ar + ar^2 + ar^3)$	**B1**	May have 12 in place of *a*.
	$9(1 - r^n) = 1$ where $n = 3,4$ or 5	**M1**	Correctly deals with *a* and correctly eliminates '$1 - r$'
	$r^4 = \frac{8}{9}$ oe	**A1**	
	(5th term =) $10\frac{2}{3}$ or 10.7	**A1**	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

Question	Answer	Marks	Guidance
11(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right]$ [× 4] $\left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right]$ [× 4]	B1B1B1	B1 B1 for each, without × 4. B1 for ×4 twice.
	$\left(\frac{2}{\sqrt{4x+1}} - \frac{18}{\left(\sqrt{4x+1}\right)^3} \text{ or } \frac{8x-16}{(4x+1)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of (4x +1) correct.
	$\int y dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷ 4] + $\left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]$ [÷ 4] (+C)	B1B1B1	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6} + \frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of (4x +1) correct.
		6	
11(ii)	$\frac{dy}{dx} = 0 \rightarrow \frac{2}{\sqrt{4x+1}} - \frac{18}{(4x+1)^{\frac{3}{2}}} = 0$	M1	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x + 1 = 9$ or $(4x + 1)^2 = 81$	A1	Must be from correct differential.
	$x = 2$, $y = 6$ **or** M is (2, 6) only.	A1	Both values required. Must be from correct differential.
		3	

Question	Answer	Marks	Guidance
11(iii)	Realises area is $\int y\, dx$ **and** attempt to use their 2 and sight of 0.	***M1**	Needs to use their integral and to see '*their 2*' substituted.
	Uses limits 0 to 2 correctly → $[4.5 + 13.5] - [\frac{1}{6} + 4.5]$ (= 13⅓)	**DM1**	Uses both 0 and '*their 2*' and subtracts. Condone wrong way round.
	(Area =) 1⅓ **or** 1.33	**A1**	Must be from a correct differential and integral.
		3	13⅓ or 1⅓ with little or no working scores M1DM0A0.