MATHEMATICS

Paper 9709/11
Paper 11

Key messages

Centres should ensure that candidates are familiar with the advice for candidates in the syllabus.
In particular, that answers to problems involving trigonometry and calculus which can be obtained directly from some types of calculator will gain no credit unless accompanied by clear working.

Diagrams in the Question Paper are often not drawn to scale and candidates cannot use the appearance of a diagram to make assumptions about geometric properties which are not stated in the question.

In some questions, even though it may not be specifically required, a sketch graph is helpful.

General comments

When a result is given, as in **6(i)** and **11(i),** the working leading to that result must be clear, unambiguous and correct for full credit to be awarded.

Many candidates clearly showed the stages in their working and the links between the stages which gave them greater opportunity to gain credit for method when errors in their working occurred.

Comments on specific questions

Question 1

(i) The quickest route to the solution was often seen from those candidates who realised n=3 would give the required term without the need to find the entire expansion. Care with the use of indices generally led to the correct value of k

(ii) Those candidates able to deduce the appropriate term in **(i)** were usually able to complete this part in a similar way, as were those who had found a correct complete expansion.

Question 2

(i) It was expected that candidates would eliminate x and use the discriminant of the resulting quadratic equation to find c. However, many chose to equate the gradient of the curve and the line. Those candidates familiar with implicit differentiation used it effectively on the curve equation to successfully find the only value of y which could be a coordinate of P. From this, correct values of x and c followed.

(ii) Those candidates who chose the differentiation route often found the correct solution in **(i)** and only had to quote it here. Those who had used the discriminant in **(i)**, were usually able to solve their equation in y and go on to find x.

In both parts the candidates who chose to eliminate y in **(i)** often found the resulting algebra difficult to use to reach correct final answers.

Question 3

The formulae for sector area and arc length in terms of an angle expressed in radians were well known and almost always quoted correctly. Often small sketches clearly informed the working. The calculation of the arc length was often seen and very few, having reached this point, failed to add the 2r to reach the final simplified answer.

Question 4

(i) Using the given information, it was expected that the gradient of AB would be used to find the gradient and equation of BC. Although this route was often seen, other methods used included the use of the scalar product of AB and BC and calculation of the lengths AB, BC and CA to substitute into the Pythagoras' theorem equation. The assumption that AB=BC gained no credit without proof.

(ii) Most successful attempts to this part used the equations of AD and CD. Again, it was possible to use lengths and Pythagoras' theorem and this method was also used successfully.

Question 5

(i) Although the negative coefficient of x^2 confused some candidates, many were able to find the correct values of a and b

(ii) The value of b from **(i)** was all that was required in this part. The coordinate of the maximum point was not an acceptable answer.

(iii) The composite function was used effectively to find the required quadratic equation. Most candidates realised the composite function would definitely be correct if they used the given expression for f(x) rather than their completed square form and most realised that the zero root was a valid solution.

Question 6

(i) Nearly all the solutions seen started by expressing tan in terms of sin and cos and squaring the left hand side of the identity. Those candidates who chose to use a common denominator before squaring were often able to apply the trigonometric identity to the denominator and obtain the required factors to complete the simplification to the right hand side. To gain full credit the factorisation and simplification had to be clearly demonstrated.

(ii) Many candidates realised the result from **(i)** should be applied here and those who noted that sinθ had been replaced by sin2θ often went on to find both angles correctly. Those who used sinθ were only able to gain partial credit for their two resulting answers.

Question 7

(i) The components in the y and z directions were often found correctly for both vectors but the calculation of the component in the x direction proved to be very challenging for both the required vectors.

(ii) The use of the vectors from **(i)** in the scalar product equation appeared to be widely understood and many good answers were seen. As the angle GMA was required, it was not acceptable to find the acute angle between the two vectors.

Question 8

(a) The sum to infinity formula was well known as was the relationship between successive terms of a GP.

Most candidates appreciated that the value of the common ratio had to be numerically less than one for the existence of a sum to infinity. Many completely correct solutions were seen.

(b) Both schemes required the use of the S_n formula and some good answers were seen applying this. The number of attempts at this part compared to **(a)** suggests that some candidates found it more difficult to identify the requirements of this part of the question.

Those candidates who chose to list and add all 24 terms had to reach accurate answers to gain any credit.

Question 9

(i) Many candidates found the correct end points of the range and many of those expressed the range correctly with acceptable notation.

(ii) A considerable number of sketches showed the correct basic shape and symmetry of the required curve but fewer responses gained full credit because of the lack of an approximately zero gradient at the limits of x.

(iii) Those candidates who had a reasonable sketch and noted the instruction to 'state the largest' value of p were usually able to deduce the required value. A number thought it useful to find an expression for $g^{-1}(x)$ here and credit was given for this provided it was quoted in **(iv).**

(iv) A lot of good answers were seen for this part. However, some candidates misread the question and used p or π instead of x.

Question 10

(i) The need to integrate the second derivative to obtain the gradient function and the need to integrate the gradient function were well understood and the integration of terms was usually completed without fault. The requirement of a constant was not always appreciated and when it was, some candidates used dy/dx=6 rather than dy/dx=0 (as the question indicated) in the first integration. A number of candidates thought the same constant was generated by both integrations.

(ii) Those candidates with a correct gradient function invariably found the required second value of x.

(iii) Various methods were used successfully to determine the nature of the turning points. Most common was the sign of the second derivative, then the change in sign of the gradient and then reference to the shape of a cubic equation with a positive coefficient of x^3.

Question 11

(i) This question was accessible to many candidates with full credit awarded to many of those who attempted it. Candidates were often able to differentiate successfully and a good understanding of the relationship between the gradient of a straight line and its normal was often demonstrated. The technique of finding the equation of a straight line through a point and with a given gradient was well understood.

(ii) Most candidates who attempted this question gained at least partial credit. However, the rigorous approach required to achieve full credit was often lacking. The majority gained some credit for an attempt at integration and many were able to integrate successfully. Many candidates gave incorrect limits of integration. Successful candidates often chose the more straightforward method of finding the area of a triangle using ½ bh, rather than using integration. The assumption that a limit of zero always gives a zero result after substitution was less evident than in previous examinations. It was possible to find the required area by using a combination of areas between the curve and the y-axis and some candidates were able to successfully complete the question in this way.

MATHEMATICS

Paper 9709/12
Paper 12

Key messages

Now that papers are scanned in for marking it is very important that candidates use a black pen. If candidates write in pencil and then attempt to overwrite their work in pen their solutions are often very difficult to read. Similarly the work of those candidates who attempt to erase their pencil workings is very difficult to mark as the scanner picks up the original erased work as well as the new attempt. It is much better if rough working is not done separately but the whole attempt at a question is written in pen, in the space provided, and only diagrams are drawn in pencil.

Candidates should read the questions carefully at least twice and then extract the relevant information from them. Not fully comprehending the information often leads to candidates not gaining full credit.

General comments

The paper seemed to generally be very well received by candidates and many good and excellent scripts were seen. The paper contained a number of questions which were reasonably straightforward and gave all candidates the opportunity to show what they had learned and understood. It also contained some questions which provided more of a challenge, even for strong candidates. The vast majority of candidates appeared to have sufficient time to complete the paper.

Comments on specific questions

Question 1

This question proved to be a very accessible start to the paper, with many candidates demonstrating a good knowledge of the binomial expansion. Candidates were often able to write down the relevant term and evaluate it correctly. Weaker candidates sometimes did not include the minus sign or found the coefficient of $\frac{1}{x}$ instead of x.

Question 2

Most candidates demonstrated a very good understanding of the idea of a perpendicular bisector and full credit was very commonly gained in this question. Some weaker candidates obtained the wrong gradient for AB, did not find the gradient of the perpendicular, or didn't use the midpoint.

Question 3

In **(i)** most candidates realised the need to use the chain rule and did so correctly, although a significant number of candidates did not use the fact that the x-coordinate was decreasing and therefore obtained a positive, rather than a negative, solution. Weaker candidates sometimes differentiated again rather than using the $\frac{dy}{dx}$ given.

In **(ii)** the vast majority of candidates knew to integrate and then attempted to find the + c. The mistakes that did occur were integrating incorrectly, forgetting about the + c or using the equation of a straight line instead of finding the equation of the curve.

Question 4

(i) in particular proved to be challenging for candidates. Many candidates knew that $\sin^2 x + \cos^2 x = 1$ was required and those who started with this statement rather than $a^2 + b^2$ were generally successful. In **(ii)** candidates who used the identity $\frac{\sin x}{\cos x} = \tan x$ to obtain an equation in a and b only, were usually successful, although some errors occurred in the subsequent re-arrangements.

Question 5

Many fully correct solutions to this question were seen, with candidates generally aware of the formula required for an arc length in radians and subsequently able to formulate the correct equation using the information provided in the question. Common errors were: using 180°, $\frac{\pi}{2}$ or 2π rather than π; forgetting to add the two radii for each sector; putting the 2 on the wrong side of the equation; and using the formula for area rather than perimeter.

Question 6

Those candidates who considered the given equations and thought about their likely shape were more successful in each part of this question than those who only substituted values. For example, for the curve, those who considered the cosine curve rather than just substituting 0 and 2π were more successful in **(i)**. Similarly re-arranging the line into the form $y = mx + c$ and then considering the y-intercept and gradient was the most successful approach. In **(ii)** many good curves and lines were drawn although some lines were not straight (a ruler should be used for straight lines) and some cosine curves did not appear to 'level off' at 0 and 2π. A significant number of candidates omitted **(iii)** although the majority did see the connection with the number of intersections of the graphs in **(ii)**.

Question 7

This question was very well answered by candidates, particularly finding the inverse of f in **(i)**, and solving the equation in **(ii).** In **(i)** weaker candidates did not obtain an expression for the inverse of g. Some candidates either didn't know how to obtain the value of x when $g^{-1}(x)$ was undefined or forgot to do so. In **(ii)** most candidates gained full credit although some weaker candidates evaluated $\mathrm{fg}\left(\frac{7}{3}\right)$ or made mistakes in forming or solving the equation.

Question 8

(i) was very well done, with many candidates gaining full credit, although the other parts, particularly **(iii)**, proved to be more challenging. In **(i)** the vast majority of candidates successfully used the fact that when the given angle is 90°, the scalar product will be 0, although some candidates unnecessarily found the magnitudes of the vectors concerned. In **(ii)** most candidates used the magnitudes correctly although some did not consider the negative square root, or rounded incorrectly. Candidates found **(iii)** was more challenging and some candidates did not form the required vectors correctly, misinterpreted the given information, or did not form the unit vector once ***OC*** had been correctly found.

Question 9

This multi-stage question proved to be a real challenge for many candidates, although many fully correct solutions were also seen. Some candidates did not appear to have correctly interpreted the information given, and made incorrect steps such as equating the two curves or assuming that x was equal to 3 in the second curve as well as the first. Candidates who correctly interpreted the information given in the question often make significant progress. After differentiating the first curve there were a number of different correct methods that could be used, including: equating the tangent to the second curve and using the discriminant of the subsequent equation equated to 0 or differentiating the second curve and equating the two gradients. Some candidates' solutions were fully correct except that they did not find k or the coordinates of P. Candidates who drew a sketch of the situation were often successful and this approach is encouraged.

Question 10

It was important in this question, as always, to read the information given carefully. Many candidates who did this were able to formulate the correct equations in each part of the question and solve them. Common errors in the formulation were: in **(i)** making the sum of the first 10 terms equal to the sum of the first 5 or first 15 terms; in **(ii)** forming an equation where the 10^{th} term was 36 less, rather than 36 more, than the 4th term; and in **(iii)** multiplying the sum to infinity, rather than the sum of the first 4 terms, by 9. A number of candidates rounded prematurely in **(iii)** and did not obtain the correct final answer.

Question 11

Many completely correct solutions to this question were seen. In particular, **(i)** was very well done, with candidates successfully applying the standard rules for the differentiation and integration of this type of function. Candidates sometimes forgot to multiply and divide by 4 or did one but not the other. In **(ii)** most candidates realised that the stationary point was needed but many did not successfully solve their differential set to 0. A number of candidates did not use the fact that both terms in the equation were a function of $(4x + 1)$, so ended up with an equation that was difficult to solve. Those who did use this were often able to obtain and correctly solve a linear or quadratic equation. In **(iii)** candidates generally knew to use the integral from **(i)** but sometimes had the wrong limits or did not show all necessary working. When definite integrals are evaluated, it is important that both of the limits can clearly be seen to have been substituted into the integral. A significant number of candidates did not attempt this final part.

MATHEMATICS

Paper 9709/13
Paper 13

Key message

There is usually a question (such as **Question 3**) which requires candidates to find an angle, or lengths of lines or arcs, or areas of regions by various calculations. It is sometimes the case that calculations appear without it being made clear exactly what the calculation is attempting to find. Examiners can only award method marks, for example, if it is clear what is being attempted.

An answer unsupported by correct working will not gain credit; some candidates are omitting this essential working. This was most noticeable in **Question 10(ii)** in which the actual process of integration needed to be shown followed by the process of finding the difference of the two numbers obtained by substituting the limits. The use of calculator routines for supplying the answers is not acceptable.

General comments

The paper was generally well received by candidates and many very good scripts were seen. Almost all candidates seemed to have sufficient time to finish the paper. Accuracy of numerical answers remains an issue either due to rounding errors or premature approximation. Unless specified otherwise, apart from angles given in degrees which need to be given correct to 1 decimal place, answers should be given correct to 3 significant figures. Candidates often forget that in order to achieve this it is usually necessary to carry *more* than 3 significant figures in their working. This is particularly necessary in **Question 3** and **9(iii)**. Another issue is that some scripts are particularly difficult to read and give the impression that perhaps some of the answers are written in pencil and then superimposed in ink which makes it unclear.

Comments on Specific Questions

Question 1

Part (i) proved to be a good opening question which was well received by almost all candidates. **Part (ii)** was designed to test how well candidates could use the result of **part (i)** to solve a quadratic inequality and instructed candidates to ‘Hence find the set of values of *x* .....’. Candidates should be advised to read the question carefully and identify the importance of the word 'Hence'. Some candidates disregarded the result from **part (i)** and formed a 3-term quadratic, solved this and then inserted inequality signs, which was an inefficient approach and did not satisfy the requirements of the question.

Question 2

Part (i) was generally well answered with the majority achieving full marks. Some candidates omitted the minus signs attached to the first and third terms or made other arithmetic errors. A number of candidates either misinterpreted what was required or did not read the question carefully enough and showed that the first three terms were the same as that given in the question. **Part (ii)** was generally well answered.

Question 3

Both parts of this question were answered quite well, although some long and rather complicated methods were seen in places. A significant number of candidates used degrees throughout the question, despite one of the angles being given as $\frac{1}{5}\pi$. This made the calculations more complex and therefore more prone to error. A certain amount of accuracy was often lost in the conversion to degrees. In **part (i)**, most candidates used trigonometry in their solution but some used a combination of trigonometry and Pythagoras' theorem, while others found the area of triangle *ABC* in two different ways in order to find the length of *AD*. In **part (ii)**, a few candidates, having found the correct two areas subtracted them, whilst others subtracted both areas from the area of triangle *ABC*, effectively finding the unshaded area.

Question 4

Many candidates found **part (i)** challenging. Some candidates gave an expression for gf(x) and then repeated this in **part (ii)**. Other candidates inserted 3 and 7 into f(x) but did not recognise that this gave the answer and did not associate 8 and 24 with a and b, sometimes giving their answer as $8 \leqslant x \leqslant 24$. **Part (ii)** was answered very well by most candidates. A few candidates did not simplify the expression and therefore did not gain the mark. Some candidates got to the correct answer and then multiplied by the denominator which usually made **part (iii)** more challenging. **Part (iii)** was quite well answered; there were some slips in algebraic manipulation where a few candidates left the inverse in terms of y.

Question 5

Most candidates were able to show they understood the context of the question although a significant number of candidates did not recognise that the weekly weight loss was decreasing and therefore the common difference was negative. In **part (i)** the mark was given for an (unsimplified) expression for the total weight loss using either the correct value (–0.02) for d, or (+0.02). The majority of candidates were able to do this successfully although some candidates wrote down an expression for the weight loss in the xth week rather than the total weight loss after x weeks. In **part (ii)** candidates were required to equate their answer to **part (i)** to 13 and solve a quadratic equation. A good proportion who did not score the mark in **part (i)** were able to start again in **part (ii)** and to score at least one of the 2 marks available. One mark was also usually scored by those candidates who were using $d = +0.02$. Candidates who had obtained the correct equation were usually able to obtain 15.1 but many left this as their answer without realising that n had to be an integer. In **part (iii)**, most candidates obtained 10.1 for the total weight loss after 20 weeks but some did not realise that a further calculation was required. The expected approach was to find the sum to infinity (12.5) and to conclude that since this was less than the target of 13, the target could never be reached. Some candidates set the sum to n terms equal to 13 and simplified to reach the equation $0.92^n = -0.04$. The correct conclusion was that this equation has no solution and therefore he can never reach his target.

Question 6

The first three parts asked for answers in terms of **i**, **j** and **k**, however many candidates used column vector notation, which was not penalised. A significant proportion of candidates did not keep to the conventional order of **i**, **j**, **k** but wrote them down in the order in which they occurred in the route that was used. This did not matter for the first three parts but caused errors to be made in **part (iv)** when the scalar product was calculated. Some candidates thought that F was vertically above C which caused errors in **part (i)**. In **part (ii)** some candidates tried to find the length of FN which introduced $\sqrt{5}$ into their answer. Most candidates were able to score the one mark available – either by obtaining the correct answer or from the method mark available for obtaining an answer equal to the sum of **parts (i)** and **(ii)**. **Part (iv)** was answered very well with most candidates achieving at least 3 marks.

Question 7

Nearly all candidates achieved the correct answer in **part (i)**. In **part (ii)** most candidates achieved the correct answer, although some candidates had 400 instead 20 in the equation. **Part (iii)** was also quite well answered. Almost all candidates equated AC^2 and BC^2 and then correctly achieved the given answer. There were some arithmetic and sign errors seen and correct answers following these errors; this received no credit. **Part (iv)** was answered quite well. Candidates who did not obtain the correct answers in **parts (ii)** or **(iii)** were usually able to obtain 2 method marks.

Question 8

The most successful approach for this question was to obtain two simultaneous equations by substituting $\frac{dy}{dx} = 0$ when $x = -1$ and when $x = 3$ to give the values of a and b. This was followed by integrating the found expression for $\frac{dy}{dx}$, remembering to include the constant of integration which can be found to be –3 by substituting (–1, 2). Finally, the value of k was found by substituting (3, k). Many candidates followed this route and scored full marks. A far less common, but elegant method of finding a and b was to recognise that $\frac{dy}{dx} = 3(x+1)(x-3)$ and equating coefficients with the given expression for $\frac{dy}{dx}$. Some candidates started by integrating the given expression for $\frac{dy}{dx}$ without first evaluating a and b. This will eventually lead to four equations in four unknowns and relatively few candidates were successful when following this approach. A common error made by some candidates was to forget the constant of integration and also to start by substituting $\frac{dy}{dx} = 2$ when $x = -1$ and $\frac{dy}{dx} = k$ when $x = 3$.

Question 9

This question was quite challenging, although strong candidates were able to score well. A significant proportion of candidates made no attempt at all or some of the parts of this question. In **part (i)**, candidates were often able to get at least one of the end-points of the range. In **part (ii)**, it was expected that candidates would recognise that the three functions represented straight lines parallel to the x-axis, respectively at the top of the range, the bottom of the range and in the middle of the range. More able candidates evidently did recognise this, but answers from other candidates gave the impression that they were guessing. In **part (iii)**, the given diagram and the earlier parts of the question should have alerted candidates to expect 4 solutions. Candidates should be advised to check they have obtained all possible solutions within the range, as some candidates presented only 2 solutions. The surest way is to find 4 solutions for $2x$ in the range $0 \to 2\pi$ and to halve these solutions to find the required solutions for x. The process of dividing by 2 has the potential of losing accuracy and this question underlines the need to work to an accuracy of 4 significant figures in early work in order to achieve 3 significant figure accuracy in the answer.

Question 10

Many candidates obtained fully correct answers for **part (i)**. Most candidates were aware of the need to differentiate and the majority did this accurately. In **part (ii)** the need to integrate was almost always understood and in the most part was done accurately. Candidates who chose to integrate to find the area of the trapezium were more prone to error by using this method; some candidates also made errors when finding the equation of the tangent in **part (i)**. A few candidates attempted to find the area related to the y-axis but the limits were more awkward and this often proved to be more erroneous. In **part (iii)** most candidates tried to find $\frac{dy}{dx}$ by using the chain rule. Candidates should be advised to set working out clearly, as the awarding of partial marks was difficult in places with the use of this method. Those who understood that $\frac{dy}{dx} = \frac{1}{2}$ were mostly able to progress to the right answer. There was a significant proportion of candidates who made no attempt at **part (iii)**.

MATHEMATICS

Paper 9709/21
Paper 21

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. Candidates should also be aware of the degree of accuracy noted as many candidates did not gain accuracy marks due to inaccurate working and final answers. It is also important that each candidate ensures that they have fulfilled the demands of each question and have set their work out clearly, showing sufficient steps in their working.

General comments

Candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on the additional sheet provided. It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word 'exact' in the mathematical context. A wide range of marks was obtained. There were some exemplary scripts, showing a thorough understanding of the syllabus demands and objectives, but other scripts where candidates gained very few marks.

Comments on specific questions

Question 1

It was essential that candidates showed each step of their working as the demand of the question was to 'Show' a given result. If the correct law of logarithms was used to obtain $\ln\frac{\left(x^3-4x\right)}{\left(x^2-2x\right)}$, it was essential that $\frac{\left(x^3-4x\right)}{\left(x^2-2x\right)}$ was written in terms of linear factors, clearly indicating which of these were common factors that could be cancelled to obtain the given result. Algebraic long division of x^3-4x by x^2-2x was also acceptable. A common error that was seen was $\frac{\ln\left(x^3-4x\right)}{\ln\left(x^2-2x\right)}$.

Question 2

(i) Many completely correct solutions were seen. Many other solutions showed correct critical values but did not then show the correct inequality. The most popular method was to square each side of the given inequality and obtain the critical values from the resulting quadratic equation. For those that chose to consider pairs of linear inequalities or equations, errors in signs were more common.

(ii) Very few completely correct solutions were seen. Most candidates did not link this part of the question with the first part of the question, although the word 'Hence' indicated this link. It was expected that x in the result from part one be replaced by $3^{0.1n}$ and then the upper limit from the result in **part (i)** be used to obtain the $3^{0.1n}=4$ or $3^{0.1n}<4$. Of the candidates that reached this stage and solved for n, most left their final answer as $n=12.6$, rather than the required 12, not checking the form of the answer asked for.

Question 3

Provided that candidates realised that implicit differentiation was involved, most were able to gain some marks. Errors in the differentiation of individual terms were common in the given product and the constant 11. Most candidates were able to get a value for the gradient from their work and then use it correctly to obtain an equation for the normal. Arithmetic slips were quite common.

Question 4

(a) Many correct solutions were seen. It was important that candidates realised that they needed to make use of the identity $\tan^2 3x = \sec^2 3x - 1$ to rewrite the integrand. Common errors included errors in the coefficient of $\tan 3x$ or the reversal of terms in the identity.

(b) A significant number of candidates omitted dividing each term in the numerator by e^x and were then unable to produce any correct integration. Of those candidates who did correctly divide by e^x, most were able to obtain the correct integral (there were errors in some of the coefficients) and candidates obtained full marks provided they had simplified $-\frac{1}{2}+4$ and left the answer in exact form as required. Candidates who resorted to the use of their calculator did not gain the final accuracy mark as their answer was not in exact form. This highlights the need to appreciate the mathematical implication of the word 'exact' and to check that the demands of the question have been fully met.

Question 5

(i) Many candidates produced clear, well set out, solutions, obtaining the correct value of *a* and of *b*. It was intended that both the factor theorem and the remainder theorem be used, thus obtaining two simultaneous equations that could be easily solved. Some candidates attempted algebraic long division, being more likely to make errors, especially when equating their remainders to either zero or 27. Synthetic division was also a common method used and the same problems occurred as in algebraic long division. The majority of candidates gained full marks used the remainder and factor theorem.

(ii) Provided that a correct response had been obtained for **part (i)**, most candidates were able to obtain the linear factors correctly. It was important that full working was shown, usually by first obtaining a quadratic factor by using $x - 2$ and either by algebraic long division or by observation. For those candidates who had incorrect values for *a* and for *b*, there was the availability of a method mark for attempting a correct method with their incorrect values. It was not intended that calculators be used in this part of the question.

Question 6

(i) The majority of candidates recognised that they needed to differentiate the given function using the quotient rule, and obtained both marks available for this. It was then important that the value of x at the point where the curve crosses the x-axis be identified. A significant number of candidates appeared to think that if they have differentiated they needed to equate their result to zero and attempt to solve. For those who did find the required value, most were able to obtain the correct gradient.

(ii) This part did require equating the derivative obtained in **part (i)** to zero but then rearranging the result to obtain the given equation. Few candidates were able to do this, although many did attempt a rearrangement but not using the correct method.

(iii) Most candidates were able to gain at least partial credit. Many candidates were able to obtain correct iterations, but some did not complete a sufficient number of iterations. It is a good idea to indicate in the working that the last two iterations round to the same 3 significant figures number. Some candidates, as in previous sessions, did not write their final answer to the required level of accuracy, again highlighting the need to ensure that the demands of the question have been met.

Question 7

(i) Many candidates were able to produce a correct solution. However, each step in the process must be shown. It is advised that candidates always start with the left hand side of the given expression and work with this in order to obtain what is written on the right hand side.

(ii) Very few candidates were able to make any credit on this part. Most resorted to the use of their calculator which did not gain them any marks. The word 'Hence' indicates that the result from **part (i)** needs to be used in **part (ii)**. The result had to be rearranged and the value of θ replaced by $15°$. The result of $2\text{cosec}30°$ and then $\dfrac{2}{\sin 30°}$ could then be obtained and the value of 4 confirmed.

(iii) Very few candidates were able to make any meaningful attempt at this part unless they realised that they had to again make use of the result from **part (i)** to obtain a quadratic equation in terms of $\text{cosec}\dfrac{\phi}{2}$ Of the candidates whoe did this, many were able to obtain the two positive solutions but often omitted or made errors with the two negative solutions. Answers in degrees should always be given to one decimal place unless the angle is exact.

MATHEMATICS

Paper 9709/22
Paper 22

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. Candidates should also be aware of the degree of accuracy noted as many candidates do not gain accuracy marks due to inaccurate working and final answers. It is also important that each candidate ensures that they have fulfilled the demands of each question and have set their work out clearly, showing sufficient steps.

General comments

Candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on the additional sheet provided. It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word 'exact' in the mathematical context. A wide range of marks was obtained. There were some exemplary scripts, showing a thorough understanding of the syllabus demands and objectives.

Comments on specific questions

Question 1

The majority of candidates were able to make a reasonable attempt at this question using the factor theorem with many obtaining full marks, Common errors included slips in arithmetic, not equating the result to zero and the substitution of $x = 1$ rather than $x = -1$.

Question 2

(i) Many completely correct solutions were seen. The most popular method was to square each side of the given equation and obtain the solutions from the resulting quadratic equation. For those candidates who considered pairs of linear equations, errors in signs were more common. Some candidates mistakenly thought that an inequality was involved and gave their final answers in the form of inequalities.

(ii) Very few candidates realised the implication of the word 'Hence', with many starting the question again. The candidates who did recognise the link between the two parts and therefore replaced x in **part (i)** with e^{3y} very often attempted to give a result for $e^{3y} = -\frac{1}{7}$ as well, so did not gain the final accuracy mark.

Question 3

Most candidates recognised that they needed to differentiate a quotient and did so correctly. The candidates who chose to re-write the equation as $y = 3x(\ln x)^{-1}$ and differentiate a product tended to be less successful as incorrect simplification later led to errors.

Many candidates equated their derivative to zero and attempted to solve, but did not recognise the need to give exact answers. Some candidates resorted to the use of a calculator for both of the coordinates. Others, having obtained an exact value for the x-coordinate then used a calculator to determine the value of the y-coordinate. Some candidates omitted the y-coordinate completely.

Question 4

(a) Unless candidates recognised that they need to make use of the double angle formula to write $2\cos^2 x$ as $\cos 2x + 1$, they could only gain credit for the correct integration of $4\sin 2x$. Many candidates integrated $4\sin 2x$ incorrectly with results of $4\cos 2x$, $-4\cos 2x$ and $8\cos 2x$ all being commonly seen. Although many candidates applied the limits correctly to a correct integration, many of these candidates resorted to the use of a calculator, not recognising the need for an exact answer.

(b) Very few completely correct solutions were seen. The main problem appeared to be involving the value of h, many of which were incorrect. This then often led to the incorrect x values being used. Many candidates did not appear to know the formula for the trapezium rule. Some candidates who did know the formula applied it incorrectly.

Question 5

(i) The majority of candidates were able to find the correct quotient using algebraic long division. However many made errors with the remainder, forgetting to subtract a correct -4 from zero, thus giving the remainder of 4. A common error was to give a remainder of -4.

The candidates who attempted to use synthetic division often ended up with a quotient of $2x^2 - 8$, having omitted to take into account that they were dividing by a factor of $2x + 1$.

(ii) Very few candidates recognised the significance of the word 'Hence' so many candidates did not attempt to use the result from **part (i)**. For those who did, it was important that they wrote the integrand $\dfrac{2x^3 + x^2 - 8x}{2x + 1}$ as $x^2 - 4 + \dfrac{4}{2x + 1}$ before attempting integration. As few candidates did this, even fewer were able to manipulate the results from applying the limits to obtain an answer in the form required. Some candidates integrated correctly but were unable to deal with the term -3 by writing it as $\ln e^{-3}$.

Question 6

(i) Most candidates attempted to rearrange the equation $1 = 4t^2 e^{-t}$ to obtain the given result, but a significant number of candidates did not deal with the term e^{-t} correctly.

(ii) Most candidates were able to gain at least partial credit. Many candidates were able to obtain some correct iterations, but some of these candidates did not complete a sufficient number of iterations. It is a good idea to indicate in the working that the last two iterations round to the same 5 significant figures number. Some candidates, as in previous sessions, did not write their final answer to the required level of accuracy, again highlighting the need to ensure that the demands of the question have been met. Some candidates, after obtaining sufficient correct iterations, gave a final answer of 0.714 rather than the correct answer. There were also some candidates who did their iterations correct to 4 significant figures but gave their final answer to 3 significant figures.

(iii) Attempts at parametric differentiation were usually successful provided $\dfrac{d}{dt}\left(e^{-2t}\right)$ and $\dfrac{d}{dt}\left(e^{-t}\right)$ were correct. The need to differentiate a product was not recognised by some candidates, but most candidates recognised that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Question 7

(a) (i) Most candidates were able to produce a correct form, showing a good understanding of the method required for this syllabus requirement. Some candidates, however, gave α as $\frac{\pi}{4}$ thus highlighting the need for candidates to ensure that they have fully met the demands of the question.

(ii) Provided a correct form had been obtained in **part (i)**, most candidates were able to use the correct order of operations and obtain a correct answer. For those candidates who had made errors in **part (i)**, partial credit was available for a correct approach.

(b) To make meaningful progress in this part, candidates needed to be working in terms of $\tan x$. The candidates who attempted to work in terms of $\sin x$ and $\cos x$ were unable to progress to obtain an equation that could be solved. It is essential that candidates recognise that when a solution is becoming ever more complex, an error has probably been made and if no errors are immediately obvious, then perhaps a different approach should be attempted.

For those candidates who worked with $\tan x$, many obtained a quadratic equation in $\tan x$. and many obtained the results $\tan x = \frac{1}{3}$ and $\tan x = -2$. However, some candidates gave their solutions in degrees rather than radians, while others had problems obtaining a solution for $\tan x = -2$ in the given range.

MATHEMATICS

Paper 9709/23
Paper 23

Key messages

It is important that candidates are familiar with the rubric on the front of the examination paper as it will remind them of important facts which tend to be forgotten during the examination itself. Candidates should also be aware of the degree of accuracy noted as many candidates do not gain accuracy marks due to inaccurate working and final answers. It is also important that each candidate ensures that they have fulfilled the demands of each question and have set their work out clearly, showing sufficient steps.

General comments

Candidates appeared to have sufficient time to work through the paper and also sufficient space in which to answer their questions. It was pleasing to note that when extra space was needed for an answer, work was done on the additional sheet provided. It should also be noted that many candidates did not obtain some of the accuracy marks available as they did not appreciate the implication of the word 'exact' in the mathematical context. A wide range of marks was obtained. There were some exemplary scripts, showing a thorough understanding of the syllabus demands and objectives.

Comments on specific questions

Question 1

The majority of candidates were able to make a reasonable attempt at this question using the factor theorem with many obtaining full marks, Common errors included slips in arithmetic, not equating the result to zero and the substitution of $x = 1$ rather than $x = -1$.

Question 2

(i) Many completely correct solutions were seen. The most popular method was to square each side of the given equation and obtain the solutions from the resulting quadratic equation. For those candidates who considered pairs of linear equations, errors in signs were more common. Some candidates mistakenly thought that an inequality was involved and gave their final answers in the form of inequalities.

(ii) Very few candidates realised the implication of the word 'Hence', with many starting the question again. The candidates who did recognise the link between the two parts and therefore replaced x in **part (i)** with e^{3y} very often attempted to give a result for $e^{3y} = -\frac{1}{7}$ as well, so did not gain the final accuracy mark.

Question 3

Most candidates recognised that they needed to differentiate a quotient and did so correctly. The candidates who chose to re-write the equation as $y = 3x(\ln x)^{-1}$ and differentiate a product tended to be less successful as incorrect simplification later led to errors.

Many candidates equated their derivative to zero and attempted to solve, but did not recognise the need to give exact answers. Some candidates resorted to the use of a calculator for both of the coordinates. Others, having obtained an exact value for the x-coordinate then used a calculator to determine the value of the y-coordinate. Some candidates omitted the y-coordinate completely.

Question 4

(a) Unless candidates recognised that they need to make use of the double angle formula to write $2\cos^2 x$ as $\cos 2x + 1$, they could only gain credit for the correct integration of $4\sin 2x$. Many candidates integrated $4\sin 2x$ incorrectly with results of $4\cos 2x$, $-4\cos 2x$ and $8\cos 2x$ all being commonly seen. Although many candidates applied the limits correctly to a correct integration, many of these candidates resorted to the use of a calculator, not recognising the need for an exact answer.

(b) Very few completely correct solutions were seen. The main problem appeared to be involving the value of h, many of which were incorrect. This then often led to the incorrect x values being used. Many candidates did not appear to know the formula for the trapezium rule. Some candidates who did know the formula applied it incorrectly.

Question 5

(i) The majority of candidates were able to find the correct quotient using algebraic long division. However many made errors with the remainder, forgetting to subtract a correct -4 from zero, thus giving the remainder of 4. A common error was to give a remainder of -4.

The candidates who attempted to use synthetic division often ended up with a quotient of $2x^2 - 8$, having omitted to take into account that they were dividing by a factor of $2x + 1$.

(ii) Very few candidates recognised the significance of the word 'Hence' so many candidates did not attempt to use the result from **part (i)**. For those who did, it was important that they wrote the integrand $\dfrac{2x^3 + x^2 - 8x}{2x + 1}$ as $x^2 - 4 + \dfrac{4}{2x + 1}$ before attempting integration. As few candidates did this, even fewer were able to manipulate the results from applying the limits to obtain an answer in the form required. Some candidates integrated correctly but were unable to deal with the term -3 by writing it as $\ln e^{-3}$.

Question 6

(i) Most candidates attempted to rearrange the equation $1 = 4t^2 e^{-t}$ to obtain the given result, but a significant number of candidates did not deal with the term e^{-t} correctly.

(ii) Most candidates were able to gain at least partial credit. Many candidates were able to obtain some correct iterations, but some of these candidates did not complete a sufficient number of iterations. It is a good idea to indicate in the working that the last two iterations round to the same 5 significant figures number. Some candidates, as in previous sessions, did not write their final answer to the required level of accuracy, again highlighting the need to ensure that the demands of the question have been met. Some candidates, after obtaining sufficient correct iterations, gave a final answer of 0.714 rather than the correct answer. There were also some candidates who did their iterations correct to 4 significant figures but gave their final answer to 3 significant figures.

(iii) Attempts at parametric differentiation were usually successful provided $\dfrac{d}{dt}\left(e^{-2t}\right)$ and $\dfrac{d}{dt}\left(e^{-t}\right)$ were correct. The need to differentiate a product was not recognised by some candidates, but most candidates recognised that $\dfrac{dy}{dx} = \dfrac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Question 7

(a) (i) Most candidates were able to produce a correct form, showing a good understanding of the method required for this syllabus requirement. Some candidates, however, gave α as $\frac{\pi}{4}$ thus highlighting the need for candidates to ensure that they have fully met the demands of the question.

(ii) Provided a correct form had been obtained in **part (i)**, most candidates were able to use the correct order of operations and obtain a correct answer. For those candidates who had made errors in **part (i)**, partial credit was available for a correct approach.

(b) To make meaningful progress in this part, candidates needed to be working in terms of $\tan x$. The candidates who attempted to work in terms of $\sin x$ and $\cos x$ were unable to progress to obtain an equation that could be solved. It is essential that candidates recognise that when a solution is becoming ever more complex, an error has probably been made and if no errors are immediately obvious, then perhaps a different approach should be attempted.

For those candidates who worked with $\tan x$, many obtained a quadratic equation in $\tan x$. and many obtained the results $\tan x = \frac{1}{3}$ and $\tan x = -2$. However, some candidates gave their solutions in degrees rather than radians, while others had problems obtaining a solution for $\tan x = -2$ in the given range.

MATHEMATICS

Paper 9709/31
Paper 31

General comments

The response to this paper was varied. Some candidates showed a good knowledge of all topics on the specification and gave good answers to all ten questions. However, a large proportion of candidates offered no response at all to the last two questions (Vectors and Complex Numbers) which accounted for 23 of the 75 marks unavailable.

Most candidates scored well on **Question 3** (Implicit Differentiation), **Question 6(i)** (Trigonometric Identity), and **Question 8(i)** (Partial Fractions). Candidates found **Question 5** (Differentiation and Integration of Trigonometric Functions) and **Question 7** (Differentiation and numerical methods) more challenging.

Key messages

- Prepare all the topics on the syllabus.
- Show clear working.
- Write clearly and do not overwrite one solution with another. This results in an illegible script once scanned.
- Check your algebra and arithmetic carefully.
- If a question asks you to obtain a given answer, then take particular care to show full working.
- If a question asks for an exact answer, then decimal working is not appropriate.

Question 1

The majority of candidates showed some understanding of how to apply the trapezium rule, with several obtaining the correct answer. The most common error was to ignore the modulus signs and use ordinates –3, –2, 0 and 4. Some candidates used the wrong number of ordinates, or used ordinates covering a wider interval than that specified. A few candidates used 0, 1, 2 and 3 in place of the ordinates. Some candidates made unsuccessful attempts to integrate the function.

Question 2

Most candidates used at least one law of logarithms correctly, and many obtained the correct quadratic equation in x. The correct final answer was often seen, but it was sometimes accompanied by the invalid solution $x = 0.70$. Neither $In(2x - 3)$ nor $In(x - 1)$ are defined when $x = 0.70$. The most common errors in the working occurred when candidates started with $ln\, x - ln(x-1) = ln\left(\frac{x}{x-1}\right)$ before dealing with the 2. A minority of candidates dealt with the 2 and then simply deleted the logarithms to obtain $2x - 3 = x^2 - (x - 1)$.

Question 3

Most candidates demonstrated some knowledge of implicit differentiation and found the correct derivative with respect to x of at least one of the terms $3xy^2$ and y^3. Many candidates reached the correct final answer. The most common errors were to introduce an additional term $\frac{dy}{dx} = \cdots$ at the start of their work or to not differentiate the 1 on the right hand side of the equation. There were also errors in rearranging the equation and in the arithmetic in substituting the given coordinates. A small number of candidates used the point (3, 1) rather than (1, 3).

Question 4

Most candidates were aware that $cot\theta = \frac{1}{tan\,\theta}$, but some found dealing with $cot(\theta + 45°)$ challenging. Many candidates obtained the correct first step of $\frac{1}{tan\,\theta} - \frac{1-tan\,\theta}{tan\,\theta+1} = 3$ but there were many sign errors and algebra slips in rearranging this to obtain a quadratic equation in $tan\theta$. Candidates who obtained a correct quadratic equation usually reached the correct answers.

Question 5

(i) A small number of candidates recognised $\frac{1}{sin^2\,\theta}$ as $cosec^2\theta$ and immediately gave a correct form of the answer. The majority of candidates worked from $(sinx)^{-2}$, many reaching a correct form of the answer. Some candidates made an error in reducing the power and obtained the incorrect form $-2cos\theta\,sin{-}1\theta$.

(ii) A large number of candidates did not take the key first step of separating the variables in order to integrate. Those who did separate the variables correctly usually obtained $\frac{1}{2}x^2$ but only the stronger candidates recognised the function in θ or saw the link with part **(i)**.

Question 6

(i) The majority of candidates were familiar with this result and how to obtain it. Many fully correct solutions were seen.

(ii) Most candidates did not use the result from part **(i)** to tackle this integral. Those who did this usually obtained an answer involving $cosx$ and $cos3x$ and went on to use the limits correctly. There were some incorrect attempts to integrate such as $\int sin^3\,x\ \ dx = \frac{1}{4}sin^4\,x\,(+c)$. The wording 'showing all necessary working' meant that the working had to be seen in order for candidates to score all of the marks and candidates could not earn 4 marks by just writing down the answer.

Question 7

(i) The majority of candidates understood that if they were comparing the gradients of the two functions then they needed to start by differentiating. The derivative of $4\,cos\,\frac{1}{2}x$ was usually correct, but there were several sign errors in differentiating $(4-x)^{-1}$. Many candidates equated the product of their derivatives to –1. Only a minority of candidates reached the given answer correctly.

(ii) There were two common approaches used by candidates: either they considered an equation of the form $f(a) = 0$, or they worked with an equation of the form $f(a) = a$. Many candidates tackled the latter form but were hoping to find a sign change. A minority of candidates were working in degrees rather than radians.

(iii) Several candidates offered no attempt to apply the iterative formula, but those who did usually followed the instructions regarding accuracy and reached the correct answer. A few candidates did not understand the basic process and substituted a succession of values 2.0, 2.1, 2.2, 2.3 etc.

Question 8

(i) The majority of candidates showed good understanding of how to split the given fraction into partial fractions. The solutions varied considerably in length, but many candidates obtained the correct answers. The candidates who started by comparing the coefficients of powers of x usually had a correct method, but often made slips in the arithmetic.

(ii) A significant number of candidates offered no attempt to find the expansion of $f(x)$. A small number attempted to use the McLaurin expansion, with varying degrees of success, usually due to errors in differentiation. Most preferred the binomial expansion. There was evidence of understanding of the process, but many errors in dealing with the constants were seen: 2^{-1} and 3^{-1} often became 2 and 3.

Question 9

(i) Of the candidates who attempted this question, most found at least one of the vectors $\overrightarrow{BC}$, $\overrightarrow{BD}$ and $\overrightarrow{CD}$. The most popular method for finding the equation of the plane was to use the vector product of two of these vectors.

Some candidates used the alternative of forming and solving four equations in *a*, *b*, *c* and *d*. Both methods were subject to arithmetic errors.

(ii) Many candidates attempted to find the angle. Some candidates were able to determine that the vector **k** was perpendicular to the plane *OABC*, others found two vectors in the plane and used the vector product. The process of using the scalar product to find an angle was well understood by those candidates who attempted to use it.

Question 10

(i) Many of the candidates who attempted this question understood how to express a complex number in the form $re^{i\theta}$. Both values were usually found correctly. Those candidates who worked with this form to find the modulus and argument of u^4 often obtained the correct values. Candidates who started with the given form for *u* and attempted to find the fourth power often made errors in their working or gave an angle in the wrong quadrant for the argument.

(ii) Candidates who worked with the modulus-argument form for *u* often verified the given result very quickly. Cubing the original form was more popular, but prone to algebra and arithmetic slips. Many candidates were aware that complex roots of the equation with real coefficients occur in conjugate pairs, so the other complex root was often stated correctly.

(iii) For those candidates who attempted to draw the Argand diagram, the basic structure of the diagram was often correct, with the appropriate region being shaded. The main issues were with accuracy, with the circle not passing through the origin, or the line and the circle not intersecting on the imaginary axis.

MATHEMATICS

Paper 9709/32
Paper 32

General comments

The candidate response to this paper was very varied. There were many candidates who offered strong responses to all ten questions, some candidates who showed a good level of understanding of the specification but lacked the arithmetic and algebra skills to complete the work accurately, and some candidates who demonstrated only a vague understanding of the topics examined.

The candidates scored particularly well in **Question 1** (binomial expansion), **Question 3** (trigonometric equation), **Question 7** (differential equation) and **Question 8** (partial fractions and integration). The questions that candidates found the most challenging were **Question 5** (complex numbers), **Question 6(i)** and **(ii)** (circular measure), **Question 9(ii)** (equation of a plane) and **Question 10** (trigonometric identity and calculus).

Key messages

- Show clear working.
- Write clearly and do not overwrite one solution with another. Overwriting solutions can result in an illegible script once scanned.
- Check your algebra and arithmetic carefully.
- Take care in your use of notation, and particularly in the use of brackets.
- If a question asks you to obtain a given answer then take particular care to show full working.
- If a question asks for an exact answer then decimal working is not appropriate.

Comments on specific questions

Question 1

The majority of candidates showed a good level of understanding of the binomial expansion. The most common error in expanding $(1+3x)^{\frac{1}{3}}$ was to use x in place of $3x$. Some candidates only expanded as far as the term in x^2, meaning that they had insufficient terms to complete their solution. Multiplication by $(3 - x)$ to find the coefficient of x^3 was often completed successfully. Several candidates stated the term in x^3 correctly but did not go on to state the coefficient of x^3.

A small number of candidates attempted the McLaurin expansion, which was a valid alternative method although involved rather more work and increased opportunities for arithmetic errors.

Question 2

Those candidates who recognised 9^x as $(3x)^2$ usually went on to form and solve a correct quadratic equation. Many candidates rejected the possibility of $3^x = -3$ and obtained the correct final answer. The usual ways to solve $3^x = 4$ were to start with $xIn3 = In4$ or to use $x = log_3 4$.

Candidates who did not recognise the equation as a quadratic in 3^x often started with an incorrect use of logarithms to obtain the incorrect equation $xIn9 = xIn3 + In12$. Another common error was to use $9^x = 3 \times 3^x$.

Question 3

The majority of candidates chose to rewrite the given equation as a quadratic in $tan\theta$, with many reaching the correct equation $tan^2\ \theta\ = \frac{1}{5}$. Some candidates did not then use the square root, and some only obtained one answer correctly because they did not consider both square roots. The alternatives of starting with $cot\ 2\ \theta = \frac{cos\ 2\theta}{sin\ 2\theta}$ and using the double angle formulae to form an equation in sinθ or in cosθ were quite common and usually successful.

Question 4

Most candidates applied the quotient rule or the product rule to find the derivative of $\frac{x}{1+ln\,x}$. The chosen rule was usually applied correctly. Many candidates simplified $\frac{1+ln\,x-\frac{x}{x}}{(1+ln\,x)^2}$ correctly, but there were a minority of candidates who 'cancelled' $(1\ +\ Inx)$ to obtain an incorrect answer such as $\frac{-1}{1+ln\,x}$. There was some confusion between the tangent and the normal, but several candidates reached the correct equation $\frac{ln\,x}{(1+ln\,x)^2} = \frac{1}{4}$. Only the stronger candidates went on to reach a correct quadratic in lnx. There were errors due to incorrect use of brackets and notation: $(lnx)^2$ was often written as ln^2x and this then became lnx^2. Some candidates who reached the correct quadratic in lnx then used the substitution $x\ =\ lnx$ to simplify their equation and stopped work when they had found a value for their x. Several candidates who obtained the correct solution for x then went into decimals so they did not obtain an exact value for y.

Question 5

(i) The majority of candidates were aware that the conjugate of the given root would also be a root of the equation. A few candidates changed both signs, and some did not include the i in their response.

(ii) The most popular approach was to start by substituting the given root into the equation to find the value of k. The question specifically ruled out the use of a calculator. Those candidates who simply wrote down $\left(-1+\sqrt{3}i\right)^3 = 8$, with no working shown, gained no credit. The common alternative approach was to use the two known roots to form the quadratic factor $x^2 + 2x\ +\ 4$ and then use the linear factor $kx\ +\ a$ to compare coefficients and solve for k. When successful, this approach was very efficient because it gave the third root at the same time. However, the common error was to combine the quadratic factor with a linear factor $x\ +\ a$, in which case no further progress was possible. Some candidates obtained the correct linear factor and stated that this was the root. Several candidates used their calculators to state the third root and gave no supporting working, so no marks were scored.

Question 6

(i) The simplest solution to this question was to use the isosceles triangle *OAB* to form two identical right angled triangles and then use $\frac{\frac{1}{2}AB}{r} = cos\,x$. Several candidates simply wrote down $\frac{AB}{2r} = cos\,x$ with no justification, and gained no credit. It was quite common to see longer solutions involving the sine rule or cosine rule. Many candidates offered no solution.

(ii) Candidates needed to equate half the area of the semicircle to the area of the sector. The given answer enabled several candidates to find their errors and correct their solutions. The most common errors occurred in the area of the sector, with many candidates using an angle of x rather than$2x$. There was a lot of evidence of misuse of factors of 2, with not all errors being traced back to their source. Some candidates who started with a correct equation lost the factor of x from the $16x$ in the denominator in the course of their working.

(iii) There were two common approaches used by candidates: either they considered an equation of the form $f(a) = 0$, or they worked with an equation of the form $f(a) = a$. Many candidates were unable to distinguish between the two forms and tackled the latter but were hoping to find a sign change. There were several calculation errors in combining the surd and the inverse trigonometric function. A few candidates worked in degrees rather than radians.

(iv) Many of the candidates who calculated values of the function correctly reached the correct solution having used the required level of accuracy. Candidates who used an initial value of 1.25 or 1.5 were more likely to reach the correct final answer than those who started from 1. This was largely due to the fact that candidates kept applying the iterative formula until two successive answers rounded to the same value correct to 3 decimal places. In cases where an initial value of 1 was used, the convergence was too slow for this to give the correct answer. When the convergence is slow, candidates should continue beyond the point where they think that they have found the root to ensure that the final figure is correct.

Question 7

(i) The majority of candidates started correctly, by separating the variables and attempting to integrate. The separation was usually completed correctly, and most candidates recognised the need to use integration by parts to integrate the function in x. Several candidates had $\int \frac{1}{e^y} dy$ and mistakenly believed this to be $ln(e^y)$. Those candidates who started with the form $\int e^{-y} dy$ were more likely to reach a correct answer. Many candidates went on to find the correct value for the constant of integration. The final step, going from an expression for $-e^{-y}$ to an expression for y was more challenging for some candidates in dealing with the negatives and using logarithms.

(ii) Candidates needed to be aware that $ln(1 - x)$ is only defined for values of x less than 1. Many answers were not expressed correctly, and it was commonly incorrectly stated that $ln(a)$ could not be negative.

Question 8

(i) Many candidates started correctly on the partial fractions. There were a few slips in the algebra and the arithmetic, but many fully correct solutions were seen. Candidates are advised to split the fraction as far as possible. Those candidates who opted for the partial form of $\frac{A}{2x+1} + \frac{Bx+C}{(2x+3)^2}$ could gain full marks in part **(i)** but they still had a lot of work to do before they could complete part **(ii)**.

(ii) Many candidates recognised the exact derivatives and integrated to obtain answers of the correct form. Those candidates who lost the 2 in the first log integral usually repeated the error in the second log integral. There were some sign and coefficient errors in integrating $(2x + 3)^{-2}$. It was rare for candidates who started with the two term form of the partial fractions to make any progress beyond integrating the first term. Many candidates with incorrect coefficients in their integration provided the correct answer, but the working often didn't contain sufficient detail to support the answer.

Question 9

(i) Many candidates were able to find the point of intersection of two lines. The majority of candidates who formed the equation of the line *AB* correctly went on to form and solve simultaneous equations and demonstrated clearly that the two lines did not intersect. There was one very common error: candidates who reached a correct equation $3\lambda = 2$ often concluded that $\lambda = \frac{3}{2}$.

(ii) Several candidates found half of the vector $\overrightarrow{AB}$ rather than the position vector of the midpoint of *AB*. It was generally understood that the direction vector of the line *AB* was perpendicular to *m*, and several candidates were able to use this, with their midpoint, to find an equation for *m*. Candidates who did not appreciate the significance of the line being perpendicular to the plane tried to use the vector product to find a normal vector, but did not start with directions in the plane so were not successful. Several candidates showed understanding of the correct method for finding the point of intersection of a line with a plane, and those who used all the information correctly usually reached the correct final answer.

Question 10

(i) Most candidates gave a correct expansion of at least one of $sin(3x + x)$ and $sin(3x - x)$. Some candidates then went on to use these correctly to derive the given result. Several candidates did not link their work to $sin4x$ and $sin2x$.

(ii) Although most candidates were aware of the need to find the value of $\int sin\,3\,x\,cos\,x\,dx$, many were unable to use the result of part **(i)** with this task. As well as completing the integration correctly, candidates also needed to determine the upper limit for their integral. It was common to see the incorrect value $\frac{\pi}{2}$ being used. Only the stronger candidates reached the final answer correctly. A number of candidates did not 'show all necessary working' and no marks were given for an answer that was not supported by correct working.

(iii) Many candidates understood that they needed to find $\frac{dy}{dx}$ in order to find the coordinates of *M*. Several did not follow the instruction to use the result of part **(i)** and started by differentiating the original form of the function. This never resulted in an equation in $cos2x$, but could have been followed through to find the x-coordinate of *M*. Several candidates who started with the correct form $2cos4x + cos2x$ rearranged this correctly to obtain an expression in $cos2x$, provided that they used a correct form of the double angle formula. Some candidates never obtained an equation in $cos2x$ because they used the double angle formula on both terms at the same time. If they followed this through, they could reach the final answer by forming a quartic in $cosx$. Several candidates who were solving a correct equation gave the final answer as the x-coordinate of the maximum point, rather than of *M*.

MATHEMATICS

Paper 9709/33
Paper 33

General comments

The standard of work on this paper was often high. However, some candidates found it challenging to answer the questions involving a given answer, for instance **Question 2**, **Question 4(ii)** and **Question 6(ii)**.

Questions, or parts of questions, that were generally well answered were **Question 1** (laws of logarithms), **Question 3(i)** (trigonometrical identity), **Question 3(ii)** (integration), **Question 5** (differential equation), **Question 6(iii)** (numerical iteration), **Question 7(i)** (product rule), **Question 7(iii)** (solution of trigonometrical equation), **Question 8(i)** and **(ii)** (complex numbers, modulus and argument), **Question 9(i)** (partial fractions), **Question 9(ii)** (binomial expansions), **Question 10(i)** (perpendicular distance from point to line) and **Question 10(ii)** (equation of line that lies in a plane).

Questions that proved to be more challenging were **Question 2** (definite integral using integration by parts), **Question 4(i)** and **(ii)** (quotient rule and solution of quadratic equation in e^a), **Question 6(i)** and **(ii)** (roots of quartic equation), **Question 7(ii)** (use of $cos(A + B)$ in finding an expression for a stationary point) and **Question 8(iii)** (Argand diagram).

In general the presentation of the work was good, though there were some rather untidy scripts. Candidates should bear in mind that scripts will be scanned for marking and they should use a black pen, appropriately sized lettering and symbols, and present their work clearly. Candidates should avoid using ink that is absorbed into the paper and then appears on the reverse side as this can make it difficult to read the pages when scanned.

It was pleasing to see that many candidates were aware of the need to show sufficient working in their solutions such as in the context of solving a quadratic equation and substituting limits into a definite integral.

Key messages

- Know what is meant by an exact answer.
- Apply the method asked for in the question.
- Use brackets correctly when manipulating algebraic expressions.
- Present all detailed working when the answer is given in the question.
- Write clearly and do not overwrite one solution with another.

Comments on specific questions

Question 1

Many fully correct answers were seen, and only a few candidates were unable to present their final answer to the accuracy required. Candidates were, however, asked to use logarithms to solve the equation. This should have been done immediately to obtain a linear equation in x. Many candidates started instead by using the laws of indices, then took logarithms to non-standard bases. If correctly done, this received full credit but it often led to errors. A significant number of candidates omitted the brackets around $3 - 2x$, introducing errors.

Question 2

Most candidates succeeded with the initial integration by parts but poor presentation often resulted in sign errors and arithmetic errors. In many cases, different parts of the second integration by parts were scattered over the page and limits were introduced on different lines. It would have been much clearer to obtain the complete indefinite integral before substituting any limits. Many candidates who did reach the correct indefinite integral then just wrote down the given answer. However, candidates needed to show all of the details of substituting both limits into all three terms.

Question 3

(i) Many candidates could have saved time by choosing carefully the identity for $cos2\theta$, i.e. different versions in the numerator and the denominator. However, despite this most candidates obtained the given answer correctly, showing all the necessary working.

(ii) In this question, too, detailed working was required for a given answer. Candidates generally showed sufficient working except at the end where several ln operations were often combined into the given answer without showing intermediate steps. Sign errors were seen in the indefinite integral. Some candidates chose to reverse the limits to take account of the negative sign. In this method, the details needed to be clearly shown to justify the change of sign.

Question 4

(i) Most candidates applied the correct quotient or product rule, but some made sign errors in differentiating exponential functions. To prove that the derivative was negative, it was necessary to reduce the expression to a single term in the numerator and a squared term in the denominator.

(ii) To make any progress with obtaining the given equation, candidates needed to have a correct expression from part **(i)**. Many candidates with a correct expression were unable to make any progress, for example inverting terms individually to change the equation from one in e^{-a} to one in e^{a}. Answers without sufficient detail of how they were obtained did not receive any credit. Although few candidates obtained the equation correctly, most realised it was a quadratic in e^{a} and were able to solve it. Candidates needed to check any constraints on the domain, in this case $x > 0$, giving a clear reason why one of the solutions had to be rejected. Most candidates did not reject one of the roots, and many did not give the final answer in exact form.

Question 5

Most candidates scored well on this question provided they had separated the variables correctly. Most of the errors seen were either incorrect exponentiation or an incorrect coefficient for the $ln(y^2 + 5)$ term, e.g. 2 instead of $\frac{1}{2}$.

Question 6

(i) A large number of candidates thought that *b* was 6. Few candidates showed any working towards their answer, even when it was correct. Candidates needed to realise that *b* was an integer root of the quartic equation then, using the factor theorem, substitute a few integer values to determine y at these points. So, for example, they should have shown working: $y(2) = -20$ and $y(4) = 94$ or $y(3) = 0$.

(ii) Many candidates tried unsuccessfully to work backwards from the given answer. From part **(i)** they knew $(x - 3)$ was a factor of the quartic equation and could find the cubic factor either by inspection or by long division.

(iii) Of the candidates who realised that they needed to start with an initial value of *a* < 1, nearly all of them produced completely correct answers. Sign errors were occasionally seen in the final answer.

Question 7

(i) This was another question where most candidates obtained full marks. However, some candidates expanded $sin(x + \frac{\pi}{3})$ before differentiating. This was inefficient since it required the use of the product rule twice instead of once. If candidates combined trigonometrical functions too, they also needed the chain rule and the possibility of errors increased. This unnecessary work led candidates to further problems in part **(ii)**.

(ii) The unnecessary work undertaken in part **(i)** often made it difficult for candidates to recover to $cos(2x + \frac{\pi}{3}) = 0$ at the stationary point. In contrast, candidates who differentiated straight away in part **(i)**, without any further work, could immediately express their expansion as $cos(x + \frac{\pi}{3} + x) = cos(2x + \frac{\pi}{3}) = 0.$

(iii) The expression in part **(ii)** led most candidates to at least one of the required answers in part **(iii)**. Most candidates obtained the other correct answer, but some looked in the wrong quadrant.

Question 8

(i) Most candidates could express *u* as $x + iy$ and few errors were seen.

(ii) Although most candidates obtained the correct modulus, a large number of candidates had the argument in the wrong quadrant. A diagram is often helpful for checking the answer is correct.

(iii) It was essential that the scales on the Real and Imaginary axes were identical, otherwise the position of *u* would be distorted. The perpendicular bisector was usually correct. In many cases, the circle was not centred at the origin nor was its radius indicated. It was essential that the coordinates of *u* and the radius of the circle were clearly shown.

Question 9

(i) This question was answered well by the majority of candidates. There were some miscopying errors such as writing $(1 - x^2)$ or $(1 + x)^2$ instead of $(1 - x)^2$. Another error was introduced when multiplying throughout by $(1 - x)^3(3 + x)$ instead of $(1 - x)^2(3 + x)$.

(ii) This question was also generally answered well. The main errors seen were in dealing with the constant 3 from $\frac{1}{3 + x}$, usually resulting incorrectly in $\frac{1}{1+\frac{x}{3}}$, and in being unable to handle the $-x$ in the expansions of $\frac{1}{1-x}$ and $\frac{1}{(1-x)^2}$.

Question 10

(i) Candidates often scored full marks on this question. Errors that did occur were usually sign errors. However, there were some candidates who incorrectly used the vector OP in their working instead of $OP - (\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k})$.

(ii) Many candidates answered this question well. The two approaches were either to consider two points on the line *l* since they were also in the plane, or to consider one point on the line (in the plane) and the direction vector of *l* (perpendicular to the plane). Either method led to two equations that could be solved simultaneously.

MATHEMATICS

Paper 9709/41
Paper 41

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Cases where this was not adhered to were seen in **Question 3**, **Question 4** and **Question 6**. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3sf.
- When answering questions involving an inclined plane, a force diagram can help candidates ensure they include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable on this paper in **Question 3, Question 4** and all parts of **Question 6**.
- In questions such as **Question 5** in this paper, where velocity is given as a quadratic function of time, calculus must be used and it is not possible to apply the equations of constant acceleration.

General comments

The paper was well done by a number of candidates although a wide range of marks was seen.

The presentation of the work was good in most cases and, as the papers are now scanned, it is important to write answers clearly using black pen.

In **Question 1, Question 3** and **Question 4**, angles were given in terms of the sine or the tangent of the angle and these can often lead to exact answers. There is no need in such questions to evaluate the angle itself.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Question 1** was found to be the easiest question whilst **Question 5** and **Question 6(ii)(c)** proved to be the most challenging.

One of the rubrics on this paper is to take g = 10 and it has been noted that virtually all candidates are now following this instruction. In fact in some cases, such as here in **Question 3(i)** and **Question 4(i),** it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

Most candidates made a reasonable attempt at this question. The main errors here were to approximate the angles rather than to use the given information to find the sine and cosine of α and θ so that all calculations were exact. It was then impossible to produce an accurate value of the forces and these forces could not be shown to be in equilibrium due to rounding errors. Most candidates attempted to resolve forces horizontally and vertically, and attempted to demonstrate the state of equilibrium.

Question 2

(i) This question involved use of the constant acceleration formulae and most candidates made an attempt. The most straightforward approach was to use the equation $v = u + at$ with v = 0, u = 25 and $a = -g = -10$. Substitution of these values gave the required value for the time t taken to reach the greatest height. An error that was often seen was to take $a = g$ which led to a negative value of t so full credit could not be awarded. This error could have been avoided by stating which direction was being taken as positive e.g. upwards or downwards.

(ii) There were several methods available to find the required time. It had to be remembered here that the initial point of projection was 3 m above ground level and so it was necessary to find when the particle had travelled upwards for 20 m from projection. One method was to use the given information to find the times at which the particle was 20m above its initial position by using the equation $s = ut + \frac{1}{2}at^2$ with $s = 20$, $u = 25$ and $a = –10$. This gave a quadratic equation which, when solved, gave two values of t; the times at which the particle was 20 m from its initial position on the way upwards and on the way down. The difference between these values gave the required time. Many candidates attempted to use this method but often used $s = 23$ rather than $s = 20$.

(iii) Again there were several methods available to find the value of h. One method was to realise that P would take 0.5 s to reach the highest point and 0.5 s to return to the height h. Since P took 2.5 seconds to reach the highest point, the height h was reached after 2 seconds of motion. By using the constant acceleration equation $s = ut + \frac{1}{2}at^2$ with $u = 25$, $t = 2$ and $a = –10$, the distance travelled by P in the first 2 seconds could be found. It had to be remembered that it was necessary to add 3 m to this height since the question asked for the height above the ground. Many candidates did in fact forget to add the extra 3 m. Another error seen was to merely substitute $t = 1$ into the equation $s = ut + \frac{1}{2}at^2$ since 1 second was referred to in the question.

Question 3

(i) In this question the lorry was moving at constant speed and hence had zero acceleration. The driving force, DF, therefore had to exactly balance the two forces due to resistance and the component of the weight of the lorry down the hill. Once this driving force was found, the relationship $\text{DF} = \frac{P}{v}$ had to be used, where P was the power and v was the speed of the lorry. Substitution of the given values produced the required answer. Errors that were seen included omitting the weight component of the lorry, or using the mass of the lorry, 12 000 kg, rather than the weight, 12 000g N, when finding this component. Also the slope of the hill was given exactly and there was no requirement to evaluate the angle of the slope. In fact, if an approximation to the angle itself was used, the exact given answer could not be achieved.

(ii) In this part of the question candidates had to use the fact that when the speed of the lorry was 5 ms^{-1}, the resistance was 1500 N. If this information was used in the given expression for resistance as kv^2 then k could be found. Some candidates continued to use the component of the weight when attempting to find k, but the given expression only related to the resistance.

(iii) In this situation the lorry moved along a straight level road and so there was no direct effect of the weight. The driving force, DF, had to exactly balance the resistance in the form $60v^2$. Again use could be made of the relationship $\text{DF} = \frac{P}{v}$ and this gave rise to a cubic equation for v which could be solved to give the required speed. Many candidates used this method but some did not realise that the given expression for resistance should be used in this part.

Question 4

(i) This question included a given exact answer and the tangent of the angle involved was given. In cases such as this it is vital that the angle itself is not used but that the given information is used to find $\sin\theta$ and $\cos\theta$ exactly as fractions. As the particle was on the point of moving up the plane, by resolving forces along the plane it could be seen that the force of 20 N was exactly balanced by the weight component and the friction force, F. Since $F = \mu R$ it was necessary to resolve forces perpendicular to the plane in order to find the normal reaction, R. Once this had been found the equilibrium equation along the plane could be solved for μ. Many candidates used this method correctly but some did in fact find the angle and hence lost some accuracy and were not able to obtain the exact value for μ.

(ii) In this part the motion was governed by two forces, the component of the weight down the plane and the effect of the friction force acting up the plane. Use of Newton's second law enabled the acceleration to be found. Errors seen here were to take both forces to be acting down the plane or to ignore the effect of the component of the weight. In some cases the mass rather than the weight was used when finding the weight component down the plane.

(iii) The most direct method to find the required work done against the friction force, F, was to find the distance, d, moved in the first 2 seconds of motion and then use WD = $F \times d$. Most candidates attempted to use this method. An alternative approach was to use energy principles, but this involved finding both PE lost and KE gained. When using the first method an error that was often seen was to use $d \cos \theta$ in the formula for work done when in fact the friction force moved directly parallel to the plane.

Question 5

(i) There were several ways of finding the minimum velocity of P. One was to use calculus and differentiate the velocity to find acceleration, a, and then set $a = 0$ to find the time at which the velocity was minimum and then substitute this value of t into the given expression for v. Alternatively the method of completing the square for the quadratic expression for velocity could be used and this enabled the time at which the minimum occurred to be found and gave the required value of v directly. As the expression for v was a quadratic, it was also possible to sketch the graph and by calculating where $v = 0$, the minimum could be found by using symmetry. Most candidates used one of these methods to find the required minimum value of v. This part was well done by most candidates.

(ii) Most candidates correctly realised that integration of the expression for velocity was needed to find the total distance travelled by P in the first 8 seconds. However, many candidates did not realise that the velocity-time graph was partly above the t-axis and partly below. This needed to be taken into account when performing the integration. It was first necessary to find where $v = 0$ so that the different regions of positive and negative velocity could be found. This happened at $t = 2$ and $t = 6$. Two regions had positive velocity, namely $0 < t < 2$ and $6 < t < 8$ and the middle region, $2 < t < 6$, was where velocity was negative. Three separate integrals had to be used to find the total distance with the sign of the middle region being reversed as it gave a negative contribution. The majority of candidates did not consider this but merely integrated the velocity between $t = 0$ and $t = 8$. This gave the displacement but not the required distance.

Question 6

(i) In this question, since the particles were in equilibrium, the tension in the string was balanced by the weight of the 0.2 kg particle and it was also balanced by the component of the weight of the 0.4 kg particle down the plane. Equating these two expressions for the tension gave the sine of the required angle and hence the angle could be found. Most candidates solved this correctly. An error that was seen was to transpose sine and cosine when resolving the weight of the 0.4 kg particle.

(ii) (a) Since the angle given in this part was less than that found in **part (i)**, the 0.2 kg particle would move downwards. It was necessary to write down Newton's second law for both particles. It was also possible to replace one of these equations with one that is applied to the whole system. This produced two equations in the two variables, T, the tension in the string and, a, the acceleration. These could then be solved to give the required values. Most candidates used this method. Errors seen were to forget to include g when dealing with the weight of the particles and confusing sine and cosine when resolving.

(b) Most candidates who found the acceleration in **part (ii)(a)** attempted this part and generally it was well done. The particle started from rest and had a known acceleration and so use of the equation $v^2 = u^2 + 2as$ with $u = 0$, $a = 1.05$ and $s = 0.5$ gave the required value of v. Most candidates attempted this part and even those who had found an incorrect value for a in the previous part were able to gain credit for using a correct method.

(c) This question specifically asked for a solution using energy rather than Newton's second law. There were several different possible approaches. However, candidates found this to be the most difficult question on the paper. The most straightforward approach was to consider only the motion of particle A after particle B reached the ground. In question 6**(ii)(b)** the speed of particle A at the instant that particle *B* reached the ground was found. In the subsequent motion of particle *A*, all of its kinetic energy was transformed into potential energy. There is no tension in the string during this motion, but many candidates used the value of the tension found in 6**(ii)(a)**. If *d* was the extra distance along the plane moved by *A* after *B* reached the ground then the PE gain was PE = $0.4g \times d \sin 20$ and this had to be equated to an expression for the initial kinetic energy in the form KE = $\frac{1}{2} \times 0.4 \times 1.03^2$. Equating PE gain = KE loss gave the required value of *d*. It had to be remembered that the question asked for the total distance moved by *A* and this was 0.5 + *d*. Errors made by candidates included use of tension during this motion and consideration of kinetic and potential energy of *B* even though it was stationary.

MATHEMATICS

Paper 9709/42
Paper 42

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Question 1**, **Question 2**, **Question 3, Question 4** and **Question 5**. Candidates are advised to carry out all working to at least 4 significant figures if a final answer is required to 3sf.
- When answering questions involving an inclined plane, a force diagram can help candidates to ensure they have included all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 3** and **Question 4**.
- In questions such as **Question 7** in this paper, where velocity is given as a quadratic function of time, calculus must be used and it is not possible to apply the equations of constant acceleration.
- If an angle is given in terms of the sine, cosine or tangent of the angle then there is no requirement to evaluate the angle. When needed, all other trigonometric values can be obtained from the given information. There are examples of this in this paper in **Question 3**, **Question 4** and **Question 6**

General comments

The paper was generally well done by many candidates, although a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

In **Question 3, Question 4** and **Question 6**, values of trigonometric functions were given and these can often lead to exact answers.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Question 5** was found to be the easiest question whilst **Question 7(iii)** proved to be by far the most challenging.

One of the rubrics on this paper is to take $g = 10$ and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

Comments on specific questions

Question 1

This was a standard problem on force systems and the most straightforward approach was to resolve forces horizontally and vertically. This was the method used by the majority of candidates. Most candidates performed well on this question. Errors were seen due to the mixing up of sine and cosine particularly when evaluating components of the 17 N force. Another error often seen was accuracy being lost because of premature rounding. Candidates should again be reminded that in questions such as this they should keep all intermediate calculations to 4 or 5 decimal places in order to produce a final answer correct to 3sf. It was not possible to use Lami's theorem in this problem since there were 4 forces acting.

Question 2

This question involved motion that required the application of the constant acceleration equations. The most direct method, which was applied by the majority of candidates, was to apply the equation $s = ut + \frac{1}{2}at^2$ to the motion over the first 5 seconds using $t = 5$, and to apply the same equation for the first 8 seconds using $t = 8$. This gave a pair of simultaneous equations in the variables u and a. Solving these equations produced the required solution. An error that was seen was candidates attempting to apply the equation to the final 3 seconds of motion but wrongly using the same u as for the first 5 seconds. It was possible to apply the equation to the final 3 seconds but u had to be replaced by $u + 5a$. Another error that was seen was candidates misreading the given information, thinking that the 160 m referred to a distance in addition to the first 80 m, and applying the equation $s = ut + \frac{1}{2}at^2$ with $s = 240$ and $t = 13$. Overall this question was well done by the majority of candidates. Although it was possible to determine u and a exactly as fractions, some candidates used decimals and in some cases did not give answers correct to 3sf.

Question 3

This was a question in which the angle, θ, was given in terms of tan θ which meant that it was easy to find sin θ and cos θ exactly without calculating the angle. There were two different methods to approach this problem. In both cases it was necessary first to find the friction force. Resolving forces perpendicular to the plane gave the normal reaction, R, as $R = 13g \cos\theta$. Since motion was taking place $F = \mu R = 0.3 \times 13g \cos\theta$. One method of solution was then to resolve forces along the plane and since there was no acceleration, the force T N exactly balanced the friction force plus the component of the weight down the plane. This gave the value for T. Application of work done = $T \times 2.5$ gave the required answer. Another method was to use the work-energy principle which in this case stated that the work done by T was equal to the sum of the work done against friction and the gain in potential energy. Most candidates made a good attempt at this question. An error seen when using the relationship WD = $T \times 2.5$ was to wrongly also multiply this by cos θ. Some candidates made sign errors either when resolving forces or when combining work done against friction and potential energy and others mixed up sine and cosine of the angles.

Question 4

This was another question with the angle given in terms of sin θ and so it was not necessary to calculate the angle explicitly. There were two approaches that could be used here. Either Newton's second law could be applied to the motion of the car or alternatively the work-energy principles could be used. The majority of candidates chose to use Newton's second law. If this approach was adopted then there were three force terms to consider: the given driving force; the given resistive force; and the component of the weight down the plane. This combination of forces needed to be equated to $1250a$ and solved for a in both the up and down cases. An error seen when candidates applied this method was to omit one of the force terms or to use incorrect signs when combining them. Once the value of a had been found, application of the constant acceleration equations such as $v^2 = u^2 + 2as$ could be used to find the required speed. If the work-energy method was used then the equation consisted of 4 terms, namely the work done by the driving force, the work done against the resistance force, the change in potential energy and the change in kinetic energy. Both up and down motion involved the same terms but they needed to be combined with the correct signs, and errors in sign were the main cause of incorrect solutions. The kinetic energy term involved the required speed v and the equation could be solved to find this speed in each case. Another error that was frequently seen was the inclusion of a potential energy and a work done by the weight term, so including the potential energy twice. This question was well done by a large number of candidates.

Question 5

(i) This question included a given answer so candidates needed to show all of their working. Since the acceleration was required it was best to apply Newton's second law to both particles or to one particle and to the system. This produced two equations involving the tension, T, in the string and the acceleration, a, of each particle. Solving the equations gave the required results. Most candidates produced very good solutions to this question. An error that was seen on a few occasions was to think that the tension acting on particle A was different to the tension acting on particle B. Since the question asked to show the acceleration was $\frac{10}{3}$, it was important that candidates did not simply use their calculator to solve the equations as this is not a method for showing such a result and some detail of the method of solution had to be seen.

(ii) There were several methods available to find the maximum height of B. It was necessary to find the speed of particle B as particle A reached the ground and this could be achieved by using the equation $v^2 = u^2 + 2as$ with $u = 0$, $a = \frac{10}{3}$ and $s = 0.5$. Solving this gave the required speed, which then became the initial speed of the subsequent motion of B as it moved when the string became slack. Particle B then moved under the influence of gravity until coming to rest. Further use of the equation $v^2 = u^2 + 2as$ with u taken as the initial speed, $v = 0$ and $a = -g = -10$ and the extra height travelled after A reached the ground was found as $s = \frac{1}{6}$. The maximum height of B above the ground was found using $0.5 + 0.5 + \frac{1}{6}$. Many candidates made a good start to this part of the question but forgot to add the extra 0.5 m that particle B had travelled up to the point when A reached the ground.

Question 6

This question proved to be difficult for some candidates. Two situations were described in the question and it was necessary to write down the equations which described the motion in each case. The resistive force was given in terms of the velocity of the particle. In the first case, where the particle moved on a horizontal road, the speed was given as 18ms^{-1} and the driving force could be evaluated from the given information as $\frac{36000}{18} = 2000\text{ N}$. The driving force in this case was exactly balanced by the resistive force, leading to the equation of motion as 2000 = 18A + B. Many candidates did not substitute the value of $v = 18$ into their equation. In the second case the car moved up a hill with a given slope and in this case the driving force of $\frac{21000}{12} = 1750\text{ N}$ was balanced by a combination of the resistance and the component of the weight of the car down the slope. The value $v = 12$ had to be used and the resulting equation of motion took the form 1250 = 12A + B. These two simultaneous equations could then be solved for A and B. The candidates who substituted the relevant values of v into their equations found this to be a straightforward problem but a large number of candidates seemed to be confused by the fact that the resistance force was not constant and were reluctant to substitute a value for v. The condition $v > 2$ was given in the question, since this guaranteed that the resistance was positive. However, some candidates thought that it was necessary to apply this condition and several cases were seen where values of $v = 2$ and $v = 3$ were wrongly used.

Question 7

(i) Candidates were asked to sketch the velocity-time graph for the two particles. Although most candidates sketched the correct straight line graph for the motion of particle Q, a significant number of candidates chose to represent the quadratic expression for P as a set of connected straight lines rather than the correct quadratic curve passing through (0,0) and (3,0). In order to fully represent the nature of the motion it was vital to annotate the axes with values at the critical points and often this also was not seen.

(ii) There were several different ways of showing that P and Q met after 5 seconds. In all cases the displacements had to be found. For particle P this involved integrating the quadratic function. For particle Q, either integration could be used or the displacement at t = 5 could be evaluated by calculating the area under the straight line from $t = 0$ to $t = 5$. If integrals were used for both then one method was to evaluate the definite integral with limits of $t = 0$ and $t = 5$ and to show that both of these equated to 25. An alternative method was to equate the two integral expressions for the displacement of P and of Q. This led to a cubic equation which could be factorised as $t(t + 1)(t - 5) = 0$ and hence it could be shown that $t = 5$ is where the displacements were the same. Most candidates made a good attempt at this question. However some equated the two given velocities which was not what was required here but was an approach for **Question 7(iii)**.

(iii) This question proved to be the most difficult on the paper for almost all candidates. There were two different approaches which could be taken. Either the expressions for displacement found in 7**(ii)** could be subtracted to find the distance between the particles at any time or it could be realised that the maximum distance apart happened when the two velocities were equal. These two methods of approach are equivalent. If the distance between the particles was found, then this expression had to be differentiated and set to zero to find its maximum value. This was equivalent to setting the two velocities to be equal. This gave a quadratic equation in t which, when solved, gave the time at which the maximum distance occurred. It was found to be at t = 3.19 seconds (to 3sf). This value of t could then be used in the expressions for the two particles and the values subtracted to determine the maximum distance. Very few candidates scored full marks on this part. A common error was to think that the maximum occurred at t = 3. This was very close to the actual value but came from an incorrect method and hence did not score marks.

MATHEMATICS

Paper 9709/43
Paper 43

Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper (e.g. **Question 2(i)** and **Question 3(i)**). Candidates are reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers (e.g. **Question 2**).
- Candidates are advised to check that when forming an equation using either Newton's Second Law or an energy approach, there are no missing or extra terms – in particular, due to the component of weight on an inclined plane (e.g. **Question 3** and **Question 5**).

General comments

This paper provided the opportunity for candidates to apply their knowledge to a variety of routine and more difficult problems. Much work of a very high standard was seen. **Question 1**, **Question 2(i)** and **Question 4(i)** were found to be the easiest questions, whilst **Question 4(ii)**, **Question 5(ii)** and **Question 6(iii)** were found to be the most challenging.

Comments on specific questions

Question 1

This question was straightforward for most candidates. Almost all drew a correct shaped graph. Acceleration and deceleration were occasionally represented by curved lines. Time values of 24 and 30 were occasionally seen instead of 29 and 35. The value 2.1 was sometimes shown and used as the maximum velocity rather than the acceleration of the bus. Key time values such as 29 s were sometimes missing from the graph. The area property for calculating distance was well known and frequently applied accurately. Some candidates calculated the trapezium area directly whilst others opted for a lengthier solution, totalling the distances for the three separate stages of the journey or applying 'suvat' formulae to each stage.

Question 2

(i) The great majority of candidates resolved the forces and calculated both components accurately. The y-component (8.98) was sometimes seen as 9.0 or 9, either due to the approximation of each component before combining or due to a final answer corrected to two instead of the three significant figures required. A few candidates found x- and y-components for each force without combining them. Other errors included a miscalculation of 65° as 75° and a sine/cosine mix.

(ii) Most solutions made use of the components found in **part (i)** to calculate a relevant angle. A clear statement of the direction was expected but was sometimes missing. The use of a diagram was sometimes helpful in clarifying an incomplete description. A number of candidates calculated the magnitude of the resultant rather than the direction of the resultant. A few found the resultant and then used sin or cos instead of tan in a lengthier method to obtain the angle, sometimes losing accuracy through premature approximation.

Question 3

This question was frequently well answered with $P = Fv$ being applied accurately to both situations. The understanding of F as the driving force rather than '*ma*' or resistance was essential for this question.

(i) Candidates usually attempted to apply $P = Fv$. The most common error was to use an incorrect 'driving force' with at least one of the components missing e.g. $\frac{30000}{v} - 1550 = 1400 \times 0.4$ (no component of weight). The answer was sometimes given as 9.7, correct to two significant figures, instead of 9.72, correct to three significant figures as required. Occasional sign errors or sine/cosine errors were also seen. A very small number of candidates attempted to use $F = Pv$ instead of $P = Fv$.

(ii) Candidates were expected to interpret 'steady speed' to mean 'no acceleration' and to adjust the driving force accordingly. Some candidates repeated the driving force from **part (i)** including an acceleration term. Others omitted a term from their equation using e.g. $\frac{P}{40} = 1550$ (no component of weight).

Question 4

Whilst **part (i)** was a routine pulley problem, **part (ii)** was challenging for many and this part of the question was a good differentiator.

(i) The majority of candidates applied Newton's Second Law to each particle and solved the resulting simultaneous equations successfully. Since 3 ms^{-2} was mentioned in the question, some candidates assumed $a = 3$, rather than showing $a = 3$ as required. Some showed $a = 3$ and omitted to find the tension in the string. Candidates should be aware that in a 'show that' question, stating a calculator solution for simultaneous equations may not be sufficient.

(ii) A variety of approaches were seen. Many candidates found accurately either the common height 1.375 m or the 'attached' distance 0.375 m. Some candidates realised that the break occurred halfway between 1 m and 1.75 m, whilst others formed and solved various equations. Those who formed a single equation for each particle after the break were often able to obtain the correct time difference although some assumed a velocity of 0 ms^{-1} at the 'break' position and others found no time difference by solving $1.375 = 1.5t_B + \frac{1}{2}gt_B^2$ instead of $1.375 = -1.5t_B + \frac{1}{2}gt_B^2$. Those who split the unattached motion for particle B into several stages made more errors, such as omitting one stage of motion e.g. $t_B = 0.15 + 0.395$ instead of $t_B = 0.15 + 0.15 + 0.395$.

Some candidates oversimplified the situation by assuming that each particle moved with $a = g$ from the initial position, solving $1.75 = \frac{1}{2}gt_A^2$ and $1 = \frac{1}{2}gt_B^2$. Others assumed that the string broke when particle A reached the ground, rather than when particles A and B were at the same height. A very small number of candidates recognised that the time difference could be found most simply by calculating the journey time for particle B to travel from break position to maximum height and back again.

Question 5

(i) **Part (i)** was frequently well answered with PE gain equated to KE loss to obtain the distance moved up the plane. Some candidates found the height gained rather than the required distance. Others formed the equation in h (height) and then used trigonometry ($h/\sin 30^\circ$) whilst others found the distance directly. Errors with trigonometry led to incorrect answers, such as 10 m from $20\sin 30^\circ$ instead of $20/\sin 30^\circ$. Some candidates formed a three term equation, including both potential energy and work done against the component of weight without realising that these are the same. Sign inconsistencies were also seen such as $\frac{1}{2} \times 18 \times 20^2 = -18 \times 10 \times d \sin 30^\circ$; $d = 40$. Although the question specified 'use an energy approach', some candidates solved the problem using Newton's Second Law and '*suvat*' formulae.

(ii) Candidates were expected to consider the two stage journey up and down the rough plane. Those who did this often successfully obtained 12.6 ms^{-1}, either using an energy approach or Newton's Second Law or a combination of both e.g. an energy equation up the plane and use of Newton's Second Law on the way down. Those using an energy method sometimes included an extra term in their equation, with repeated potential energy as in **part (i)**. Candidates needed to be aware that the direction of friction was reversed for the downhill motion, to avoid a sign error.

Many candidates misinterpreted the situation and solved e.g. $\frac{1}{2} \times 18v^2 = 18g\sin 30^\circ \times s + 0.25 \times 18g\cos 30^\circ \times s$ using $s = 40$ m from **part (i)**, suggesting that the particle travelled up a smooth plane and then returned down a rough plane of equal distance.

Question 6

The majority of candidates recognised that integration was needed. Fully correct solutions to **parts (i)** and **(ii)** were often seen. In **part (iii)** candidates frequently mistook distance for displacement.

(i) Many candidates integrated $6t - 12$ twice to obtain $v(t)$ and $s(t)$ as expected and then solved simultaneous equations to obtain p and q. Occasional errors in integration or in the solution of the simultaneous equations led to incorrect p and q values. Some candidates ignored 'show that …' and solved the simultaneous equations without using calculus to make a connection with $a = 6t$-12. Others showed that $s = t^3 - 6t^2 + pt + q$ but omitted to find p and q. The constants of integration were sometimes treated incorrectly e.g. by attempting to find C from $v = 3t^2 - 12t + C$ or assuming $C = 0$ before integrating a second time.

(ii) $t = 1$ and $t = 3$ were frequently found as correct solutions to the correct $v(t) = 0$. A few candidates mistakenly solved $a = 0$ instead of $v = 0$. Others oversimplified the quadratic equation assuming $v = 3t^2 - 12t$ and thus solving $3t^2 - 12t = 0$.

(iii) Whilst candidates often integrated $v(t)$, they frequently overlooked the implications of **part (ii)** when finding distance rather than displacement. A common incorrect solution was thus: distance = $\int v dt$ = 4 m from using the limits 0 and 4 instead of integrating for three separate time intervals. An alternative approach used $s(t)$ from **part (i)** to calculate $s(0)$, $s(1)$, $s(3)$ and $s(4)$. Whilst this led to a correct solution (4 + 4 + 4), those who totalled the four displacements (1 + 5 + 1 + 5) instead of three distances mistakenly obtained 12 m. Some found the three distances of 4 m but erroneously added $s(0)$ to obtain 13 m, or alternatively totalled $s(1)$ and two distances of 4 m to obtain 13 m. A number of candidates integrated $s(t)$ instead of $v(t)$ and also mistakenly obtained 12 m.

MATHEMATICS

Paper 9709/51
Paper 51

General comments

Most candidates' work was neat and well presented. A few candidates gave answers to 2 significant figures instead of 3 significant figures as requested. Giving answers to 3 significant figures means that candidates should work to at least 4 significant figures. Most candidates used $g = 10$ as requested on the question paper.

The more challenging questions proved to be **Question 4(ii)**, **Question 5(ii)** and **Question 7(iv)**. **Question 1(i)** and **(ii)**, **Question 4(i)**, **Question 6(i)** and **(ii)** and **Question 7(i)** were generally well answered.

Key messages

- Refer to the formula booklet provided if in doubt of a particular formula.
- Answers should be given to 3 significant figures unless otherwise stated in the question.
- Write clearly and do not overwrite one solution with another.

Comments on specific questions

Question 1

This question was generally well answered by most candidates.

(i) This part of the question required candidates to resolve vertically.

(ii) Candidates needed to use the equation $F = \frac{mv^2}{r}$ in the horizontal direction.

Question 2

This question was generally well answered. Candidates were required to find the horizontal and vertical velocities after 4 seconds. Having completed that, candidates needed to use Pythagoras' theorem and trigonometry of a right-angled triangle.

Question 3

Many candidates were aware that it was necessary to take moments about the plane face of the solid. The volumes and the centres of mass of the two solids had to be found before taking moments.

Question 4

(i) Candidates generally scored well in this part of the question. Some candidates found x and y in terms of t but then didn't eliminate t correctly.

(ii) This part of the question proved to be more challenging for many candidates. One method was to differentiate the equation found in part **(i)** to find $\frac{dy}{dx}$. $\frac{dy}{dx}$ could then be replaced by $tan 15$ and the resulting equation could be solved to find the required value of x.

Question 5

(i) This part of the question was generally well answered. Candidates needed to set up a 2 term energy equation.

(ii) This part of the question was more challenging for many candidates. Candidates needed to recognise that the greatest speed occurred at the equilibrium position. With that in mind it was necessary to set up a 3 term energy equation.

Question 6

(i) This part of the question was generally well answered.

(ii) This part of the question needed candidates to use $\tan\theta = \frac{0.1}{0.2}$, where θ was the required angle.

(iii) Many candidates were aware that it was necessary to take moments about B in order to solve this part of the question.

Question 7

(i) This part of the question was generally well answered.

(ii) In this part of the question candidates were required to integrate the equation from part **(i)**. Having done that, the correct limits had to be substituted. This was generally well answered.

(iii) Most candidates realised that they needed to include another term in their equation. This term should have been $\frac{16(x-1)}{1}$, namely the tension in the string.

(iv) This part of the question was challenging for most candidates. Those candidates who managed to find an equation in part **(iii)** realised it had to be integrated. Very few candidates used the correct limits and so many candidates were not able to reach the correct conclusion.

MATHEMATICS

Paper 9709/52
Paper 52

General comments

The work of most candidates was neat and well presented. A few candidates gave answers to 2 significant figures instead of 3 significant figures as requested. When giving answers to 3 significant figures this means working to at least 4 significant figures. Most candidates now use $g = 10$ as requested on the question paper.

The more challenging questions proved to be **Question 6(i)** and **(ii)**, and **Question 7(ii)** and **(iii)**. **Question 1**, **Question 3**, **Question 4(i)** and **Question 5(i)** were generally answered well.

Key messages

- Candidates should always refer to the formula booklet provided if in doubt of a particular formula.
- Answers should be given to 3 significant figures unless otherwise stated in the question.
- Write clearly and do not overwrite one solution with another.

Comments on specific questions

Question 1

(i) k could be found by comparing the given equation with the trajectory equation quoted in the formula booklet.

(ii) Again it was necessary to use the trajectory equation from the formula booklet.

(iii) Since the horizontal velocity remains constant $x = 28\cos 30 \times 3$.

Question 2

Many candidates were able to find the centre of mass from AB and/or AG but then did not use Pythagoras' theorem to find the required distance.

Question 3

This question was generally well answered.

(i) Candidates needed to find the radius and then use $F = mr\omega^2\omega^2\ \omega^2$.

(ii) This part of the question required candidates to resolve vertically.

Question 4

(i) This part of the question was generally well answered. It was necessary to use Newton's Second Law vertically.

(ii) Firstly, candidates needed to realise that the greatest downward speed occurred when the acceleration was zero. By putting $10 - 40x - 50x^2 = 0$, $x = 0.2$ could be found. Next it was necessary to integrate the equation found in part **(i)** with the constant of integration being zero. By substituting $x = 0.2$, the velocity could be found and hence the kinetic energy. Then, by using $EE = \frac{\lambda x^2}{(2l)}$, the elastic potential energy could be calculated.

Question 5

This question was generally well answered.

(i) Candidates needed to use $T = \frac{\lambda x}{L}$ twice and then solve the two equations to find a and λ.

(ii) By using $T = \frac{\lambda x}{L}$ and Newton's Second Law horizontally, the required speed could be calculated.

Question 6

(i) This part of the question proved to be challenging for many candidates. Candidates needed to find the horizontal and vertical velocities, v_H and v_V, when $t = 4$. At this point $(v_H)^2 + (v_V)^2 = 30^2$, resulting in an equation involving θ, which when solved gave the required answer.

(ii) Candidates needed to use $s = ut + \frac{1}{2}at^2$ vertically. When completed correctly this gave $s = -33.75$. Very few candidates explained that this meant below the level of projection.

Question 7

(i) A number of candidates found $r = 0.2$ from incorrect working. Some candidates used the incorrect formula for the centre of mass of the semi-circular arc.

(ii) This part of the question was challenging for many candidates. Candidates needed to take moments about A and so AC had to be found.

(iii) Many candidates found this challenging. Firstly, candidates needed to take moments about A. This resulted in the equation $0.8Y = (0.8 - 0.2\pi) \times \frac{AB}{2} + 0.2\pi \times AO$ where O is the centre of the semicircle and Y is the distance of the centre of mass from A. Using $AB = 0.8 - 0.2$ and $AO = 0.8 - 0.2 + 0.2$, Y could be found. Finally, $\tan\theta = \frac{0.1}{Y}$ could be used to find the required angle.

MATHEMATICS

Paper 9709/53
Paper 53

General comments

Most candidates' work was neat and well presented. A few candidates gave answers to 2 significant figures instead of 3 significant figures as requested. Giving answers to 3 significant figures means that candidates should work to at least 4 significant figures. Most candidates used $g = 10$ as requested on the question paper.

The more challenging questions proved to be **Question 4(ii)**, **Question 5(ii)** and **Question 7(iv)**. **Question 1(i)** and **(ii)**, **Question 4(i)**, **Question 6(i)** and **(ii)** and **Question 7(i)** were generally well answered.

Key messages

- Refer to the formula booklet provided if in doubt of a particular formula.
- Answers should be given to 3 significant figures unless otherwise stated in the question.
- Write clearly and do not overwrite one solution with another.

Comments on specific questions

Question 1

This question was generally well answered by most candidates.

(i) This part of the question required candidates to resolve vertically.

(ii) Candidates needed to use the equation $F = \frac{mv^2}{r}$ in the horizontal direction.

Question 2

This question was generally well answered. Candidates were required to find the horizontal and vertical velocities after 4 seconds. Having completed that, candidates needed to use Pythagoras' theorem and trigonometry of a right-angled triangle.

Question 3

Many candidates were aware that it was necessary to take moments about the plane face of the solid. The volumes and the centres of mass of the two solids had to be found before taking moments.

Question 4

(i) Candidates generally scored well in this part of the question. Some candidates found x and y in terms of t but then didn't eliminate t correctly.

(ii) This part of the question proved to be more challenging for many candidates. One method was to differentiate the equation found in part **(i)** to find $\frac{dy}{dx}$. $\frac{dy}{dx}$ could then be replaced by $tan\, 15$ and the resulting equation could be solved to find the required value of x.

Question 5

(i) This part of the question was generally well answered. Candidates needed to set up a 2 term energy equation.

(ii) This part of the question was more challenging for many candidates. Candidates needed to recognise that the greatest speed occurred at the equilibrium position. With that in mind it was necessary to set up a 3 term energy equation.

Question 6

(i) This part of the question was generally well answered.

(ii) This part of the question needed candidates to use $\tan\theta = \frac{0.1}{0.2}$, where θ was the required angle.

(iii) Many candidates were aware that it was necessary to take moments about B in order to solve this part of the question.

Question 7

(i) This part of the question was generally well answered.

(ii) In this part of the question candidates were required to integrate the equation from part **(i)**. Having done that, the correct limits had to be substituted. This was generally well answered.

(iii) Most candidates realised that they needed to include another term in their equation. This term should have been $\frac{16(x-1)}{1}$, namely the tension in the string.

(iv) This part of the question was challenging for most candidates. Those candidates who managed to find an equation in part **(iii)** realised it had to be integrated. Very few candidates used the correct limits and so many candidates were not able to reach the correct conclusion.

MATHEMATICS

Paper 9709/61
Paper 61

Key messages

Candidates must work with 4 significant figures or more in order to achieve the accuracy required for a 3 significant figure answer. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method. A wrong answer with no working shown gains no credit. Candidates should label graphs and axes, including units, and choose sensible scales.

General comments

It was pleasing to see scripts from candidates taking this paper who had a good knowledge of the syllabus, although the full range of marks was seen.

It is worth noting that the 2020 Questions Papers will say 'no marks will be given for unsupported answers from a calculator'. This is particularly important to consider in the Normal distribution questions where many candidates typed the numbers into calculators and wrote the answer down. From next year this could score 0 marks if no working is shown.

Question 1

This question was found to be challenging by candidates. **Part (i)** required candidates to follow and apply some instructions. However, some candidates tried finding the mean, or applying general formulae rather than applying the specific instructions. In **part (ii)** candidates were instructed to use the numbers obtained in **(i)** to find the variance of the original raw data. They were expected to use the fact that the variance of (t – 120) is the same as the variance of t, and thus use the formula mean of squares minus mean squared. Candidates who found the variance correctly from the raw data but did not fully answer the question, were awarded a Special Case B1.

Question 2

Some candidates could answer this question well, being able to write the P(Rosa takes a plum) as an algebraic fraction. Those who drew a tree diagram were usually successful as they then knew that they had to multiply the probabilities. A few realised that P(Rosa takes a plum) was $\frac{1}{4} \div \frac{5}{8} = \frac{2}{5}$ and used this fraction to find x from an algebraic fraction. A few found an answer by inspection of the numbers, but this did not gain full credit.

Question 3

This question was well done, with many candidates scoring full marks. Candidates needed to demonstrate how they found P(X∩Y) first and then show that it was the same answer as P(X) x P(Y) to prove independence.

Question 4

This question required candidates to compare the two data sets using a measure of central tendency and ranges or interquartile ranges. As cumulative frequency curves were shown, the most straightforward method to answer the question was to use medians as the measure of central tendency. Many candidates completed unnecessary work by making tables from the data in the graph and using these to calculate mean and standard deviation. Candidates who calculated the correct statistics did not always comment on what they had found. There was some confusion about which number should be used to find the median. Some candidates used $\frac{1500}{2}$ from the axes, rather than $\frac{1400}{2}$ as given in the question. Candidates should consider whether or not their answers are sensible and possible. When told that the marks are out of 100, it is not sensible to give a median greater than this.

Question 5

(i) It was pleasing to see that many candidates recognised the Binomial distribution and were able to score a mark for one correct term. A few candidates thought 'more than 12' meant or included 'equal to 12' and others thought that they had to subtract their answer from 1.

(ii) Approximately half of the candidates recognised the Normal approximation to the Binomial. Most of these found the mean and variance correctly and remembered to use a continuity correction.

Question 6

(i) This part of the question was quite well done, the most successful candidates being those who drew a tree diagram. Some candidates did not realise that once the bell had rung Amy no longer played, so produced incorrect tree diagrams. A small number of candidates incorrectly tried to use a binomial expansion.

(ii) Many candidates were able to gain most of the marks for the table. Some candidates who did not gain full marks put Amy's gain as \$0 in the table above 0.64. Candidates needed to make sure that they read the question carefully and took careful note of the information given. In this case the P(Amy rings the bell) = 0.2 was crucial information that some candidates did not use.

(iii) This mark could be gained even if a candidate had made an error in **part (ii)**. In order to gain this mark, candidates needed to show the unsimplified values they had used to find Amy's expected gain.

Question 7

(i) When tackling this type of question, candidates must show the normal distribution standardising equation and their working. It is worth reiterating what was said in the Comments section at the beginning of this report that from next year unsupported answers from a calculator with no working will gain no marks.

(ii) This question proved to be very challenging for candidates. Very few candidates appreciated what a central 90 per cent interval looked like on the standardised normal distribution diagram, or that the z-value is represented by the probability or area on the left of the curve. Candidates also had difficulty knowing how to deal with 830 + w being the upper value, meaning the standardised value was $(830 + w - \frac{830}{\sigma})$ which simplified to $\frac{w}{120}$. Again it is important for candidates to realise that typing in the mean, sd and 90 per cent to get the upper and lower bounds of a central 90 per cent confidence interval, and then subtracting 830, will not gain them full credit.

(iii) This routine question was generally well attempted. Many candidates did not gain one of the available marks because they did not appreciate that the z-value of 0.97 should be negative and, when they found a negative standard deviation, ignored the minus sign.

Question 8

(i) Most candidates who attempted this question were able to give the answer using combinations as expected. A few used permutations. The expected method was a singe combination, but some candidates successfully added the combinations of cars and buses which gave 4 toys.

(ii) This question was more problematic for some candidates. Many candidates achieved the correct answer, but some used permutations again or multiplied the correct answer by two.

(iii) This question was generally tackled successfully by those candidates who looked at the ways of arranging the cars and the buses as factorials. Those who tried to list results had less success.

(iv) Many candidates were able to find the number of arrangements with a car at each end, but did not know how to take into account the rest of the information. Some candidates successfully listed the 10 ways the buses could be arranged within the cars. Some candidates attempted to find the number of ways with no restrictions and subtract the various ways buses could be together. This was a cumbersome method and was rarely successful.

MATHEMATICS

Paper 9709/62
Paper 62

General comments

The majority of candidates presented their solutions in a logical manner. There was sometimes a lack of structure which made determining the final answer difficult for examiners.

Candidates often needed to show sufficient method to justify their conclusions. Not communicating intended processes produced uncertainty about the final answer. Greater accuracy often needed to be used within calculations, otherwise premature approximation affected the final answer.

Many good solutions were seen for **Question 3** and **Question 5**. The context in **Question 1** and **Question 4** was found challenging by many candidates. Management of time seems to have caused difficulties for some candidates to attempt all the questions.

Key messages

- Sufficient method must be shown to justify answers.
- Candidates should present their working in a clear manner.
- Only non-exact answers should be rounded to 3 significant figures.
- Referencing to context is important when determining the most appropriate approach for a solution.

Comments on specific questions

Question 1

A number of very clear solutions were seen for this question which used a standard context for throwing dice. Others needed to more clearly communicate the information required to determine independence, presenting work in a more methodical manner with annotations provided with the workings. The most efficient approach seen was to generate an outcome space initially and use this to identify the required probabilities. The best solutions indicated clearly on the outcome space values that fulfilled the conditions of Events S and T. Candidates needed to be able to interpret statements like ‘is less than 6’ as not including 6.

The main alternative approach was to list outcomes which fulfilled the events separately and then try to compare the lists for common terms. In both approaches, a significant number of counting errors were seen which led to incorrect probabilities. Weaker candidates often assumed independence and calculated $P(S \cap T)$ using $P(S) \times P(T)$. This is a ‘circular’ argument and gained little credit.

Question 2

Many good solutions to this normal approximation question were seen. The best solutions often had a simple sketch to identify the required probability area. Almost all candidates recognised that volume is a continuous variable, so no continuity correction was required.

A number of candidates were unable to answer the question completely as the calculation of the expected number of cartridges was omitted. Candidates needed to be aware that this value was an interpretation of their exact answer, rather than a rounding, so referencing approximation was inappropriate.

Question 3

Most solutions recognised that this was binomial distribution and used the appropriate formulae and approximations.

(i) The best solutions clearly stated the outcomes that would satisfy the condition, recognised that subtraction from 1 would be the most efficient method, and provided a clear unsimplified expression with efficient use of calculators to achieve an accurate 3 significant figures result. Weaker solutions often evaluated each term and, because of premature approximation, achieved an inaccurate final answer. A few candidates attempted to sum the probabilities of all of the scenarios, but these were often successful. Candidates needed to be able to interpret conditions such as 'at least 7' to indicate that only values higher than 7 were excluded. Some poor notation was noted, with the omission of the trailing bracket, or complete omission of brackets, but candidates frequently recovered when evaluating their answer.

(ii) Many good solutions were seen. These clearly stated the initial exponential equation, and then showed an appropriate method of solution. The use of logarithmic properties was acceptable and an efficient method. Candidates needed to ensure that their final answer fulfilled the stated condition.

Question 4

Many good attempts were seen to this question. The best solutions stated the z-values appropriately, used the standardisation formula accurately, and provided clear algebraic solutions of the simultaneous equations that were generated. A sketch of the normal distribution curve often assisted in identifying whether the z-value was positive or negative. A number of solutions rounded too early in the process, especially when μ was being eliminated and σ calculated first.

Question 5

The majority of candidates provided appropriate solutions to this question. Others needed to apply the context information more accurately to understand that a sweet would not be returned to the box. Re-reading the initial information after attempting each part of a question would have been helpful to ensure that the context had been appropriately applied.

(i) Almost all candidates attempted the tree diagram which is an extension of Upper Secondary mathematical knowledge and skills. The labelling was generally clear, although at times candidates were not consistent with the ordering of outcomes which caused difficulties later. It was not uncommon to have an unexpected additional branch between the first and second sweet removal branches where the changes to the box content were stated. Weaker responses often simply considered the original contents in a non-replacement context or continued the tree diagram to consider a third sweet being removed, which reduced possible credit. The use of a ruler for the branches was seen more frequently in solutions which were successful in later parts.

(ii) Almost all candidates produced a probability distribution table that was linked with the tree diagram that was created in part **(i)**. Weaker solutions ignored the context and considered possible outcomes of up to 6 toffees being removed. Candidates needed to be aware that the total of the probabilities for all the outcomes is 1, so that no table should sum to more than 1. Where the tree diagram was inaccurate, many gained partial credit using this property.

(iii) This part was omitted by some candidates who had created a probability distribution table. Candidates needed to be aware that 'mean' and 'expected value' were often interchangeable in this context. The best solutions stated the calculation that was being attempted before evaluating.

(iv) Most solutions identified that a conditional probability was being calculated. The best solutions often included a statement of the conditional probability formula for the context, used the tree diagram in part **(i)** to identify the required probabilities, and handled the fractions accurately. Many candidates were able to interpret their inaccurate tree diagram in part **(i)** appropriately and provided sufficient working to communicate their intentions to gain credit. Weaker solutions interpreted the denominator $\mathrm{P(T)}$ as simply the probability for picking a toffee initially.

Question 6

(i) Many candidates found this question challenging. Understanding when using a box-and-whisker plot is appropriate is a requirement of the syllabus and candidates needed to be able to identify the key advantages and disadvantages of such a representation. The best responses often included several advantages including reference to the spread or shape of the data distribution as well as the more common ease of identifying the median or quartiles. The disadvantage was then normally related to the lack of other data present, so not being able to calculate the mean or mode were the more specific answers, although reference to the lack of individual data was also seen. Weaker answers often referred to the ease or difficulty of constructing the box-and-whisker plot. Uncertainty of the appropriate technical terms was noted, with the median often being referred to as the mean.

(ii) Whereas part **(i)** required a generic response, this part was placed in context and comments needed to relate to the specific data provided. Again, many candidates didn't show understanding of the appropriate technical terms, with median and mean frequently interchanged. Many candidates simply calculated the value of each measure of central tendency without making any conclusion. The best solutions identified that 768 was considerably larger than any other value thus making the mean inappropriate, and that there was no repeated value so effectively no mode, meaning that the median must be the most appropriate. Weaker answers often ignored the context and simply provided a reasoned approach with general descriptions of the median, mean and mode.

(iii) (a) Many solutions lacked the accuracy required at this level, with the use of inappropriate scales common. Candidates needed to ensure that any scale they used enabled them to communicate effectively their accurate values wherever possible. Most candidates found the median successfully, but the remaining quartiles were often incorrect. Good box-and-whisker plots were constructed using a ruler, ensuring that the diagram clearly communicated the intended value of the 5-points, with the whiskers being drawn at the middle of the box height and not entering the box itself. A linear scale using 1 cm for 20 minutes ensured accuracy and needed to be annotated with both the values linearly and labelled both 'time' and 'minutes'. Weaker solutions were often drawn freehand, omitted the labelling on the scale, and used a scale such as 2 cm for 37 minutes. Candidates needed to understand that at this level diagrams that represent statistical data need to be labelled with both units and the item that is being measured.

(b) Where the quartiles were identified in part **(iii) (a)**, most candidates were able to accurately calculate the interquartile range. Weaker responses often simply found the difference of the term values and restated the median as the answer.

Question 7

Most solutions recognised that this was a permutation and combination style question, and candidates were generally able to determine whether 'arrangements' or 'selections' were required by context. Candidates who presented their working in a clear manner were often more successful.

(a) The context for this part caused difficulties for some candidates. Successful solutions often had simple diagrams to visualise the problem, and this was a technique which provided support for some candidates. Most solutions considered loading the boats in the order stated in the question, although many solutions did not realise that once people were in a boat, they could no longer be placed in another boat, so that the pool of people being chosen from was reducing; ${}^6C_3 \times {}^6C_2 \times {}^6C_1$ was a common error. Some candidates calculated the ways of arranging the people in the boats, but then believed that the order that the boats were loaded needed to be considered, and amended their answer by multiplying by 6. Few candidates used permutations in their solutions.

(b) (i) Good solutions often included a simple 'diagram' to visualise the condition applied in the question. Most solutions realised the impact of the repeated value and divided as required. Weaker solutions often didn't multiply by 2 to allow for the order of 'odd' and 'even' to be interchanged.

(ii) Again, the best solutions included simple visual representations of the condition that was being applied. The more successful approach was to consider the arrangements of 3,7,7,7,8 and then determine how the 2s could be inserted between the numbers. A significant number of solutions didn't divide by 2 to remove the repeated value impact of the value 2.

Candidates who calculated the total number of ways the values could be arranged and then subtracted the number of ways where the 2s were together, were often less accurate. Many solutions didn't divide appropriately to remove the impact of the repeated values. A significant number of solutions didn't consider the 2s as a single item when calculating the number of ways that the values could be arranged, with the 2s together leading to some extremely complex and inaccurate approaches.

MATHEMATICS

Paper 9709/63
Paper 63

Key messages

Candidates should be advised that it is important that they show all their working. Presenting correct answers with no working is not always awarded full marks. Where calculators are used, there needs to be written work detailing the expression entered into the calculator. Candidates are advised to clearly identify their final answer and it is preferable if candidates only offer one answer.

In **Questions 4(ii)**, **7(iii)** and **7(iv)** a significant number of candidates chose much longer methods than necessary to arrive at the correct answer. In **Question 7(iv)**, if candidates used an alternative method to the required method and still arrived at the correct final answer, they forfeited one mark.

General comments

A number of candidates gave several of their answers to fewer than three significant figures. In **Question 5(i)**, some candidates seemed to think that 0.096 was correct to three significant figures. To avoid losing accuracy marks, candidates should be advised to write an answer to more than three significant figures before they do the rounding. A number of candidates lost accuracy marks through premature approximation, not always realising that if the final answer must be correct to three significant figures, their input numbers need to be correct to at least four significant figures. Premature approximation was particularly apparent in **Questions 1(i)**, **5(i)** and **5(ii)**.

Almost all candidates appreciated that the exact answers in **Questions 3(i)**, **3(ii)** and **8(iii)** were required. Most candidates also gave the required four figure z-value in **Question 1(ii)**.

Comments on specific questions

Question 1

(i) This question was well answered. Most candidates knew how to standardise with only a few introducing inappropriate continuity corrections or rooting or squaring the standard deviation. The majority found the correct area, the most common error being to subtract phi(0.882) from phi(0.882) and giving zero as the answer.

(ii) Most candidates used the tables correctly and only a small number used them backwards, resulting in their equating their standardisation expression with a probability instead of a z-value. The most common error was to equate the standardisation expression to a positive z-value and to produce a value for t that was greater than the mean.

Question 2

(i) This question was well answered. Some of the errors made by candidates included missing the first set of branches, starting the second set from three different points. There were a few slips in the calculation of the missing probabilities while others did not calculate them at all.

Another error appeared in the second set of branches for the ‘reply’, with three groups of three branches each with the probabilities 0.4, 0.15 and 0.6.

The labelling was varied but the meaning was usually clear. More able candidates knew to include a key to explain the labelling.

(iii) Many candidates were familiar with the formula for conditional probability and produced the correct answer. Some candidates misinterpreted the question and found the P(email/immediate reply) instead of P(email/not immediate reply). Candidates who found the tree diagram in part **(i)** challenging rarely made any progress with part **(ii)**.

A number of candidates did not achieve the accuracy mark in this question by giving the answer to two significant figures.

Question 3

(i) This question was well answered with almost all candidates, realising that they needed to find the number of ways of arranging nine people. A significant number gave 9! as their final answer and forgot to consider the fact that Mr Keene and Mr Uzuma could be arranged in two ways.

(ii) Most candidates appreciated that 5! and 2! were part of the answer and the most common mistake was to present 5! x 2! as the final answer although some candidates divided by the factorials. Other candidates then went on to consider arranging the parents and multiplied by 4!. More able candidates considered the two sets of children and the four parents as six items to be rearranged and correctly multiplied 5! by 2! by 6!.

Question 4

(i) Most candidates made a good attempt at this question and considered at least two of the three correct scenarios. A few candidates misunderstood the question and considered the situation where there are four women and two men. Other candidates included the situation with three men and three women, ignoring the word ‘twice’. Most candidates made a good attempt at this question and considered at least two of the three correct scenarios. The most common error was to omit the possibility of six men and no women.

(ii) This question was approached in a number of different ways. Generally, more able candidates **either** subtracted the number of ways of forming a committee with the two men together (10C4) from the total number of ways (12C6) **or** considered separately the number of ways with either one of the men and five others ($2 \times$ 10C5) and added this to the number of ways of forming a committee with neither of the men (10C6).

Those who tried the third method in the mark scheme where they selected a committee from a pool of 11 with either one of the two removed ($2 \times$ 11C6) and then the pool with both men removed (10C6) were less successful, most of them adding the 10C6 to their total instead of subtracting.

A few candidates found the very neat way of considering the pool without one of the men (11C6) and then adding the number of ways with the removed man and not the other man (10C5).

A significant number of candidates considered the men and women separately which required much lengthier calculations, however very few candidates were successful using this method.

Question 5

(i) Most candidates appreciated that the Binomial Distribution was required and the majority arrived at the correct final answer. A few candidates understood the wording of the question and subtracted the correct answer from 1 while others confused ‘significant figures’ with ‘decimal places’ and gave their final answer as 0.096 without ever writing a more accurate answer.

(ii) This question was well answered with most candidates recognising that a Normal Approximation was required and successfully finding the mean (204) and the variance (134.64). Some used the variance rather than the standard deviation in the standardisation expression and a significant number either forgot the continuity correction or used 190.5 instead of 189.5. A large number of candidates obtained the correct final answer with only a few finding the wrong area by not subtracting from 1.

Question 6

(i) Most candidates produced a correct probability distribution table with all the correct scores and associated probabilities. Some candidates misunderstood the word 'product' and produced a table for the possible 'sums' of the scores.

(ii) More able candidates successfully applied the correct formulae for the mean and the variance to their table in **(i)** and many obtained full marks. There was evidence of confusion between the word 'mean' and the expression E(X) with many candidates thinking they were different. It was fairly common to see E(X) divided by 15 and stated as the mean. Some would then proceed to calculate the variance correctly using E(X) while others continued with the error and divided $E(X^2)$ by 15 as well.

Candidates who simplified the P(1) to $\frac{1}{5}$ in their answer to part **(i)** frequently went on to use P(1) as $\frac{1}{15}$ in this second part.

(iii) Some candidates attempted to find P(X > 3.2) by standardising and using the normal distribution, giving an answer of 0.5. This may have been a reaction to having calculated the mean and variance. On occasion, some candidates attempted to assign a probability to the 'score' of 3.2 – 4 by finding $0.8 \times \frac{4}{15}$ and adding it to the three other probabilities.

Candidates who appreciated the discrete nature of the question often selected the relevant scores from their mean and their table.

Question 7

(i) Most candidates understood the idea of a back-to-back stem and leaf diagram, however several candidates ignored the instruction to draw Thaters School on the left. A few candidates gained full marks on this question. Other candidates did not line up the leaves carefully enough or inserted unwanted commas between the leaves. The most common mistake was to either forget the key altogether or to miss out the correct units and/or the names of the schools. Other candidates did not realise that the key needed to be in the form 'leaf/stem/leaf' for a back-to-back stem and leaf diagram and instead gave two separate keys.

(ii) Most candidates used the correct method to locate the quartiles and knew to subtract the lower quartile from the upper quartile. Many of those who did not identify the quartiles correctly, used values within the accepted ranges and were able to gain the mark for finding the difference. The most commonly seen incorrect values for the quartiles were 52 and 61 found by using position numbers 4 and 10 ($\frac{13}{4}$ and $\frac{39}{4}$ rounded up).

(iii) More able candidates identified that the quickest way to find $\Sigma(x - 60)^2$ was to subtract 60 from each of the times for Whitefay Park School, square each of the answers and sum the thirteen squares. Most candidates who used this method found the correct answer.

More complicated approaches were **either** to expand $\sum(x - 60)^2$ to $\sum x^2 - 120\sum x + 13 \times 60^2$ and find the answer by evaluating the expression **or** to calculate the Variance of x, use the fact that the variance of (x – 60) equals the Variance of x and work backwards to find $\sum(x - 60)^2$. Only the most able candidates were successful with either of these methods.

A significant number of candidates calculated $\sum(x - 60)^2$ for Thaters School instead of Whitefay Park School.

(iv) More able candidates appreciated that the easiest way to find the variance of (x – 60) was to divide the answer to **(iii)** by 13 and subtract $(\frac{46}{13})^2$.

Those who had calculated the variance to find the answer in part **(iii)** often did some more complicated calculations to arrive back at the same answer.

Candidates are advised to read the question carefully, as some candidates ignored the instruction to use their answer to **(iii)** and the fact that $\sum(x - 60) = 46$. Even if they arrived at the correct answer by calculating the variance of x using uncoded values, they were only awarded one of the two marks.

MATHEMATICS

Paper 9709/71
Paper 71

Key messages

It is important that candidates read the question carefully (see **Question 1** below) and know how to round answers to the required number of significant figures.

Solutions should be fully and neatly presented, and standard statistical notation needs to be used correctly.

If candidates use additional sheets for working, the question number must be clearly indicated.

Confusion between standard deviation and variance continues to prevent some candidates gaining marks (see **Question 1** below)

When carrying out a significance test the comparison between the test value and critical value (or equivalent) must be clearly shown in order to justify the conclusion (see comments on **Questions 2(i)** and **3(i)** below)

General comments

Some candidates found parts of this paper challenging. There were also some good scripts. **Question 5** (particularly **part (i)**) was well attempted, as was **Question 6(i)**. Questions that proved to be more challenging were **Questions 3** and **4**.

There did not appear to be any time issues for candidates on this paper.

The comments below indicate common errors and misconceptions. However, there were many full and correct solutions presented.

Comments on specific questions

Question 1

It was important that candidates read the question carefully here. The question asked for the standard deviation to be found, but many candidates left their answer as a variance. The units required were 'cents' and a few candidates worked in dollars. **Part (i)** was better attempted than **part (ii)** and in both cases many candidates calculated the mean correctly but errors were made in calculating the standard deviation.

Question 2

This was a reasonably well attempted question. Some candidates omitted or gave incorrect hypotheses and many didn't state a necessary assumption. The comparison between the test value ($z = 2.552$) and the critical value ($z = 2.326$), or equivalent, must be clearly shown either as an inequality statement or on a fully labelled diagram so that the conclusion drawn is fully justified. The conclusion needed to be in context and not definite. In **part (ii)** there were many incorrect answers or statements that were too vague (for example, 'it' is Normal is too vague, it must be clear that it is the 'population' that is Normal).

Question 3

This question proved to be challenging for many candidates. A significant number of candidates omitted Hypotheses, or gave incorrect ones, used incorrect Binomial distributions, or attempted to work with other incorrect distributions. The conclusion needed to be in context, not definite and fully justified with a valid comparison shown.

In **part (ii)** many candidates incorrectly thought that the probability of a Type I error was 0.05.

Question 4

Many candidates found **part (a)** of this question demanding and were not sure how to approach the questions as the relevant information was in diagram form. Candidates with a good understanding of probability density functions approached the questions, as expected, using basic understanding. Some candidates preferred to work with an equation for $f(x)$, and those who were able to find the correct equation were usually able to reach correct answers. However, many candidates did not use the information on the diagram to deduce $f(x)$.

Part (b) proved to be more accessible than **part (a)**, and the integration of $g(t)$ was reasonably well attempted.

Question 5

This question was quite well attempted, particularly **part (a)(i)**. A common error in **part (a)(i)** was to include, or omit, terms in the expression for $P(2 \leqslant X \leqslant 4)$. Similarly in **part (a)(ii)** errors included extra or omitted terms in the required expression as well as incorrect values for λ, though many candidates did realise that $\lambda = 4.6$ was the value required. **Part (a)(iii)** was also well attempted, errors here included omission of, or a wrong continuity correction, and errors were made when standardising (often an incorrect standard deviation being used).

Part (b) was very well attempted. Candidates were able to set up and solve a correct equation in λ.

Question 6

Part (i) of this question was particularly well attempted. Candidates were able to correctly explain why the given sample was not random. **Part (ii)** required the description of a method that would result in a random sample and was not as well answered. Many candidates suggested alternative places to visit to collect the sample rather than considering a method that gave each person in the town equal chance of being selected. Calculating the confidence interval in **(iii)** was reasonably well attempted; errors included incorrect values for z and candidates should note that the final answer must be written as an interval. Candidates found **Part (iv)** very challenging, though some candidates gained partial credit for attempting to find z (= 2.056) and some candidates correctly reached 0.980 but then thought that x was 98 rather than 96.

MATHEMATICS

Paper 9709/72
Paper 72

Key messages

It is important that candidates know how to round answers to the required number of significant figures (see, in particular, comments on **Questions 1** and **3** below)

If candidates use additional sheets for working, the question number must be clearly indicated.

Standard statistical notation needs to be used correctly.

Confusion between standard deviation and variance continues to prevent some candidates gaining marks (see **Question 2** below).

When carrying out a significance test, the comparison between the test value and critical value (or equivalent) must be clearly shown in order to justify the conclusion (see comments on **Questions 3** and **7** below).

General comments

This was a reasonably accessible paper for candidates. There were some very good scripts but also some scripts which demonstrated that candidates had found the paper very challenging. **Question 3** (particularly **part (i)**) was well attempted as was **Question 6**. Questions that proved to be more demanding were **Questions 2** and **7**.

As mentioned above, it is important that candidates can round correctly to 3.s.f. There were occasions where only 2 (or even 1) s.f. were given with no indication of more accurate figures; candidate who do this are unable to gain accuracy marks.

There did not appear to be any time issues for candidates on this paper, and presentation was generally good.

The comments below indicate common errors and misconceptions. However, many full and correct solutions were presented.

Comments on specific questions

Question 1

Question 1(i) was well answered, though many candidates gave their answer as 0.084 rather than 0.0842. If there was no greater accuracy seen here candidates did not gain the mark as 3.s.f. was required. Candidates were either confusing significant figures with decimal places, or thought that the zero after the decimal point was significant.

Parts (ii) and **(iii)** were reasonably well attempted, though manipulation of factorial notation and indices caused confusion for weaker candidates.

Question 2

Candidates found this question very challenging, particularly **part (ii)**.

In **part (i),** some candidates did not describe the distribution (Normal) fully and did not give the parameters. Others confused standard deviation and variance, correctly working out one, but stating it was the other. Standard notation for the Normal distribution was not always used correctly here. There were also candidates who did not know what the question required.

Very few candidates gave a correct answer in **part (ii)**. Statements such as 'it should be Normal' were quite common but did not gain credit. Candidates had to have made it clear that the population (i.e. *X* here) was Normal in order for the distribution to be exact. Incorrect answers involving the central limit theorem or values for *n* were often seen.

Question 3

Part (i) was particularly well attempted, with the majority of candidates able to correctly calculate unbiased estimates of the mean and variance. Confusion between the two formulae for the unbiased variance was not as prevalent as has been the case in the past, with the majority of candidates using the formula given on the formula sheet. Very few candidates gave the biased estimate of the variance. Once again, there was confusion by some candidates in giving a final answer correct to 3 significant figures.

There were many full and complete solutions for **part (ii)**, but in some cases the hypotheses were omitted or incorrect and errors were made when standardising (for example not using $\sqrt{50}$). In order to justify the conclusion, a comparison between the test value and the critical value had to be clearly shown either as an inequality statement or on a fully labelled diagram; some candidates did not show this comparison. It is not sufficient to merely state 'compare'. The conclusion to the test should be written in context and should not be definite.

Question 4

This question was reasonably well attempted. Many candidates scored highly, though few candidates gain full credit. Errors included using an incorrect variance, incorrectly using a continuity correction or calculating an incorrect area (> 0.5 rather than < 0.5). Very few candidates realised that their calculated probability needed to then be doubled for the final answer.

Question 5

Part (i) was usually well attempted, though '*p*' was sometimes omitted, or λ or μ incorrectly used. Weaker candidates did not always realise what was required. In **part (ii)** candidates often did not choose the required Binomial distribution $B(40,0.1)$. Whilst the confidence interval in **(iii)** was generally well attempted, there were candidates who either incorrectly used 0.1 and 0.9, or did not work with proportions. **Part (iv)** required candidates to check if 0.1 was in their CI and make a conclusion from this. Comments such as 'it' lies in the CI are too vague and did not gain credit, and some candidates did not give both the required parts of the answer.

Question 6

Questions on probability density functions are usually well attempted and this question was no exception. In **part (i)** most candidates realised that the requirement was to integrate $f(x)$ with limits 1 and *b* and equate to 1. The resulting equation in '*a*' and '*b*' was usually correctly rearranged to the form given. **Part (ii)** was also well attempted, though weaker candidates confused the values of 0.5 and the median. Instead of integrating from 1 to 1.5 and equating to 0.5, the integral was incorrectly evaluated from 1 to 0.5 and equated to the median 1.5. In **part (iii)** it was required to integrate $xf(x)$ with limits 1 and *b*. This was usually well attempted, though weaker candidates were unable to correctly carry out the integration of $xf(x)$ (i.e. $\frac{1}{x}$).

Question 7

This question was found to be very challenging by candidates. In **part (i)** many candidates did not fully understand that 320 was the number of passengers who bought tickets, or, alternatively, the number of seats on the plane. It was not simply the number of passengers or, as many candidates thought, the number of passengers who bought tickets but did not turn up. A Poisson distribution with $\lambda = 3.2$ was required in **part (ii)**. Errors included an incorrect value for λ, an extra term, or a missing term, in the expression for $P(3,4,5)$ or even an incorrect expression involving $P(< 6)$ and $Pr(> 2)$. The justification for using the Poisson approximation in **(iii)** was often not fully correct or was not in context. It was not sufficient to just say $np < 5$, np needed to be stated as 1.6 to be 'in context'. **Part (iv)** was poorly attempted, with candidates using a range of methods: from the correct Poisson distribution to incorrect Poisson distributions or more usually trying to incorrectly model the situation with an invalid Normal distribution. Some candidates only included one term in their (correct form of) calculation and lost marks accordingly. The final conclusion for the test should be fully justified, written in context and not definite.

MATHEMATICS

Paper 9709/73
Paper 73

Key messages

Candidates appeared well prepared for this paper and the standard of responses was generally high. Answers were given to sufficient accuracy and in most cases if candidates offered two solutions to a question they indicated which they considered correct. The responses to questions involving significance testing were in the main very well done. Candidates should ensure that in 'show that' questions the conclusion is clearly stated.

General comments

Candidates nearly always score well on questions involving the Normal and Poisson Distributions, such as **Questions 4** and **7** on this paper. There continues to be scope for improvement on questions involving sampling (**Question 3** on this paper). Solutions were almost invariably well presented, and answers were given to sufficient accuracy. Candidates should use the additional page in the answer booklet before asking for extra paper.

Comments on specific questions

Question 1

Nearly all candidates were familiar with the form of a confidence interval of a proportion, and were able to calculate the mean and variance that were needed to substitute into the correct expression. Very few errors were made in identifying the value of z required, and therefore most candidates scored highly.

Question 2

This question was well answered by most candidates. The requirement to substitute into the formula for the unbiased estimator of the variance, a formula that is given in the formula booklet, was met by most candidates. They were then, in most cases, able to rearrange in order to find Σx and then use this to find the mean. A small number of candidates stopped at the Σx stage and so did not gain the final mark.

Question 3

This 3 part question tested sampling in context. In **part (i)** candidates were required to identify why the proposed method was not random. Those candidates who answered correctly normally did so by pointing out that Person D had more chance than the others of being picked. In **part (ii)** candidates were required to suggest how to change the situation to make the method of choosing a random process. Correct answers were those that ensured that only 1 number of the dice should be allocated to each person. A significant number of responses ignored the requirement to use the 6 sided die and suggested using a 4 sided one. These candidates did not score the mark. In **part (iii)** candidates were required to identify that there were 6 possible pairs of the 4 people and indicate 1 side of the die should be allocated to each pair. Throughout the question it was clear that some candidates tried to suggest an alternative method of choosing e.g. by using random number tables. Candidates must read the question with great care, as here it clearly specified that a 6 sided dice was the method being used.

Question 4

The majority of candidates scored well on this question, which was testing the topic of combinations of Normal Variables. A minority of candidates made errors in finding the variance of the contents of the box, with errors generally being of 2 types. Some candidates treated the variances given as if they were standard deviations, while others saw the need to treat the total of e.g. 10 cans as a multiple, rather than a sum, of their individual variances. Those candidates who worked with the correct values for the mean and variance of the total weight almost invariably standardised correctly.

Question 5

Responses to this question, on significance testing, were very varied. Candidates were expected to calculate the mean of the 6 values given in order to test against a known mean and standard deviation. Some candidates only gained the mark for finding the mean of the 6 values given, whilst other candidates were able to state the correct Hypotheses, scoring a second mark. A significant number of candidates calculated the variance of the sample given, rather than using the stated variance, when calculating the test statistic for the mean. There were some candidates who did not include $\sqrt{6}$ in this calculation. The final 2 marks required the calculation of the maximum significance level (α) of this test. This was found by calculating $2(1 - \Phi(z))$ and could be scored following through an incorrect z. Only a minority of candidates scored the full 6 marks.

Question 6

This 3 part question, testing continuous random variables, was answered well by most candidates. In **part (i)** candidates were required to show that the integral of the pdf gave 1 for all values of a, and conclude that therefore the function was a pdf for all values of a. Most candidates were able to show that the integral evaluated to 1, although many then failed to draw the required conclusion. In **part (ii)** candidates needed to show that if the median was 2 then 'a' was 2.52 correct to 3sf. Most candidates showed sufficient working, including showing that $a^3 = 16$, and thus deduced the correct value for a. It is important for candidates to note that when answers are given, full correct working must be shown. In **part (iii)** candidates were required to find the mean of the pdf. The vast majority of candidates were able to do this, although a small minority of candidates did not evaluate $x\mathrm{f}(x)$ prior to integration. Some candidates did not include the constant at this stage ($3/a^3$), or incorrectly evaluated it.

Question 7

The use of the Poisson, and the Normal distribution to approximate a Binomial, was tested in this question. The three parts were answered well by most candidates. In **part (i)** candidates were required to calculate $P(X \geqslant 3)$ using a suitable approximation. Given the distribution B(70,0.04), this approximation was Po(2.8). Most candidates scored the full 3 marks. A few candidates used the Binomial, while some others included an extra term in $1 - P(X \leqslant 2)$.

In **part (ii)** candidates were required to find $P(X = 6)$ using a suitable approximation given B(70,0.04) and B(50,0.06) which, added, represented the total of missed appointments in a day. Nearly all candidates who did not gain any credit were those who incorrectly attempted to use a normal distribution.

In **part (iii)** candidates were expected to identify that the expected number of missed appointments in 10 days was approximated by N(58,58). While the majority of candidates who used the correct distribution scored full marks, a significant number of candidates did not make the correct continuity correction needed for at least 50, making no correction or using 49.5.

Question 8

The final question on this paper was on significance testing of a Binomial distribution. This question was answered well by many candidates, and better than many similar questions in previous years. Nearly all candidates correctly stated the Hypotheses, and many then correctly calculated the test statistic $P(X \geqslant 6)$ given Bin(10,0.25). A clear test comparing this value with 0.01 then needed to be performed and a conclusion indicating that there was insufficient evidence to conclude the spinner was biased was required. Many candidates showed fully correct working at all stages. A minority of candidates only calculated $P(X = 6)$ as their test statistic, making their test invalid.

Parts (ii) and **(iii)** tested a full understanding of significance testing. **Part (ii)** required candidates to identify that since 6 was not in the critical region, the type 1 error was $P(X \geqslant 7)$ and **part (iii)** required candidates to realise that a type II error would occur if $P(X \leqslant 6)$ given B(10,0.6)). Many candidates scored full marks for the whole question, but the progressive level of difficulty of this question caused problems for some candidates.