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MATHEMATICS

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MARK SCHEME

Maximum Mark: 75

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This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

- AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
- AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
- CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
- CWO Correct Working Only – often written by a ‘fortuitous’ answer
- ISW Ignore Subsequent Working
- SOI Seen or implied
- SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

- MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
- PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Question	Answer	Marks
1	Use law of the logarithm of a power or a quotient	M1
	Remove logarithms and obtain a correct equation in x . e.g. $x^2 + 1 = ex^2$	A1
	Obtain answer 0.763 and no other	A1
	Total:	3
2	<i>EITHER:</i> State or imply non-modular inequality $(x-3)^2 < (3x-4)^2$, or corresponding equation	(B1)
	Make reasonable attempt at solving a three term quadratic	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	<i>OR1:</i> State the relevant critical inequality $3-x < 3x-4$, or corresponding equation	(B1)
	Solve for x	M1
	Obtain critical value $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	<i>OR2:</i> Make recognizable sketches of $y = x-3 $ and $y = 3x-4$ on a single diagram	(B1)
	Find x -coordinate of the intersection	M1
	Obtain $x = \frac{7}{4}$	A1
	State final answer $x > \frac{7}{4}$ only	A1)
	Total:	4

Question	Answer	Marks
3(i)	Use correct formulae to express the equation in terms of $\cos \theta$ and $\sin \theta$	M1
	Use Pythagoras and express the equation in terms of $\cos \theta$ only	M1
	Obtain correct 3-term equation, e.g. $2\cos^4 \theta + \cos^2 \theta - 2 = 0$	A1
	Total:	3
3(ii)	Solve a 3-term quadratic in $\cos^2 \theta$ for $\cos \theta$	M1
	Obtain answer $\theta = 152.1^\circ$ only	A1
	Total:	2
4(i)	State $\frac{dy}{dt} = 4 + \frac{2}{2t-1}$	B1
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1
	Obtain answer $\frac{dy}{dx} = \frac{8t-2}{2t(2t-1)}$, or equivalent e.g. $\frac{2}{t} + \frac{2}{4t^2-2t}$	A1
	Total:	3
4(ii)	Use correct method to find the gradient of the normal at $t = 1$	M1
	Use a correct method to form an equation for the normal at $t = 1$	M1
	Obtain final answer $x + 3y - 14 = 0$, or horizontal equivalent	A1
	Total:	3

Question	Answer	Marks
5(i)	State $\frac{dy}{dt} = -\frac{2y}{(1+t)^2}$, or equivalent	B1
	Separate variables correctly and attempt integration of one side	M1
	Obtain term $\ln y$, or equivalent	A1
	Obtain term $\frac{2}{(1+t)}$, or equivalent	A1
	Use $y = 100$ and $t = 0$ to evaluate a constant, or as limits in an expression containing terms of the form $a \ln y$ and $\frac{b}{1+t}$	M1
	Obtain correct solution in any form, e.g. $\ln y = \frac{2}{1+t} - 2 + \ln 100$	A1
	Total:	6
5(ii)	State that the mass of B approaches $\frac{100}{e^2}$, or exact equivalent	B1
	State or imply that the mass of A tends to zero	B1
	Total:	2

Question	Answer	Marks
6(i)	<i>EITHER:</i> Substitute $x = 2 - i$ (or $x = 2 + i$) in the equation and attempt expansions of x^2 and x^3	(M1)
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR1:</i> Substitute $x = 2 - i$ in the equation and attempt expansions of x^2 and x^3	(M1)
	Substitute $x = 2 + i$ in the equation and add/subtract the two equations	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR2:</i> Factorise to obtain $(x - 2 + i)(x - 2 - i)(x - p)$ $\left(= (x^2 - 4x + 5)(x - p) \right)$	(M1)
	Compare coefficients	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR3:</i> Obtain the quadratic factor $(x^2 - 4x + 5)$	(M1)
	Use algebraic division to obtain a real linear factor of the form $x - p$ and set the remainder equal to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	<i>OR4:</i> Use $\alpha\beta = 5$ and $\alpha + \beta = 4$ in $\alpha\beta + \beta\gamma + \gamma\alpha = -3$	(M1)
	Solve for γ and use in $\alpha\beta\gamma = -b$ and/or $\alpha + \beta + \gamma = -a$	M1
	Obtain $a = -2$	A1
Obtain $b = 10$	A1)	

Question	Answer	Marks
	<i>OR5:</i> Factorise as $(x - (2-i))(x^2 + ex + g)$ and compare coefficients to form an equation in a and b	(M1)
	Equate real and/or imaginary parts to zero	M1
	Obtain $a = -2$	A1
	Obtain $b = 10$	A1)
	Total:	4
6(ii)	Show a circle with centre $2 - i$ in a relatively correct position	B1
	Show a circle with radius 1 and centre not at the origin	B1
	Show the perpendicular bisector of the line segment joining 0 to $-i$	B1
	Shade the correct region	B1
	Total:	4
7(i)	Use quotient or chain rule	M1
	Obtain given answer correctly	A1
	Total:	2
7(ii)	<i>EITHER:</i> Multiply numerator and denominator of LHS by $1 + \sin \theta$	(M1)
	Use Pythagoras and express LHS in terms of $\sec \theta$ and $\tan \theta$	M1
	Complete the proof	A1)
	<i>OR1:</i> Express RHS in terms of $\cos \theta$ and $\sin \theta$	(M1)
	Use Pythagoras and express RHS in terms of $\sin \theta$	M1
	Complete the proof	A1)
	<i>OR2:</i> Express LHS in terms of $\sec \theta$ and $\tan \theta$	(M1)
	Multiply numerator and denominator by $\sec \theta + \tan \theta$ and use Pythagoras	M1
	Complete the proof	A1)
	Total:	3

Question	Answer	Marks
7(iii)	Use the identity and obtain integral $2 \tan \theta + 2 \sec \theta - \theta$	B2
	Use correct limits correctly in an integral containing terms $a \tan \theta$ and $b \sec \theta$	M1
	Obtain answer $2\sqrt{2} - \frac{1}{4}\pi$	A1
	Total:	4
8(i)	State or imply the form $\frac{A}{3x+2} + \frac{Bx+C}{x^2+5}$	B1
	Use a relevant method to determine a constant	M1
	Obtain one of the values $A = 2, B = 1, C = -3$	A1
	Obtain a second value	A1
	Obtain the third value	A1
	Total:	5
8(ii)	Use correct method to find the first two terms of the expansion of $(3x+2)^{-1}, (1+\frac{3}{2}x)^{-1}, (5+x^2)^{-1}$ or $(1+\frac{1}{5}x^2)^{-1}$ [Symbolic coefficients, e.g. $\binom{-1}{2}$ are not sufficient]	M1
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction. The FT is on A, B, C . from part (i)	A1FT + A1FT
	Multiply out up to the term in x^2 by $Bx + C$, where $BC \neq 0$	M1
	Obtain final answer $\frac{2}{5} - \frac{13}{10}x + \frac{237}{100}x^2$, or equivalent	A1
	Total:	5
9(i)	<i>EITHER:</i> Find \overline{AP} for a general point P on l with parameter λ , e.g. $(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	(B1
	Equate scalar product of \overline{AP} and direction vector of l to zero and solve for λ	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	A1)

Question	Answer	Marks
	<i>OR:</i> Find \overline{AP} for a general point P on l with parameter λ , e.g. $(8 + 3\lambda, -3 - \lambda, 4 + 2\lambda)$	(B1)
	Differentiate $ AP ^2$ and solve for λ at minimum	M1
	Obtain $\lambda = -\frac{5}{2}$ and foot of perpendicular $\frac{3}{2}\mathbf{i} + \frac{3}{2}\mathbf{j} + 3\mathbf{k}$	A1
	Carry out a complete method for finding the position vector of the reflection of A in l	M1
	Obtain answer $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	(A1)
	Total:	5
9(ii)	<i>EITHER:</i> Use scalar product to obtain an equation in a , b and c , e.g. $3a - b + 2c = 0$	(B1)
	Form a second relevant equation, e.g. $9a - b + 8c = 0$ and solve for one ratio, e.g. $a : b$	M1
	Obtain final answer $a : b : c = 1 : 1 : -1$ and state plane equation $x + y - z = 0$	(A1)
	<i>OR1:</i> Attempt to calculate vector product of two relevant vectors, e.g. $(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \times (9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1)
	Obtain two correct components	A1
	Obtain correct answer, e.g. $-6\mathbf{i} - 6\mathbf{j} + 6\mathbf{k}$, and state plane equation $-x - y + z = 0$	(A1)
	<i>OR2:</i> Using a relevant point and relevant vectors, attempt to form a 2-parameter equation for the plane, e.g. $\mathbf{r} = 6\mathbf{i} + 6\mathbf{k} + s(3\mathbf{i} - \mathbf{j} + 2\mathbf{k}) + t(9\mathbf{i} - \mathbf{j} + 8\mathbf{k})$	(M1)
	State 3 correct equations in x , y , z , s and t	A1
	Eliminate s and t and state plane equation $x + y - z = 0$, or equivalent	(A1)
	<i>OR3:</i> Using a relevant point and relevant vectors, attempt to form a determinant equation for the plane, e.g. $\begin{vmatrix} x-3 & y-1 & z-4 \\ 3 & -1 & 2 \\ 9 & -1 & 8 \end{vmatrix} = 0$	(M1)
	Expand a correct determinant and obtain two correct cofactors	A1
	Obtain answer $-6x - 6y + 6z = 0$, or equivalent	(A1)
	Total:	3

Question	Answer	Marks
9(iii)	<i>EITHER:</i> Using the correct processes, divide the scalar product of \overline{OA} and a normal to the plane by the modulus of the normal or make a recognisable attempt to apply the perpendicular formula	(M1)
	Obtain a correct expression in any form, e.g. $\frac{1+2-4}{\sqrt{(1^2+1^2+(-1)^2)}}$, or equivalent	A1 FT
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	<i>OR1:</i> Obtain equation of the parallel plane through A , e.g. $x+y-z=-1$ [The f.t. is on the plane found in part (ii).]	(B1 FT
	Use correct method to find its distance from the origin	M1
	Obtain answer $1/\sqrt{3}$, or exact equivalent	A1)
	<i>OR2:</i> Form equation for the intersection of the perpendicular through A and the plane [FT on their \mathbf{n}]	(B1 FT
	Solve for λ	M1
	$ \lambda\mathbf{n} = \frac{1}{\sqrt{3}}$	A1)
	Total:	3
10(i)	Use correct product rule	M1
	Obtain correct derivative in any form $(y' = 2x \cos 2x - 2x^2 \sin 2x)$	A1
	Equate to zero and derive the given equation	A1
		Total:
10(ii)	Use the iterative formula correctly at least once e.g. $0.5 \rightarrow 0.55357 \rightarrow 0.53261 \rightarrow 0.54070 \rightarrow 0.53755$	M1
	Obtain final answer 0.54	A1
	Show sufficient iterations to 4 d.p. to justify 0.54 to 2 d.p., or show there is a sign change in the interval (0.535, 0.545)	A1
		Total:

Question	Answer	Marks
10(iii)	Integrate by parts and reach $ax^2 \sin 2x + b \int x \sin 2x \, dx$	*M1
	Obtain $\frac{1}{2}x^2 \sin 2x - \int 2x \cdot \frac{1}{2} \sin 2x \, dx$	A1
	Complete integration and obtain $\frac{1}{2}x^2 \sin 2x + \frac{1}{2}x \cos 2x - \frac{1}{4} \sin 2x$, or equivalent	A1
	Substitute limits $x = 0$, $x = \frac{1}{4}\pi$, having integrated twice	DM1
	Obtain answer $\frac{1}{32}(\pi^2 - 8)$, or exact equivalent	A1
	Total:	5