CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/11 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2015 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE
International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR–1 A penalty of MR–1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA–1 This is deducted from A or B marks in the case of premature approximation. The PA–1 penalty is usually discussed at the meeting.

1	θ is obtuse , $\sin\theta = k$		
(i)	$\cos\theta = -\sqrt{(1-k^2)}$	B1 [1]	cao
(ii)	$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ used	M1	Used, attempt at cosine seen in (i)
	$\rightarrow \tan\theta = -\dfrac{k}{\sqrt{(1-k^2)}}$ aef	A1√ [2]	Ft for their cosine as a function of k only, from part (i)
(iii)	$\sin(\theta+\pi) = -k$	B1 [1]	cao
2	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$		
(i)	$A = \dfrac{1}{2} \times (2+p) \times 2p^2 \ (= 2p^2 + p^3)$	M1 A1 [2]	Attempt at base and height in terms of p and use of $\dfrac{bh}{2}$
(ii)	$\dfrac{\mathrm{d}A}{\mathrm{d}p} = 4p + 3p^2$	B1	cao
	$\dfrac{\mathrm{d}A}{\mathrm{d}t} = \dfrac{\mathrm{d}A}{\mathrm{d}p} \times \dfrac{\mathrm{d}p}{\mathrm{d}t} = 0.02 \times 20 = 0.4$	M1 A1	any correct method, cao
	or $\dfrac{\mathrm{d}A}{\mathrm{d}t} = 4p\dfrac{\mathrm{d}p}{\mathrm{d}t} + 3p^2\dfrac{\mathrm{d}p}{\mathrm{d}t}$	[3]	
3	$(1-x)^2(1+2x)^6$.		
(i) (a)	$(1-x)^6 = 1 - 6x + 15x^2$	B2,1 [2]	−1 each error
(b)	$(1+2x)^6 = 1 + 12x + 60x^2$	B2,1 [2]	−1 each error **SC** B1 only, in each part, for all 3 correct descending powers **SC** only one penalty for omission of the '1' in each expansion
(ii)	Product of **(a)** and **(b)** with >1 term $\rightarrow$ 60 – 72 + 15 = 3	M1 DM1A1 [3]	Must be 2 or more products M1 exactly 3 products. cao, condone $3x^2$

4		$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k-3 \end{pmatrix}$		
	(i)	$OA \cdot OB = 18 - 8 = 10$ Modulus of $OA = 5$, of $OB = 7$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
		Angle $AOB = \cos^{-1}\left(\frac{10}{35}\right)$ aef	M1	All linked with modulus
		$\rightarrow \frac{10}{35}$ or $\frac{2}{7}$	A1 [3]	cao, (if angle given, no penalty), correct angle implies correct cosine
	(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$	B1	allow for $\mathbf{a} - \mathbf{b}$
		$k^2 + 4k^2 + (2k-3)^2 = 9 + 9 + 36$	M1	Correct use of moduli using their AB
		$\rightarrow 9k^2 - 12k - 45 (= 0)$	DM1	obtains 3 term quadratic.
		$\rightarrow k = 3$ or $k = -\frac{5}{3}$	A1 [4]	cao
5	**(i)**	$24 = r + r + r\theta$ $\rightarrow \theta = \frac{24 - 2r}{r}$	M1	(May not use θ) Attempt at $s = r\theta$ linked with 24 and r
		$A = \frac{1}{2} r^2\theta = \frac{24r}{2} - r^2 = 12r - r^2$. aef, ag	M1A1 [3]	Uses A formula with θ as f(r). cao
	(ii)	$(A =) 36 - (r-6)^2$	B1 B1 [2]	cao
	(iii)	Greatest value of $A = 36$	B1√	Ft on **(ii).**
		$(r = 6) \rightarrow \theta = 2$	B1 [2]	cao, may use calculus or the discriminant on $12r - r^2$

6 (i)	$y - 2t = -2(x - 3t)(y + 2x = 8t)$ Set x to 0 → $B(0, 8t)$ Set y to 0 → $A(4t, 0)$ → Area = $16t^2$	M1 M1 A1 [3]	Unsimplified or equivalent forms Attempt at both A and B, then using cao
(ii)	$m = \frac{1}{2}$ → $y - 2t = \frac{1}{2}(x - 3t)(2y = x + t)$ Set y to 0 → $C(-t, 0)$ Midpoint of CP is (t, t) This lies on the line $y = x$.	B1 M1 A1 A1 [4]	cao Unsimplified or equivalent forms co correctly shown.
7 (a)	$ar^2 = \frac{1}{3}$, $ar^3 = \frac{2}{9}$ → $r = \frac{2}{3}$ aef Substituting → $a = \frac{3}{4}$ → $S_\infty = \frac{\frac{3}{4}}{\frac{1}{3}} = 2\frac{1}{4}$ aef	 M1 A1 M1 A1 [4]	 Any valid method, seen or implied. Could be answers only. Both a and r Correct formula with $\lvert r \rvert < 1$, cao
(b)	$4a = a + 4d$ → $3a = 4d$ $360 = S_5 = \frac{5}{2}(2a + 4d)$ or $12.5a$ → $a = 28.8°$ aef Largest = $a + 4d$ or $4a = 115.2°$ aef	B1 M1 A1 B1 [4]	May be implied in $360 = 5/2(a + 4a)$ Correct S_n formula or sum of 5 terms cao, may be implied (may use degrees or radians)

8	$f: x \longmapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ for $0 \leqslant x \leqslant 2\pi$.		
(i)	$5 + 3\cos\left(\frac{1}{2}x\right) = 7$ $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$	B1	Makes $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$
	$\frac{1}{2}x = 0.84 \quad x = 1.68$ only, aef (in given range)	M1A1 [3]	Looks up $\cos^{-1}$ first, then ×2
(ii)		B1 B1 [2]	y always +ve, m always –ve. from (0, 8) to (2π, 2) (may be implied)
(iii)	No turning point on graph or 1:1	B1 [1]	cao, independent of graph in (ii)
(iv)	$y = 5 + 3\cos\left(\frac{1}{2}x\right)$	M1	Tries to make x subject.
	Order; −5, ÷3, $\cos^{-1}$, ×2	M1	Correct order of operations
	$x = 2\cos^{-1}\left(\frac{x-5}{3}\right)$	A1 [3]	cao

9	$y = x^3 + px^2$		
(i)	$\frac{dy}{dx} = 3x^2 + 2px$	B1	cao
	Sets to 0 $\rightarrow$ $x = 0$ or $-\frac{2p}{3}$	M1	Sets differential to 0
	$\rightarrow$ (0, 0) or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	A1 A1 [4]	cao cao, first A1 for any correct turning point or any correct pair of x values. 2nd A1 for 2 complete TPs
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$	M1	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve
	At (0, 0) $\rightarrow$ $2p$ +ve Minimum	A1	www
	At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$ $\rightarrow -2p$ –ve Maximum	A1 [3]	
(iii)	$y = x^3 + px^2 + px \rightarrow 3x^2 + 2px + p\ (= 0)$	B1	
	Uses $b^2 - 4ac$	M1	Any correct use of discriminant
	$\rightarrow 4p^2 - 12p < 0$		
	$\rightarrow$ $0 < p < 3$ aef	A1 [3]	cao (condone $\leqslant$)

10	$y = \dfrac{8}{\sqrt{3x+4}}$		
(i)	$\dfrac{dy}{dx} = \dfrac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef	B1 B1	Without the "×3" For "×3" even if 1st B mark lost.
	$\rightarrow m_{(x=0)} = -\dfrac{3}{2}$ Perpendicular $m_{(x=0)} = \dfrac{2}{3}$	M1	Use of $m_1 m_2 = -1$ after attempting to find $\dfrac{\mathrm{d}y}{\mathrm{d}x}_{(x=0)}$
	Eqn of normal $y - 4 = \dfrac{2}{3}(x-0)$	M1	Unsimplified line equation
	Meets $x = 4$ at $B\left(4, \dfrac{20}{3}\right)$	A1 [5]	cao
(ii)	$\int \dfrac{8}{\sqrt{(3x+4)}}\,\mathrm{d}x = \dfrac{8\sqrt{(3x+4))}}{\frac{1}{2}} \div 3$	B1 B1	Without "÷3". For "÷3"
	Limits from 0 to 4 $\rightarrow$ Area $P = \dfrac{32}{3}$	M1 A1	Correct use of correct limits. cao
	Area Q = Trapezium – P Area of Trapezium = $\dfrac{1}{2}\left(4 + \dfrac{20}{3}\right) \times 4 = \dfrac{64}{3}$	M1	Correct method for area of trapezium
	$\rightarrow$ Areas of P and Q are both $\dfrac{32}{3}$	A1 [6]	All correct.