



Cambridge International AS & A Level

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

February/March 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the equation $\ln(x^3 - 3) = 3 \ln x - \ln 3$. Give your answer correct to 3 significant figures. [3]

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- 2 The polynomial $ax^3 + 5x^2 - 4x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 2)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 2.

Find the values of a and b .

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(b) Sketch the graph of y against x for $0 < x < 2\pi$. [1]

- 5 (a) Express $\sqrt{7} \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

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6 Let $f(x) = \frac{5a}{(2x - a)(3a - x)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions.

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(b) Hence show that $\int_a^{2a} f(x) dx = \ln 6$.

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7 Two lines have equations $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$.

(a) Show that the lines are skew.

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(b) Find the acute angle between the directions of the two lines.

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8 The complex numbers u and v are defined by $u = -4 + 2i$ and $v = 3 + i$.

(a) Find $\frac{u}{v}$ in the form $x + iy$, where x and y are real. [3]

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(b) Hence express $\frac{u}{v}$ in the form $re^{i\theta}$, where r and θ are exact. [2]

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In an Argand diagram, with origin O , the points A , B and C represent the complex numbers u , v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC . [2]

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(d) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

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9 Let $f(x) = \frac{e^{2x} + 1}{e^{2x} - 1}$, for $x > 0$.

(a) The equation $x = f(x)$ has one root, denoted by a .

Verify by calculation that a lies between 1 and 1.5. [2]

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(b) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (c) Find $f'(x)$. Hence find the exact value of x for which $f'(x) = -8$. [6]

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Additional Page

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