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4037/23

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}bc \sin A$$

- 1** The functions f and g are defined as follows, for all real values of x .

$$f(x) = 2 \sin x + 3 \cos x$$

$$g(x) = e^{3x} - 1$$

- (a)** Find $fg(0)$. [2]

- (b)** Find $gg(x)$. [1]

- (c)** Solve the equation $g^{-1}(x) = \frac{1}{3} \ln 5$. [3]

- 2 Find the values of k for which the curve $y = x^2 + kx + (4k - 15)$ is completely above the x -axis. [4]

- 3 (a) Solve the following simultaneous equations.

$$3 \log_2 x + 2 \log_2 y = 24$$

$$5 \log_2 x - 3 \log_2 y = 2 \quad [5]$$

- (b) Solve the equation $\frac{2^{t+4}}{2^{1-2t}} = 512$. [4]

- 4 Find the exact value of $\int_3^5 \frac{(x-1)^2}{x^3} dx$. [6]

- 5** The curved surface area of a cylinder with radius r and height h is $2\pi rh$.

A closed cylinder has radius r cm and volume 1000 cm^3 .

- (a)** Show that the total surface area of the cylinder is $2\pi r^2 + \frac{2000}{r}\text{ cm}^2$. [3]

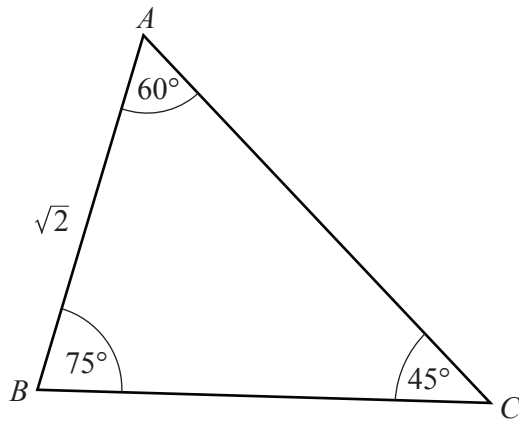
- (b)** Find the value of r which makes this area a minimum. You should show that your value of r gives a minimum for this area. [5]

- 6 A particle travels in a straight line. Its displacement, s metres, from the origin, at time t seconds, where $t > 2$, is given by $s = \ln(4t^2 - 5) - t$.

(a) Find expressions for the velocity, $v \text{ ms}^{-1}$, and acceleration, $a \text{ ms}^{-2}$, of the particle. [4]

(b) Find the time when the particle is at rest. [3]

(c) Find the acceleration at this time. [2]

7 DO NOT USE A CALCULATOR IN THIS QUESTION.

You may use the following trigonometrical ratios.

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 60^\circ = \frac{1}{2}, \quad \cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 60^\circ = \sqrt{3}, \quad \tan 45^\circ = 1$$

- (a) Given that the area of triangle ABC is $\frac{3 + \sqrt{3}}{4}$, show that $\sin 75^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$. [5]

- (b) Hence find the exact length of AC . [2]

8 (a) Show that $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = \frac{\cos x}{\sin^2 x - \cos^2 x}$. [5]

- (b) Hence solve the equation $\frac{\sin x}{\tan x - 1} - \frac{\cos x}{\tan x + 1} = 1$ for $0^\circ < x < 360^\circ$. [5]

9 A curve has equation $y = xe^{2x}$.

(a) Find $\frac{dy}{dx}$. [2]

(b) Find the equation of the normal to the curve at $x = 1$. [4]

- (c) Use your answer to **part (a)** to find the exact value of $\int_0^2 2xe^{2x} dx$. [5]

- 10 (a)** In an arithmetic progression the 5th term is 11. The 7th term is three times the 2nd term.
Find the 1st term and the common difference.

[4]

(b) A different arithmetic progression (AP) and a geometric progression (GP) have the following properties.

- The 1st terms of the AP and GP are both 3.
- The 2nd term of the AP is the same as the 3rd term of the GP.
- The 6th term of the AP is the same as the 5th term of the GP.
- The common ratio of the GP is greater than 1.

Find the common difference of the AP and the common ratio of the GP.

[6]

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