Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

5364007547

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2023**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

DC (DE) 332397

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

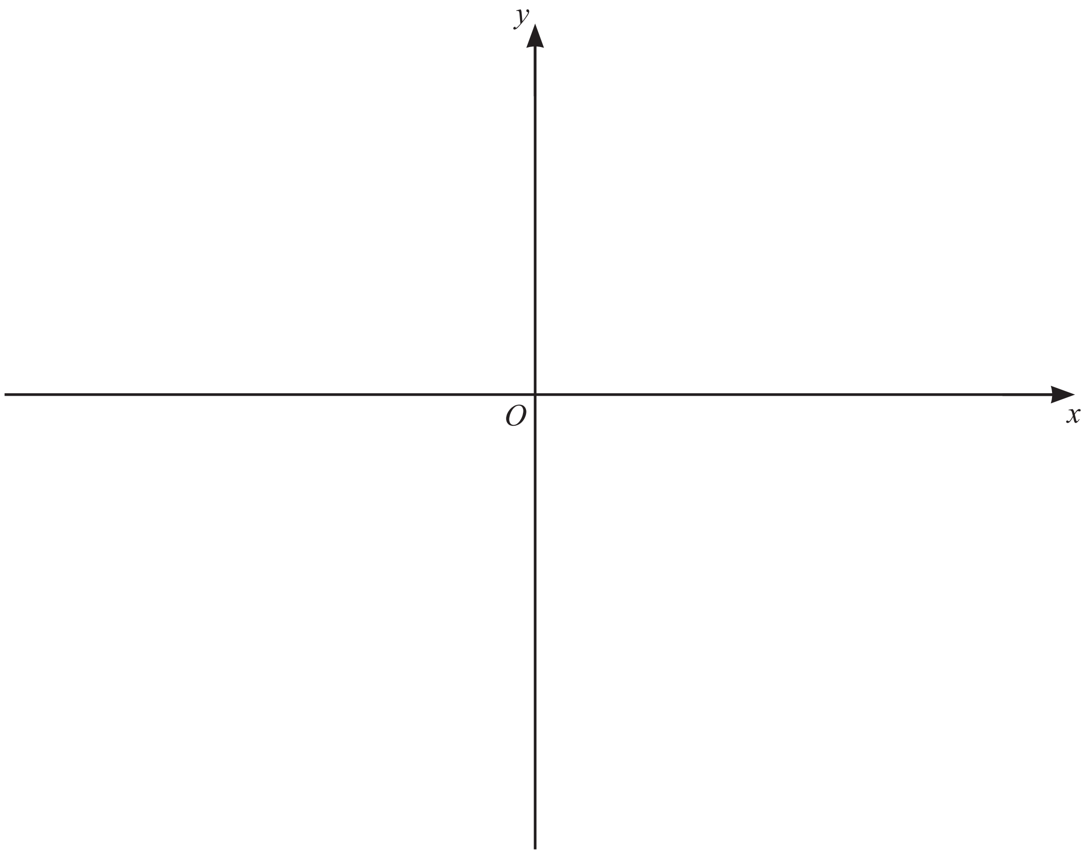
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

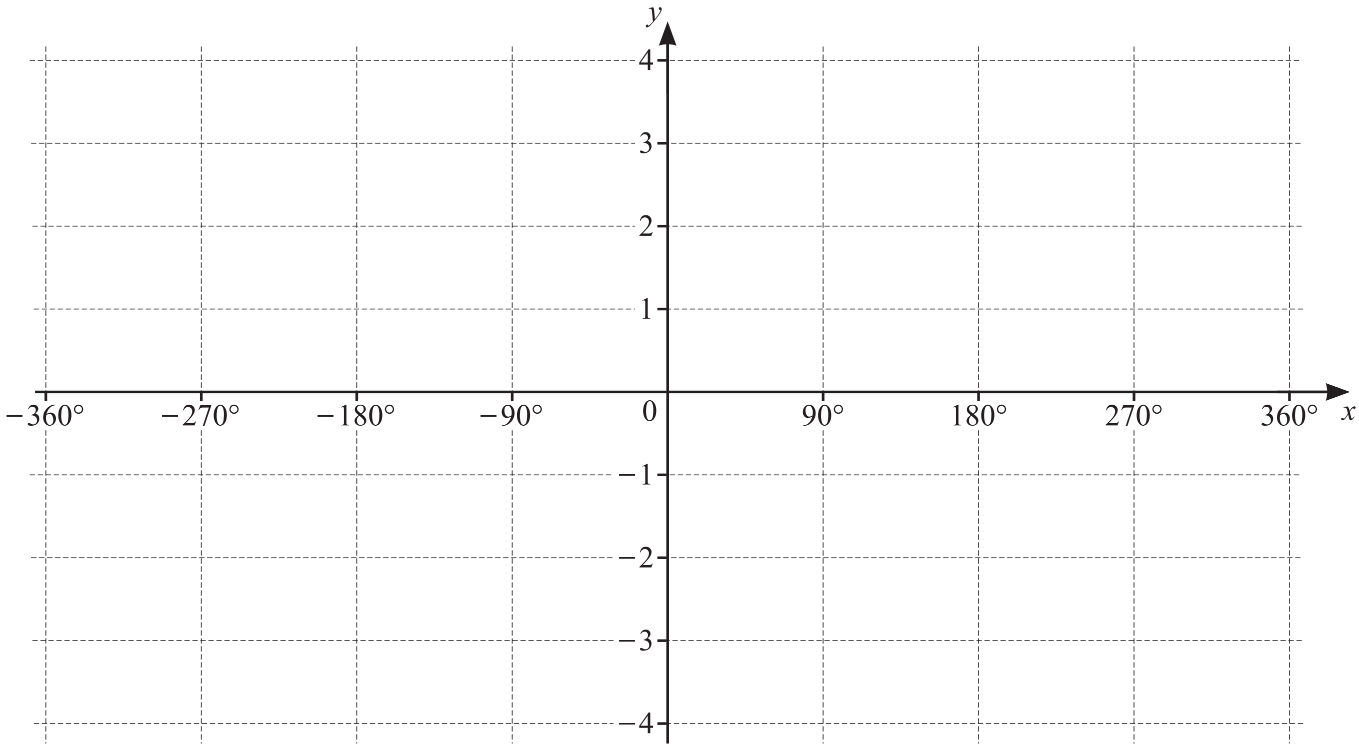
1 **(a)** On the axes, sketch the graphs of $y = 2x + 5$ and $y = |4x - 3|$, stating the intercepts with the coordinate axes. [3]



(b) Solve the inequality $|4x - 3| < 2x + 5$. [3]

2 The perpendicular bisector of the line joining the points $\left(-3, \frac{2}{3}\right)$ and $\left(6, -\frac{7}{3}\right)$ passes through the point $(2, k)$. Find the value of k. [4]

3 On the axes, draw the graph of $y = 2\sin\frac{x}{3} - 1$ for $-360° \leqslant x \leqslant 360°$. [4]



4 The polynomial P is given by $\mathrm{P}(x) = ax^3 + bx^2 + 3x + 2$, where a and b are integers. $\mathrm{P}(x)$ has a factor of $2x + 1$. $\mathrm{P}(x)$ has a remainder of -6 when divided by $x + 1$.

(a) Find the values of a and b. [5]

(b) Show that the equation $\mathrm{P}(x) = 0$ has only one real root. [3]

5 **(a)** A 5-character password is to be formed from the following 10 characters.

Letters	A B C X Y Z
Symbols	* $ # &

No character can be used more than once in any 5-character password.

(i) Find the number of passwords that can be formed. [1]

(ii) Find the number of passwords that can be formed if the password has to contain at least one symbol. [2]

(iii) Find the number of passwords that can be formed if the password has to start with two letters and end with two symbols. [2]

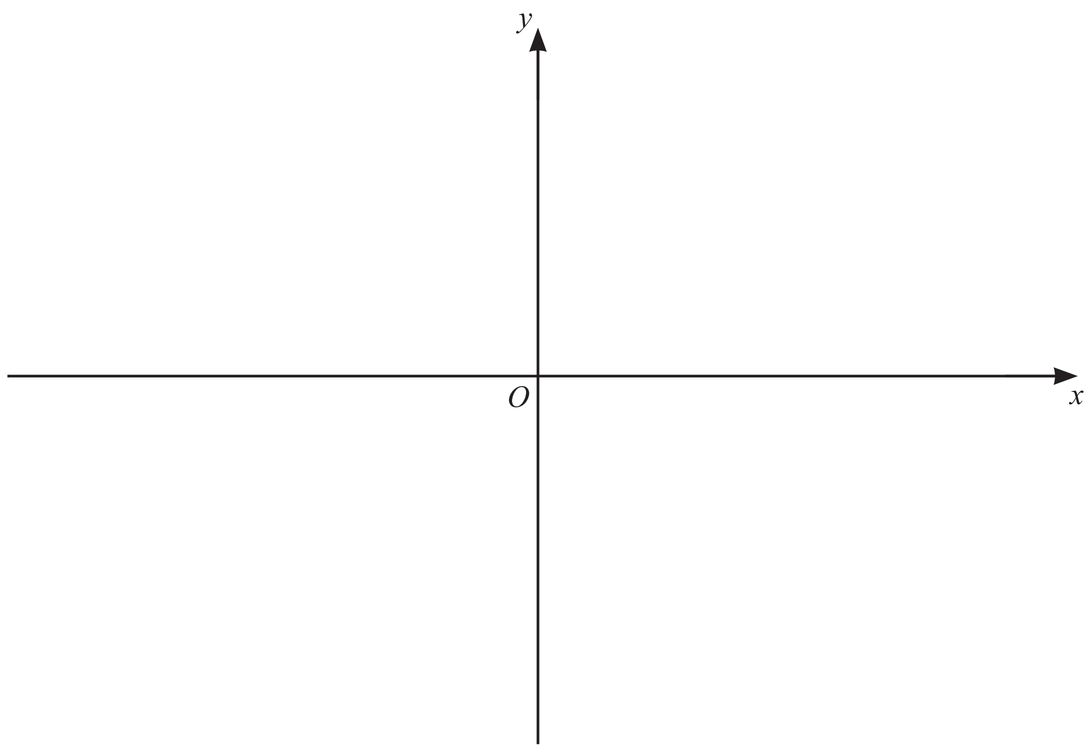
(b) A team of 8 people is to be chosen from 5 doctors, 4 teachers and 6 police officers.

Find how many possible teams have the same number of doctors as teachers. [5]

6 The polynomial $q(x)$ is given by $q(x) = -\frac{1}{3}(2x-1)(x+3)^2$.

(a) Find the x-coordinates of the stationary points on the curve $y = q(x)$. [4]

(b) On the axes, sketch the graph of $y = q(x)$ stating the intercepts with the coordinate axes. [3]



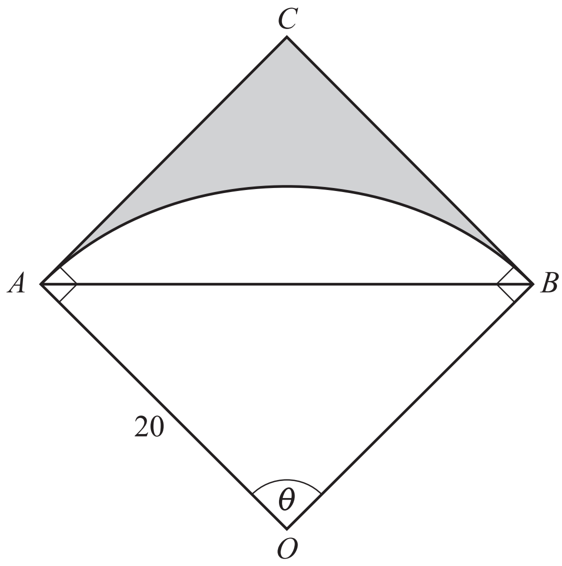
(c) Find the values of k such that $q(x) = k$ has exactly one solution. [3]

7 Solve the equation $6x^{\frac{1}{3}} - 2x^{-\frac{1}{3}} - 1 = 0$. Give your answers in exact form. [4]

8 The first three terms, in descending powers of x, in the expansion of $\left(2x^2 - \frac{1}{4x}\right)^n$ can be written in the form $256x^{16} + ax^{13} + bx^c$, where n, a, b and c are integers. Find the values of n, a, b and c. [6]

9 Given that $y = \frac{(5x+2)^{\frac{1}{3}}}{(x-1)^2}$, show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{3(5x+2)^{\frac{2}{3}}(x-1)^3}$, where A and B are integers. [5]

10 In this question, all lengths are in centimetres and all angles are in radians.

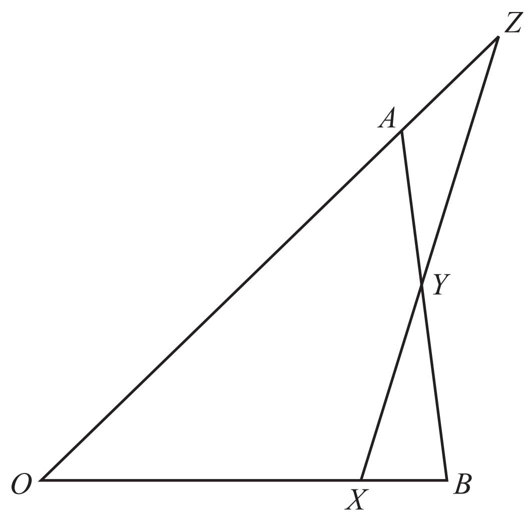


The diagram shows the sector, OAB, of a circle with centre O and radius 20. The perimeter of this sector is 65. The lines CA and CB are both tangents to the circle at the points A and B, so that the triangle ABC is isosceles, with $AC = CB$. The angle AOB is equal to θ.

Find the area of the shaded region. [9]

Additional working space for question 10.

11



In the triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The straight line XYZ is such that:

- $\overrightarrow{OX} = \frac{4}{5}\mathbf{b}$
- $\overrightarrow{AY} = \frac{1}{3}\overrightarrow{AB}$
- $\overrightarrow{AZ} = \mu\mathbf{a}$, where μ is a constant
- $\overrightarrow{YZ} = \lambda\overrightarrow{XY}$, where λ is a constant.

(a) Show that $\overrightarrow{XY} = \frac{2}{3}\mathbf{a} - \frac{7}{15}\mathbf{b}$. [3]

(b) Find $\overrightarrow{YZ}$ in terms of λ, **a** and **b**. [1]

(c) Find $\overrightarrow{YZ}$ in terms of μ, **a** and **b**. [2]

(d) Hence find the values of λ and μ, [3]

Question 12 is printed on the next page.

12 Solve the equation $3\operatorname{cosec}^2\left(\dfrac{2x}{3}-\dfrac{\pi}{3}\right)=4$, for $0 < x \leqslant 3\pi$. Give your answers in terms of π. [5]