Cambridge Assessment International Education

# Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

*1517906375*

**ADDITIONAL MATHEMATICS** **4037/12**

Paper 1 **October/November 2023**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

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***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2+bx+c=0$,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n=a^n+\binom{n}{1}a^{n-1}b+\binom{n}{2}a^{n-2}b^2+\ldots+\binom{n}{r}a^{n-r}b^r+\ldots+b^n$$

where $n$ is a positive integer and $\binom{n}{r}=\frac{n!}{(n-r)!r!}$

*Arithmetic series* $u_n=a+(n-1)d$

$$S_n=\frac{1}{2}n(a+l)=\frac{1}{2}n\{2a+(n-1)d\}$$

*Geometric series* $u_n=ar^{n-1}$

$$S_n=\frac{a(1-r^n)}{1-r}\quad(r\neq1)$$

$$S_\infty=\frac{a}{1-r}\quad(|r|<1)$$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2A+\cos^2A=1$$
$$\sec^2A=1+\tan^2A$$
$$\operatorname{cosec}^2A=1+\cot^2A$$

*Formulae for $\Delta ABC$*

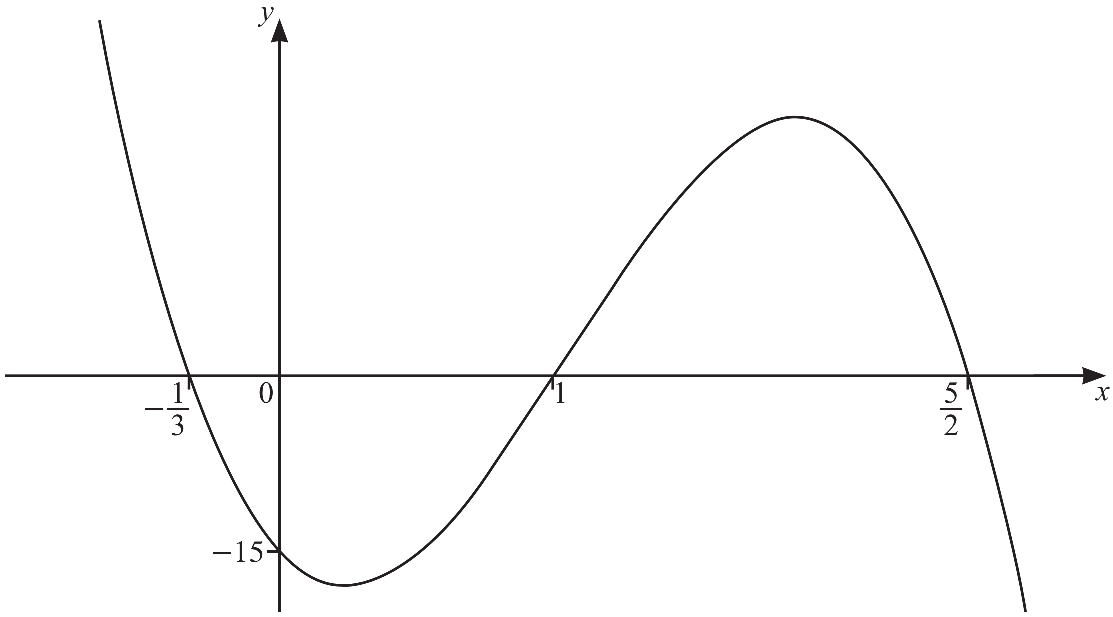
$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$a^2=b^2+c^2-2bc\cos A$$

$$\Delta=\frac{1}{2}bc\sin A$$

1




The diagram shows the graph of the cubic polynomial $y = \mathrm{f}(x)$.

**(a)** Find an expression for $\mathrm{f}(x)$ in factorised form. Write each linear factor with its coefficients as integers. [3]

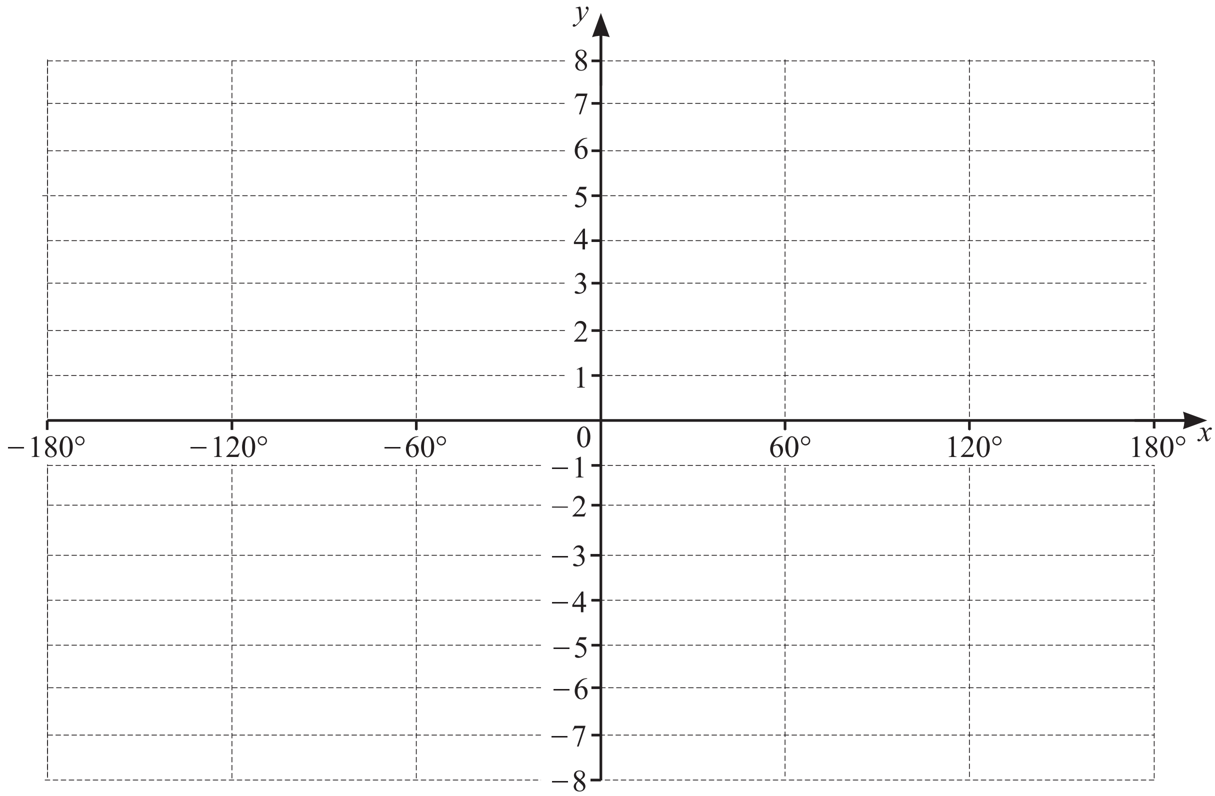
**(b)** Write down the values of $x$ such that $\mathrm{f}(x) < 0$. [2]

**2** The function g is defined by $g(x) = 5\sin\frac{3x}{4} - 2$ for all values of $x$.

**(a)** Write down the amplitude of g. [1]

**(b)** Write down the period of g in degrees. [1]

**(c)** On the axes, sketch the graph of $y = g(x)$, for $-180° \leqslant x \leqslant 180°$. [3]



3 When $\ln(y+2)$ is plotted against $x^2$ a straight line graph is obtained. The line passes through the points $(2.25, 9.37)$ and $(4.75, 3.92)$. Find $y$ in terms of $x$. [5]

**4** **(a)** It is given that the first four terms, in ascending powers of $x$, in the expansion of $\left(1-\frac{x}{2}\right)^n$ can be written in the form $1-8x+px^2+qx^3$, where $n$, $p$ and $q$ are integers. Find the values of $n$, $p$ and $q$. [5]

**(b)** Find the term independent of $x$ in the expansion of $\left(\frac{2}{x^2}+\frac{x}{3}\right)^6$, giving your answer as a rational number. [2]

**5** Solve the equation $3\sec^2\left(2\theta + \frac{\pi}{6}\right) = 4$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, giving your answers in terms of $\pi$. [5]

**6** The polynomial $p(x)$ is such that $p(x) = ax^3 + bx^2 + cx - 5$, where $a$, $b$ and $c$ are integers. It is given that $p'(0) = 12$. It is also given that $p(x)$ has a factor of $3x - 1$ and a remainder of 95 when divided by $x - 2$.

**(a)** Find the values of $a$, $b$ and $c$. [7]

**(b)** Show that the equation $p(x) = 0$ has only one real root. [3]

7 **(a)** A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit can be used only once in any 6-digit number. A 6-digit number cannot start with 0.

**(i)** Find how many 6-digit numbers can be formed. [1]

**(ii)** Find how many of these 6-digit numbers are divisible by 5. [3]

**(b)** A committee of 7 people is to be chosen from 6 doctors, 10 nurses and 8 dentists.

**(i)** Find the number of committees that can be chosen. [1]

**(ii)** Find the number of committees that can be chosen if all the doctors have to be on the committee. [1]

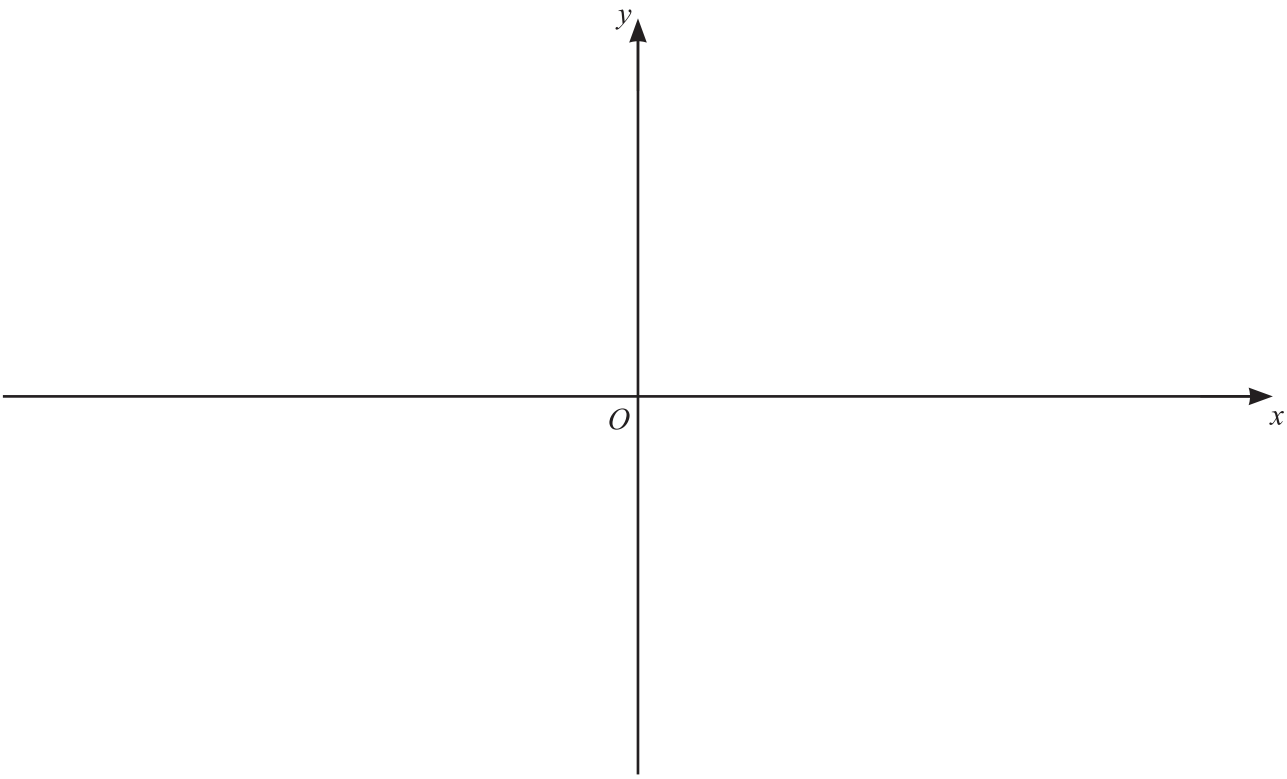
**(iii)** Find the number of committees that can be chosen if there has to be at least one dentist on the committee. [2]

**8** **(a)** It is given that $f : x \to (3x+1)^2 - 4$ for $x \geqslant a$, and that $f^{-1}$ exists.

**(i)** Find the least possible value of $a$. [1]

**(ii)** Using this value of $a$, write down the range of f. [1]

**(iii)** Using this value of $a$, sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]



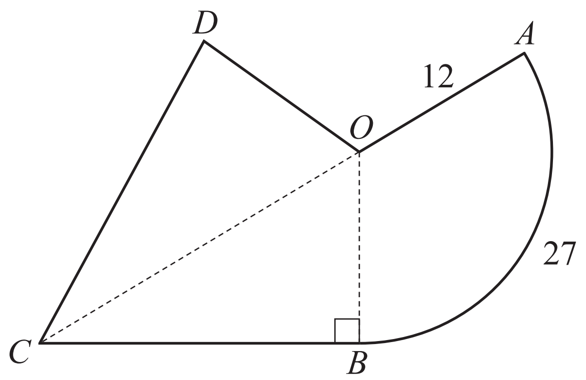
**(b)** It is given that $\quad g(x) = \ln(2x^2 + 5) \quad \text{for } x \geqslant 0,$

$$h(x) = 3x - 2 \quad \text{for } x \geqslant 0.$$

Solve the equation $hg(x) = 4$ giving your answer in exact form. [3]

**9** Solve the equation $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$ for $x > 0$. Give your answer correct to one decimal place. [4]

**10** In this question all lengths are in centimetres and all angles are in radians.

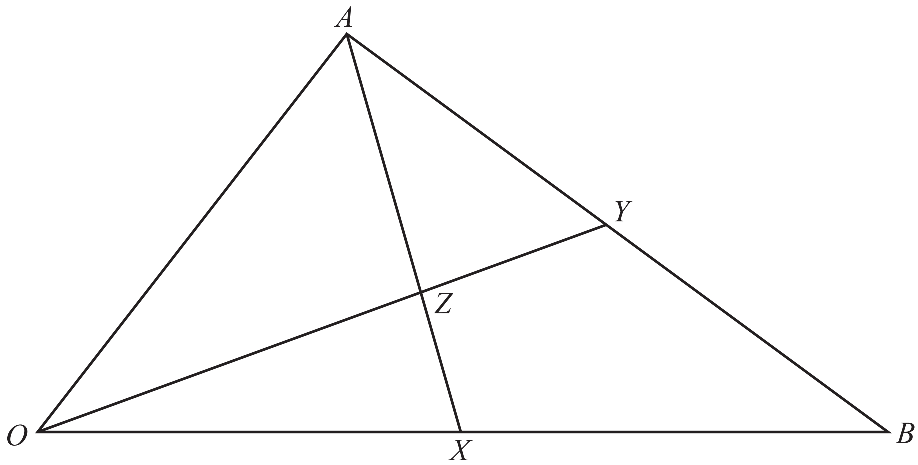



The diagram shows a badge which consists of a minor sector, $OAB$, of the circle with centre $O$ and radius 12, and a kite $OBCD$, where $OB = OD$ and $CD = CB$. The arc $AB$ has length 27. The line $OB$ is perpendicular to the line $CB$, and $COA$ is a straight line.

**(a)** Find the perimeter of the badge. [4]

**(b)** Find the area of the badge. [3]

**11**




In the triangle $OAB$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The mid-point of the line $OB$ is $X$, and the mid-point of the line $AB$ is $Y$. The lines $OY$ and $AX$ intersect at the point $Z$. It is given that $\overrightarrow{AZ} = \lambda\overrightarrow{AX}$ and $\overrightarrow{OZ} = \mu\overrightarrow{OY}$ where $\lambda$ and $\mu$ are rational numbers.

**(a)** Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\lambda$. [3]

**(b)** Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mu$. [2]

**(c)** Find the values of $\lambda$ and $\mu$. [3]

**(d)** Hence find $\overrightarrow{OZ}$ in terms of **a** and **b** only. [1]

**Question 12 is printed on the next page.**

**12** A curve has equation $y = \dfrac{\sqrt{5x-2}}{x-3}$.

**(a)** Explain why the curve does not exist when $x < \dfrac{2}{5}$. [1]

**(b)** Show that $\dfrac{dy}{dx}$ can be written in the form $\dfrac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$, where $A$ and $B$ are positive integers. [5]