Cambridge Assessment International Education

Cambridge O Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8704417121

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2022**

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

DC (DE) 318310

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where n is a positive integer and $\binom{n}{r} = \dfrac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

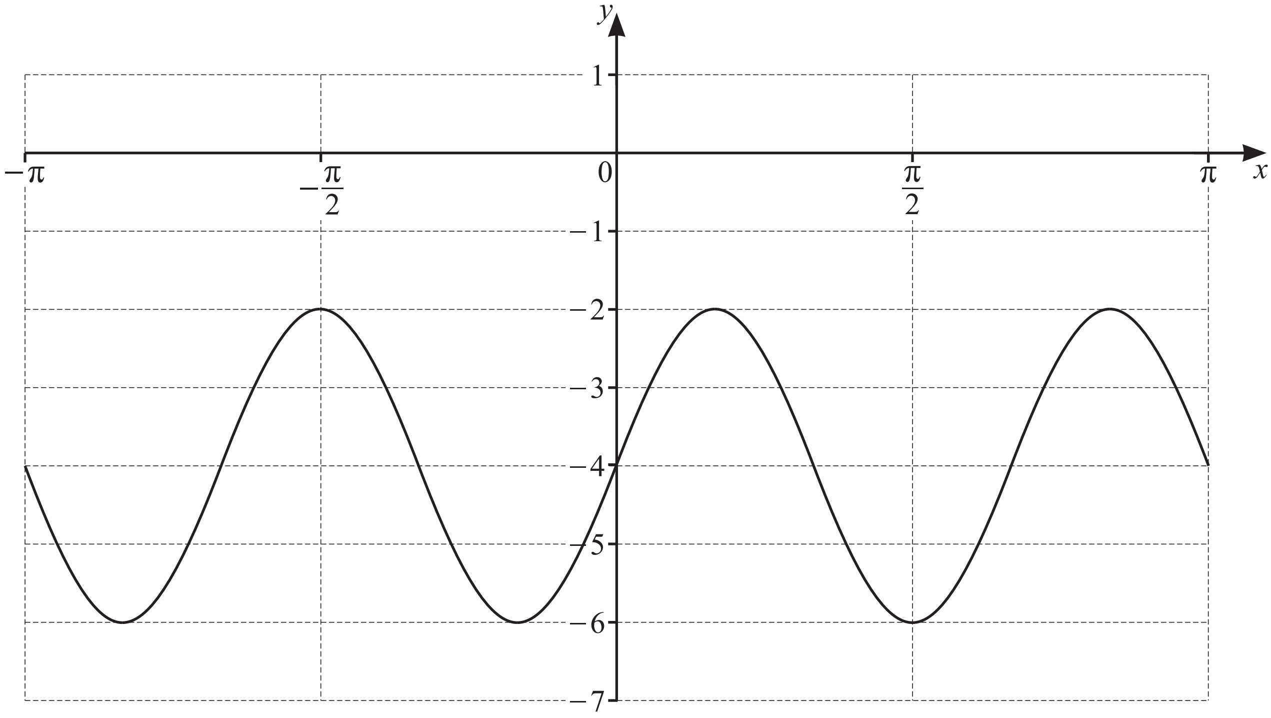
Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

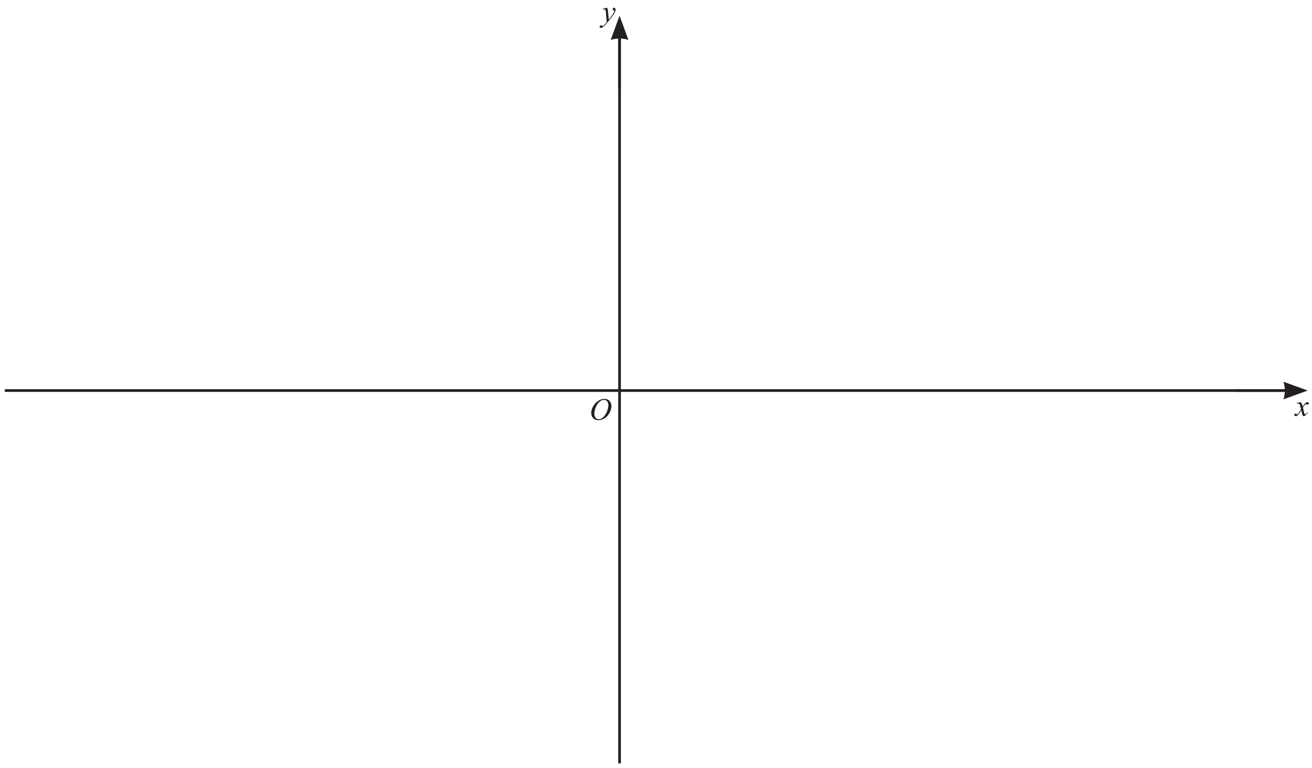
$$\Delta = \frac{1}{2}bc\sin A$$

1



The diagram shows the graph of $y = a\sin bx + c$, where a, b and c are integers. Find the values of a, b and c. [3]

2 **(a)** On the axes, draw the graph of $y = |3x^2 + 13x - 10|$, stating the coordinates of the points where the graph meets the axes. [4]



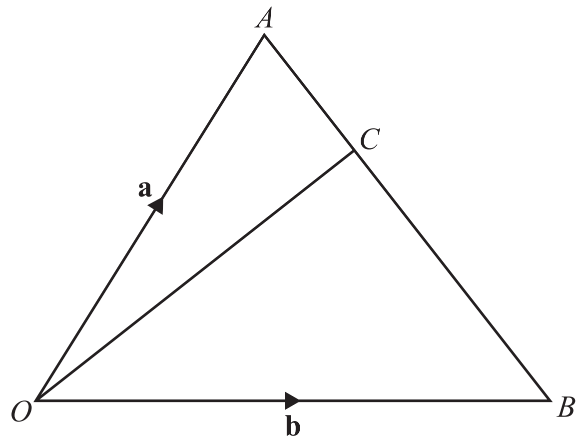
(b) Find the set of values of the constant k such that the equation $k = |3x^2 + 13x - 10|$ has exactly 2 distinct roots. [4]

3 Write $\dfrac{\sqrt{(9p^2q)} \times r^{-3}}{(2p)^3 q^{-1} \sqrt[5]{r}}$ in the form $kp^a q^b r^c$, where k, a, b and c are constants. [4]

4 Solve the equation $3\sin\left(2x + \dfrac{\pi}{4}\right) = \sqrt{3}\cos\left(2x + \dfrac{\pi}{4}\right)$, for $0 \leqslant x \leqslant \pi$. [5]

5 **(a)** Find the vector with magnitude 200 in the direction of $\begin{pmatrix} 7 \\ -24 \end{pmatrix}$. [2]

(b)

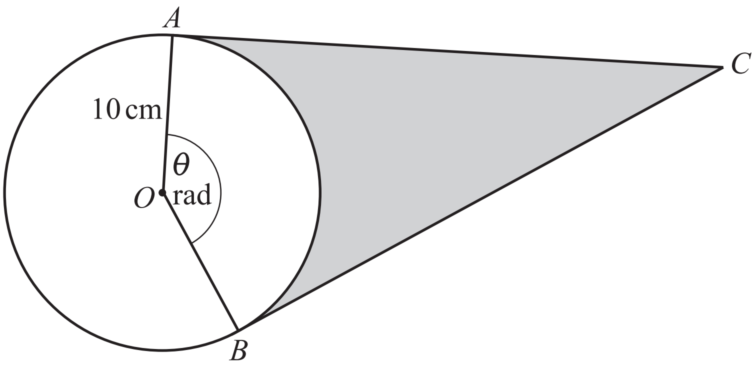


The diagram shows triangle AOB such that $\overrightarrow{OA} = \mathbf{a}$, and $\overrightarrow{OB} = \mathbf{b}$. The point C lies on the line AB such that $AC : AB = 1 : 3$. Find the vector $\overrightarrow{OC}$ in terms of **a** and **b**, giving your answer in its simplest form. [3]

(c) Given the vector equation $p\begin{pmatrix}2\\1\end{pmatrix}+q\begin{pmatrix}2\\4\end{pmatrix}=5\begin{pmatrix}-p+1\\p+q\end{pmatrix}$, find the values of p and q. [3]

6 A group of 15 people includes 3 brothers. A team of 6 people is to be chosen from this group. The three brothers must not be separated. Find the number of possible teams that can be chosen. [3]

7



The diagram shows a circle, centre O, radius 10 cm. The points A and B lie on the circumference of the circle. The tangent at A and the tangent at B meet at the point C. The angle AOB is θ radians. The length of the minor arc AB is 28 cm.

(a) Find the value of θ. [1]

(b) Find the perimeter of the shaded region. [3]

(c) Find the area of the shaded region. [3]

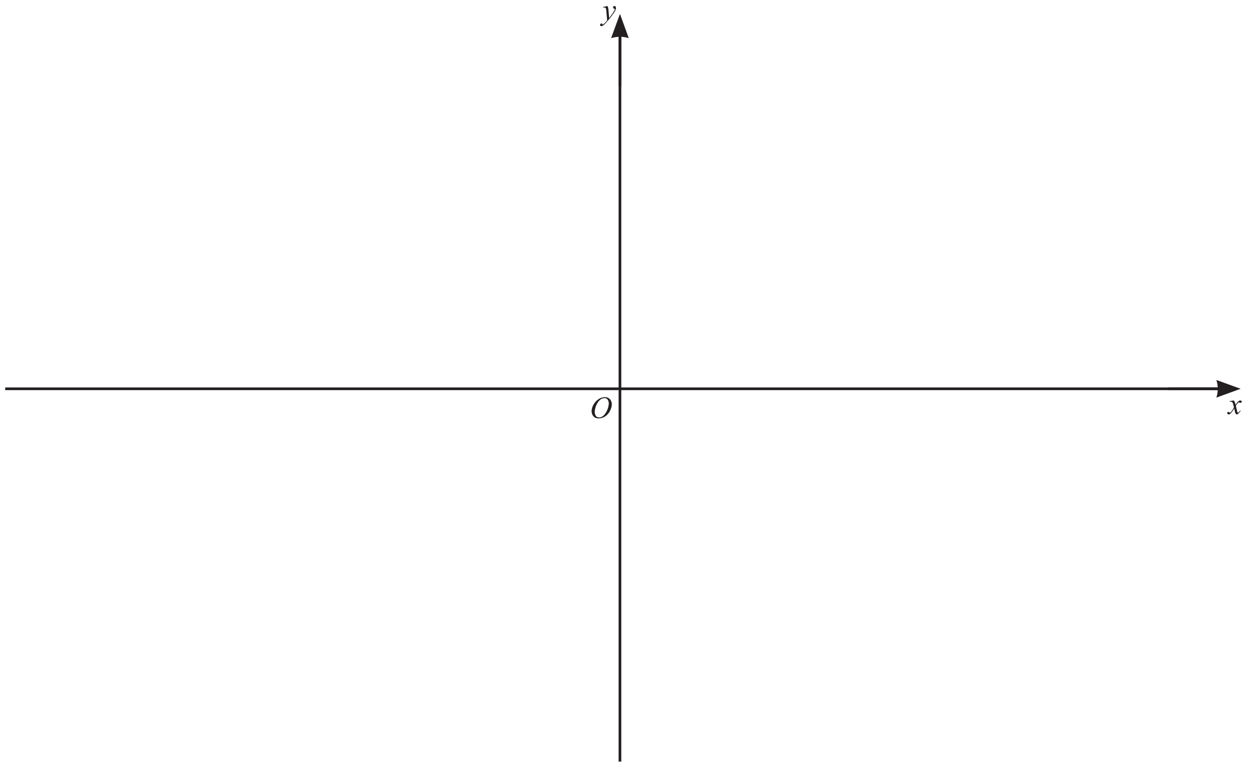
8 A function $f(x)$ is such that $f(x) = \ln(2x+3) + \ln 4$, for $x > a$, where a is a constant.

(a) Write down the least possible value of a. [1]

(b) Using your value of a, write down the range of f. [1]

(c) Using your value of a, find $f^{-1}(x)$, stating its range. [4]

(d) On the axes below, sketch the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, stating the exact intercepts of each graph with the coordinate axes. Label each of your graphs. [4]



9 **(a)** Show that $\dfrac{1}{2x+1}-\dfrac{1}{(2x+1)^2}+\dfrac{4}{4x-1}=\dfrac{24x^2+14x+4}{(2x+1)^2(4x-1)}$. [2]

(b) Hence find $\displaystyle\int_{\frac{1}{2}}^{1}\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}\,\mathrm{d}x$, giving your answer in the form $\dfrac{1}{2}\ln p+q$, where p and q are rational numbers. [7]

10 The first three terms of an arithmetic progression are $\lg x$, $\lg x^5$, $\lg x^9$, where $x > 0$.

(a) Show that the sum to n terms of this arithmetic progression can be written as $n(pn-1)\lg x$, where p is an integer. [4]

(b) Hence find the value of n for which the sum to n terms is equal to $4950\lg x$. [2]

(c) Given that this sum to n terms is also equal to -14850, find the exact value of x. [2]

11 A particle P moves in a straight line such that, t seconds after passing through a fixed point O, its displacement, s metres, is given by $s = \dfrac{(2t+1)^{\frac{3}{2}}}{t+1} - 1$.

(a) Show that the velocity of P at time t can be written in the form $\dfrac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(a+bt)$, where a and b are integers to be found. [5]

(b) Show that P is never at instantaneous rest after passing through O. [1]

12 The first three terms, in descending powers of x, of the expansion of $\left(ax+\frac{2}{5}\right)^5\left(1-\frac{b}{x}\right)^2$, can be written as $32x^5-160x^4+cx^3$, where a, b and c are constants. Find the exact values of a, b and c. [9]

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