Cambridge O Level

ADDITIONAL MATHEMATICS **4037/13**

Paper 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

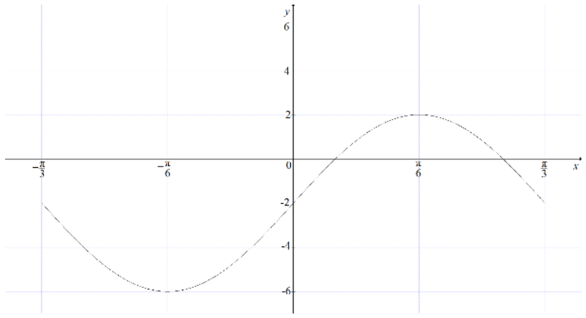
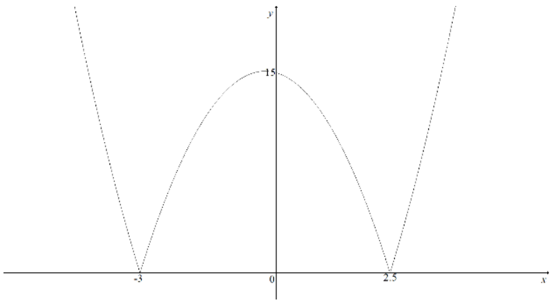
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

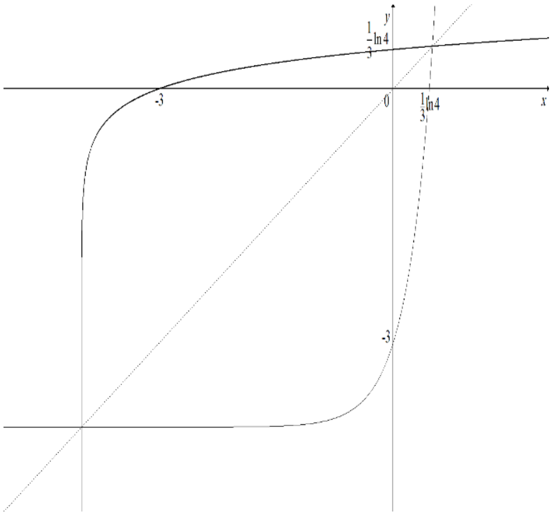
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1		**3**	**B1** for a curve starting at $\left(-\frac{\pi}{3},-2\right)$ and finishing at $\left(\frac{\pi}{3},-2\right)$ **B1** for a curve, must have implied symmetry about $\frac{\pi}{6}$ and $-\frac{\pi}{6}$, one complete cycle only. **B1** for a curve passing through (0, –2) and distinct maximum at $\left(\frac{\pi}{6}, 2\right)$ and distinct minimum at only $\left(-\frac{\pi}{6},-6\right)$
2(a)	$2\left(x+\frac{1}{4}\right)^2-\frac{121}{8}$	**2**	**B1** for $a=\frac{1}{4}$ **B1** for $b=-\frac{121}{8}$
2(b)	$\left(-\frac{1}{4},-\frac{121}{8}\right)$	**2**	**FTB1** for each, follow through on *their a* and *b* from **(a)** or **SC1** if differentiation is used ie $\frac{dy}{dx}=4x+1=0$ then $\left(-\frac{1}{4},-\frac{121}{8}\right)$
2(c)		**3**	**B1** for a correct shape. Must have the parabola part of the curve in the first and second quadrant with cusps and correct curvature and a max in the **2nd** quadrant. Ignore labelling of their maximum point if incorrect coordinates **B1** for a curve $\left(\frac{5}{2},0\right)$ and (–3, 0) **B1** for a curve (0, 15)
2(d)	$k=\frac{121}{8}$	**B1**	**FT** Follow through on *their* $-b$

Question	Answer	Marks	Guidance
3(a)	$3y^2+2y-1[=0]$ oe or $4x^2-4x-3[=0]$ oe	M1	**M1** for obtaining a 3 term quadratic equation in y or x **and** an attempt to solve
	$x=\frac{3}{2},\ x=-\frac{1}{2}$ oe	A1	
	$y=\frac{1}{3},\ y=-1$ oe	A1	Allow **A1** for the one correct pair e.g. $\left(\frac{3}{2},\frac{1}{3}\right)$ or $\left(-\frac{1}{2},-1\right)$
3(b)	$[\log_3 x+3=]\frac{10}{\log_3 x}$ oe or $\frac{1}{\log_x 3}[+3=10\log_x 3]$ oe	B1	For change of base
	$(\log_3 x)^2+3\log_3 x-10=0$ or $10(\log_x 3)^2-3\log_x 3-1=0$ $\log_3 x=-5\ \ \log_3 x=2$ or $\log_x 3=-\frac{1}{5}\ \ \log_x 3=\frac{1}{2}$	M1	**Dep** on previous **B** mark, for attempt to obtain a 3-term quadratic equation and attempt to solve to obtain 2 solutions of the form $\log_3 x=p$ or $\log_x 3=q$
	$3^{-5}\quad 3^2$ isw	2	**A1** for each
4(a)	$p'(x)=3ax^2+26x+b$ $p'(0)=b$	B1	Must see at least $p'(x)=3ax^2+26x+b$ to award the mark
4(b)	$p\left(-\frac{2}{3}\right): 8a-27c=318$ oe	M1	For use of $x=-\frac{2}{3}$, at least once **and** attempt at simplification leading to an equation in a and c only Allow one sign error.
	$p(-1): a-c=16$ oe	M1	For use of $x=-1$ **and** attempt at simplification leading to an equation in a and c only
	$a=6, c=-10$	2	**M1 dep** on both previous **M** marks and attempt to solve simultaneously to obtain both a and c **A1** for both
4(c)	$2x^2+3x-5$	B1	Allow if seen embedded i.e.: $(3x+2)(2x^2+3x-5)$ or as a quotient in long division

Question	Answer	Marks	Guidance
4(d)	$(3x+2)(x-1)(2x+5)$	**B1**	
5(a)	$r^3 = \frac{1}{8}$ soi	**M1**	Allow unsimplified $ar^{14} = \frac{1}{8}ar^{11}$ or $\frac{5r^{11}-5r^{12}}{5r^{14}-5r^{15}} = 8$ oe
	$r = \frac{1}{2}$	**A1**	
	$5 = \frac{a}{1-r}$	**M1**	For use of sum to infinity with *their* r, must be $-1 < r < 1$
	$a = \frac{5}{2}$	**A1**	
5(b)	$their(\mathbf{a}) \times \frac{\left(1-(their\ r)^n\right)}{(1-their\ r)}$	**M1**	For use of the sum to n terms
	$(their\ r)^n = 0.0002$ (12.29)	**M1**	**M1 dep** For simplification and attempt to obtain the critical value using either an equation or an inequality leading to $n =$ or $n >$
	13	**A1**	Accept $n \geqslant 13$
6(a)	$\mathrm{f} > -4$	**B1**	Allow $y > -4$ or $-4 < \mathrm{f} < \infty$ or $\mathrm{f} \in (-4, \infty)$
6(b)	$\left[\mathrm{f}^{-1}(x) =\right] \frac{1}{3}\ln(x+4)$	**2**	**M1** for a correct method to find the inverse, allow one sign error Must be in the form of $3x = \ln(y \pm 4)$ or $3y = \ln(x \pm 4)$ **A1** allow $y =$

Question	Answer	Marks	Guidance
6(c)		4	**B1** for f(x) with correct shape in quadrant 1, 3 and 4 and appropriate asymptotic behaviour **B1** for –3 on the y-axis and $\frac{1}{3}\ln 4$ on the x-axis for f(x) must have the correct shape **B1** for $f^{-1}(x)$with correct shape in quadrant 1, 2 and 3 and appropriate asymptotic behaviour **B1** for –3 on the x-axis and $\frac{1}{3}\ln 4$ on the y-axis for $f^{-1}(x)$ must have correct shape and intersect at least once
7	$\frac{1}{3}\sin 3x - 2\cos 2x + x$	2	**M1** for $a\sin 3x + b\cos 2x + x$, $a \neq \pm 3$ and $b \neq \pm 8$ **A1** all correct
	$\left(\frac{1}{3}\sin\frac{3\pi}{2} - 2\cos\pi + \frac{\pi}{2}\right) - (-2)$	M1	**Dep** on previous **M** mark for correct substitution (seen or implied) of **both** limits in x
	$\frac{11}{3}$	A1	
	$\frac{\pi}{2}$	B1	From correct substitution (seen or implied) of **both** limits in x
8(a)	1.75	B1	
8(b)	$\cos BOC = \frac{7}{25}$, $\tan BOC = \frac{24}{7}$, $\sin BOC = \frac{24}{25}$ BOC = 1.287 soi	B1	
	Arc length = r × *their* 1.287	B1	Follow through on *their BOC*
	Perimeter = 12.25 + *their* 9.009 + 14	M1	For a complete method
	35.3	A1	
8(c)	$\left(\frac{1}{2}\times 7^2 \times 1.75\right) + \left(\frac{1}{2}\times 7^2 \times \textit{their } BOC\right)$ oe or $\pi \times 7^2 - \frac{1}{2}\times 7^2 \times (2\pi - 1.75 - \textit{their}\,1.287)$	M1	For a complete method
	74.4	A1	
9(a)(i)	665 280	B1	
9(a)(ii)	221 760	B1	

Question	Answer	Marks	Guidance
9(b)	$8 \times 4 \times 3 \times 2 \times 1 \times 7$	**M1**	For either 8×7 or 4! or 24 as part of a product
	1344	**A1**	
10	$\tan(3x + 1.2)\left[= \frac{1}{\sqrt{2}}\right]$ or $\cos^2(3x + 1.2)\left[= \frac{2}{3}\right]$ or $\sin^2(3x + 1.2)\left[= \frac{1}{3}\right]$	**M1**	For an attempt to obtain an equation in $\sin(3x + 1.2)$, $\cos(3x + 1.2)$ or $\tan(3x + 1.2)$
	$x = -1.24$, -0.195, 0.852 or better	**4**	**M1 dep** for a correct attempt to obtain one correct solution **A1** for one correct solution in the range **M1 dep** for an attempt to obtain another solution within the range **A1** for 2 more correct solutions within the range and no extra solutions within the range
11	$\ln(3x+2) - \ln(2x+1) - \ln x$	**2**	**B1** for 1 correct term **B1** for the other two terms correct
	$\ln\frac{(3a+2)}{a(2a+1)} - \ln\frac{5}{3}$ $\ln\frac{3(3a+2)}{5a(2a+1)} = \left[\ln\frac{1}{5}\right]$ or $\ln\frac{(3a+2)}{a(2a+1)} = \ln\frac{1}{3}$	**2**	**M1** for application of limits correctly, **dep** on at least one **B** mark **M1** for application of log laws to obtain a single logarithm, dep on at least one **B** mark
	$a^2 - 4a - 3 = 0$ $a = 2 + \sqrt{7}$	**2**	**M1** for equating to $\ln\frac{1}{5}$ and attempt to solve resulting 3-term quadratic equation, dep on at least one **B** mark **A1** for $2 + \sqrt{7}$ must reject $2 - \sqrt{7}$

Question	Answer	Marks	Guidance
12(a)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$ $\frac{(x-1)\times\frac{2}{3}\times 6x\left(3x^2-2\right)^{-\frac{1}{3}}-\left(3x^2-2\right)^{\frac{2}{3}}}{(x-1)^2}$ or $\left(\frac{\mathrm{d}y}{\mathrm{d}x}=\right)$ $(x-1)^{-1}\times\frac{2}{3}\times 6x\left(3x^2-2\right)^{-\frac{1}{3}}-(x-1)^{-2}\left(3x^2-2\right)^{\frac{2}{3}}$	3	**B1** for $\frac{2}{3}\times 6x\left(3x^2-2\right)^{-\frac{1}{3}}$ **M1** for differentiation of a quotient or product **A1** for all terms other than $\frac{2}{3}\times 6x\left(3x^2-2\right)^{-\frac{1}{3}}$ correct
	$\frac{\left(3x^2-2\right)^{-\frac{1}{3}}}{(x-1)^2}\left(x^2-4x+2\right)$	2	**M1 dep** for attempt to factorise, must be in the form $\frac{\left(3x^2-2\right)^{-\frac{1}{3}}}{(x-1)^2}\left[ax(x-1)-\left(3x^2-2\right)\right]$ **A1** all correct
12(b)	When $x=2, \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{2}{\sqrt[3]{10}}$ oe	M1	**Dep** on the differentiation **M** mark from part **(a)** For attempt to find the value of *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2$
	$-\frac{2}{\sqrt[3]{10}}p$ or $-0.928p$	A1	
13(a)	Midpoint (10, –9)	B1	
	Gradient of l = $-\frac{5}{3}$	B1	
	Equation of l: $y+9=-\frac{5}{3}(x-10)$ oe	M1	Must be using *their* perpendicular gradient and *their* mid-point
	$y = -4$	A1	

Question	Answer	Marks	Guidance
13(b)	Attempt to use *their* R and displacement vectors or Pythagoras to find S	**M1**	May be implied by one correct coordinate If Pythagoras is used: **M1** for an attempt to reach to a 3-term quadratic with one variable using *their* equation and *their* midpoint from **(a)** e.g. $34x^2 - 680x + 646 = 0$
	(1, 6)	**A1**	
	(19, –24)	**A1**	