Cambridge Assessment International Education

Cambridge O Level

ADDITIONAL MATHEMATICS **4037/12**

Paper 1 **October/November 2022**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

[Turn over

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Maths-Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

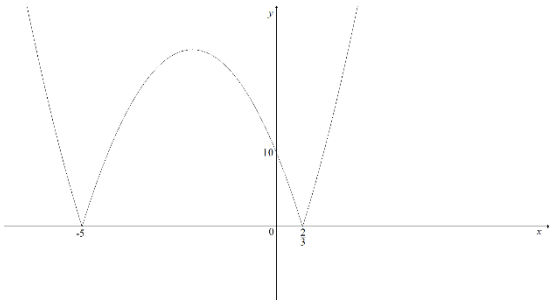
A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation '**dep**' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

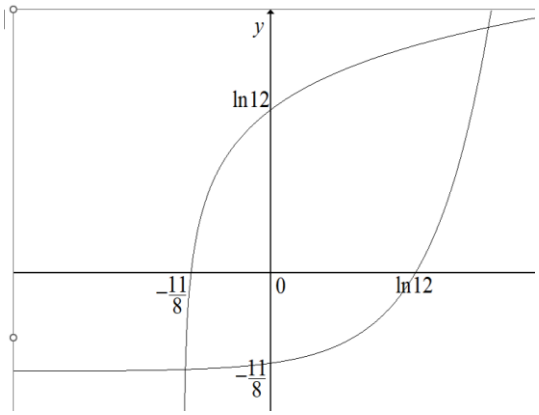
Abbreviations

awrt answers which round to
cao correct answer only
dep dependent
FT follow through after error
isw ignore subsequent working
nfww not from wrong working
oe or equivalent
rot rounded or truncated
SC Special Case
soi seen or implied

Question	Answer	Marks	Guidance
1	$a=2$	**B1**	
	$b=3$	**B1**	
	$c=-4$	**B1**	
2(a)		**4**	**B1** for a correct basic shape, allow 'construction curve' **Dep B1** for (0, 10) must have correct basic shape, must be convinced that this is the vertical intercept **B1** for (–5, 0) and $\left(\frac{2}{3},0\right)$ or (0.667, 0) or better **Dep B1** on all previous **B** marks for all correct with cusps and the correct shape for $x<-5$ and $x>\frac{2}{3}$
2(b)	Stationary point when $x=-\frac{13}{6}$ soi	**M1**	For differentiation or completing the square or use of symmetry
	$(-)\frac{289}{12}$ or $(-)24.1$ or better	**A1**	For *y*-value of stationary point, allow +ve or –ve value.
	$k>\frac{289}{12}$ or $k>24.1$ or better	**A1**	
	$k=0$	**B1**	
	Alternative		
	$3x^2+13x-(10+k)$ Using discriminant, $169+12(10+k)$	**(M1)**	Allow a sign error in $3x^2+13x-(10+k)$, but must have a term in *k* not k^2
	Critical value $(-)\frac{289}{12}$ or $(-)24.1$ or better	**(A1)**	
	$k>\frac{289}{12}$ or $k>24.1$ or better	**(A1)**	One solution only from correct work
	$k=0$	**(B1)**	

Question	Answer	Marks	Guidance
3	$\frac{3}{8}p^{-2}q^{\frac{3}{2}}r^{-\frac{16}{5}}$	**4**	**B1** for $k=\frac{3}{8}$ or 0.375 **B1** for $a=-2$ **B1** for $b=\frac{3}{2}$ oe **B1** for $c=-\frac{16}{5}$, –3.2. $-3\frac{1}{5}$
4	$\tan\left(2x+\frac{\pi}{4}\right)=\frac{1}{\sqrt{3}}$ or $\sin^2\left(2x+\frac{\pi}{4}\right)=\frac{1}{4}$ or $\cos^2\left(2x+\frac{\pi}{4}\right)=\frac{3}{4}$	**B1**	Must be from correct working Allow if $\theta=2x+\frac{\pi}{4}$ oe
	$2x+\frac{\pi}{4}=\frac{\pi}{6},\ \frac{7\pi}{6},\frac{13\pi}{6}$ $x=-\frac{\pi}{24}$	**M1**	**Dep** on previous **B1** For attempt at the correct order of operations, may be implied by a correct solution or $x=-\frac{\pi}{24}$.
	$x=\frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	**2**	**Dep M1** for an attempt to find a solution within the given range. Must be working with $\frac{7\pi}{6}$ or $\frac{13\pi}{6}$ **A1** for either
	$x=\frac{11\pi}{24}$ or $\frac{23\pi}{24}$ oe 0.458π or 0.958π 1.44 or 3.01	**A1**	For a second solution within the given range with no extra solutions within the range.
5(a)	25	**B1**	soi
	$\begin{pmatrix}56\\-192\end{pmatrix}$ or $8\begin{pmatrix}7\\-24\end{pmatrix}$	**B1**	
5(b)	$\overrightarrow{AC}=\frac{1}{3}(\mathbf{b}-\mathbf{a})$ or $\overrightarrow{CB}=\frac{2}{3}(\mathbf{b}-\mathbf{a})$ oe	**B1**	
	$\overrightarrow{OC}=\mathbf{a}+\frac{1}{3}(\mathbf{b}-\mathbf{a})$ or $\mathbf{b}-\frac{2}{3}(\mathbf{b}-\mathbf{a})$ oe	**M1**	For using $\overrightarrow{OA}+their\,\overrightarrow{AC}$ or $\overrightarrow{OB}+their\,\overrightarrow{BC}$ oe
	$\frac{2}{3}\mathbf{a}+\frac{1}{3}\mathbf{b}$	**A1**	

Question	Answer	Marks	Guidance
5(c)	$2p+2q=-5p+5$ or $p+4q=5p+5q$	**M1**	For equating like vectors to obtain at least one equation
	$p=-5,\ q=20$	**2**	**Dep M** mark for attempt to solve *their* equations to obtain both p and q **A1** for both
6	1144	**3**	**B1** With the brothers: 220 or ${}^{12}C_3$ **B1** Without the brothers: 924 or ${}^{12}C_6$
7(a)	2.8 oe	**B1**	
7(b)	$(BC=AC=)\ 10\tan 1.4$ or $\dfrac{10\sin 1.4}{\sin 0.1708}$	**M1**	
	Perimeter = $10(\textit{their}\ 2.8)+2(\textit{their}\ AC \text{ or } BC)$	**M1**	
	144	**A1**	
7(c)	Area of triangle AOC or BOC = $\dfrac{1}{2}\textit{their}(AC \text{ or } BC)\times 10$ or $\dfrac{1}{2}\textit{their}\ OC\times 10\sin 1.4$ soi	**M1**	Allow premature approximation for OC
	Area of minor sector AOB = 140	**B1**	**FT** on 50 × *their* 2.8
	Shaded area = 439 to 440	**A1**	Must have $579 \leqslant$ kite area $\leqslant 580$
8(a)	–1.5	**B1**	
8(b)	$f \in \mathbb{R}$	**B1**	Allow $y \in \mathbb{R}$, $\mathbb{R}$, $-\infty < f(x) < \infty$ oe, $f(x) \in \mathbb{R}$

Question	Answer	Marks	Guidance
8(c)	$\ln(8x+12)$ or $\ln(4(2x+3))$	**B1**	May be implied
	$f^{-1}(x)=\frac{e^x-12}{8}$ oe	**2**	**M1** for attempt to find the inverse, allow one sign error **A1** allow y = …
	Range: $f^{-1} > their(-1.5)$	**B1**	Must be correct notation, follow through on *their* **(a)** $f^{-1}(x) > their(-1.5)$, $y > their(-1.5)$
	Alternative		
	$f^{-1}(x)=\frac{e^{x-\ln 4}-3}{2}$ oe	**(3)**	**B1** for $e^{x-\ln 4}$ or $e^{y-\ln 4}$ **M1** for attempt to find the inverse, allow one sign error **A1** allow y = …
	Range: $f^{-1} > their(-1.5)$	**(B1)**	Must be correct notation, follow through on *their* **(a)** $f^{-1}(x) > their(-1.5)$, $y > their(-1.5)$
8(d)		**4**	**B1** for correct shape of $f(x)$ in quadrants 1, 2 and 3, with asymptotic behaviour **B1** for $\ln 12$ and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape. **B1** for correct shape of $f^{-1}(x)$ in quadrants 1, 3 and 4, with asymptotic behaviour **B1** for $\ln 12$ and $-\frac{11}{8}$ or -1.375 in correct position, must have a correct shape and intersect at least once with $y=f(x)$
9(a)	$\frac{(4x-1)(2x+1)-(4x-1)+4(2x+1)^2}{(2x+1)^2(4x-1)}$	**M1**	For attempt to obtain a single fraction An extra term of $(2x+1)$ throughout must be dealt with correctly before awarding **M1**
	$\frac{24x^2+14x+4}{(2x+1)^2(4x-1)}$	**A1**	Must see sufficient detail of expansion and collecting terms cso as **AG**

Question	Answer	Marks	Guidance
9(b)	$\frac{1}{2}\ln(2x+1)$	**B1**	
	$\frac{1}{2(2x+1)}$	**B1**	Allow $\frac{-(2x+1)^{-1}}{-1\times 2}$ oe
	$\ln(4x-1)$	**B1**	
	$\left(\frac{1}{2}\ln 3+\frac{1}{6}+\ln 3\right)-\left(\frac{1}{2}\ln 2+\frac{1}{4}\right)$	**M1**	For correct application of limits, must have at least one log term. Must be using individual fractions from **(a)** Fractions and log terms must be bracketed correctly and manipulated correctly
	$\frac{1}{2}\ln\frac{27}{2}-\frac{1}{12}$	**3**	**M1** for application of log laws using $\frac{1}{2}\ln 3+\ln 3-\frac{1}{2}\ln 2$ to obtain the correct form **A1** for $\frac{1}{2}\ln\frac{27}{2}$ **B1** for $-\frac{1}{12}$
10(a)	Common difference = $4\lg x$	**B1**	
	Sum to n terms = $\frac{n}{2}\left(2\lg x+(n-1)(4\lg x)\right)$	**M1**	For use of the sum formula with *their* common difference
	$n(2n-1)\lg x$	**2**	**Dep M1** for a correct attempt to rearrange to the required form **A1** cao
10(b)	$n(2n-1)=4950$	**M1**	For $n((\textit{their } p)n-1)=4950$ together with an attempt to solve to obtain n
	50	**A1**	cao
10(c)	$50(99)\lg x=-14850$	**M1**	For use of *their* n and p in a complete method to find x or use of part **(b)**
	10^{-3} or equivalent	**A1**	

Question	Answer	Marks	Guidance
11(a)	$\dfrac{\left((t+1)\times\dfrac{3}{2}\times 2\times(2t+1)^{\frac{1}{2}}\right)-(2t+1)^{\frac{3}{2}}}{(t+1)^2}$	**3**	**B1** for $\dfrac{3}{2}\times 2\times(2t+1)^{\frac{1}{2}}$ **M1** for a correct attempt at a quotient or a product **A1** for all terms apart from $\dfrac{3}{2}\times 2\times(2t+1)^{\frac{1}{2}}$ correct
	$\dfrac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(t+2)$	**2**	**M1 dep** on previous M mark for attempt to obtain in the required form
	Alternative		
	$s=\dfrac{(2t+1)^{\frac{3}{2}}-t-1}{(t+1)}$ $\dfrac{\left((t+1)\times\left(3\times(2t+1)^{\frac{1}{2}}-1\right)\right)-\left((2t+1)^{\frac{3}{2}}-t-1\right)}{(t+1)^2}$	**(3)**	**B1** for $\dfrac{3}{2}\times 2\times(2t+1)^{\frac{1}{2}}$ **M1** for a correct attempt at a quotient or a product **A1** for all terms apart from $\dfrac{3}{2}\times 2\times(2t+1)^{\frac{1}{2}}$ correct
	$\dfrac{(2t+1)^{\frac{1}{2}}}{(t+1)^2}(t+2)$	**(2)**	**M1 dep** on previous M mark for attempt to obtain in the required form
11(b)	$(2t+1)^{\frac{1}{2}}(t+2)=0$ oe has no real positive solutions so velocity is never zero	**B1**	**FT** on *their* positive linear factor Reference needs to be made to both factors.

Question	Answer	Marks	Guidance
12	$a^5x^5 + 2a^4x^4 + \frac{8}{5}a^3x^3$	**3**	**B1** for each correct term, allow when first seen
	$1 - \frac{2b}{x} + \frac{b^2}{x^2}$	**B1**	
	$a = 2$	**B1**	
	$32 - 64b = -160$	**M1**	For using *their* expansions and *their* value for *a* to obtain two terms involving x^4
	$b = 3$	**A1**	
	$\frac{64}{5} - 192 + 288 = c$	**M1**	For using *their* expansions and *their* value for *a* to obtain three terms involving x^3
	$c = \frac{544}{5}$ oe	**A1**	
	Alternative		
	$a^5 (= 32)$	**(B1)**	
	$a = 2$	**(B1)**	
	$-2ba^5 + 2a^4 = -160$ soi $32 - 64b = -160$	**(2)**	**B1** for $2a^4$ soi **M1** For using *their* expansions and *their* value for *a* to obtain two terms involving x^4
	$b = 3$	**(A1)**	
	$\frac{8}{5}a^3 - 4a^4b + a^5b^2 = c$ $\frac{64}{5} - 192 + 288 = c$	**(3)**	**B2** for both $\frac{8}{5}a^3$ and a^5b^2 **B1** for either $\frac{8}{5}a^3$ or a^5b^2 if only one correct **M1** for using *their* expansions and *their* value for *a* to obtain three terms involving x^3
	$c = \frac{544}{5}$ oe	**(A1)**	