Cambridge Assessment International Education

# Cambridge O Level

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| CANDIDATE NAME | | | | | | | | | |
| CENTRE NUMBER | | | | | | CANDIDATE NUMBER | | | | |

*3243006222*

**ADDITIONAL MATHEMATICS** **4037/22**

Paper 2 **October/November 2021**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

**INFORMATION**

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.




**[Turn over**

***Mathematical Formulae***

## 1. ALGEBRA

*Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n$$

where $n$ is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series* $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series* $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

## 2. TRIGONOMETRY

*Identities*

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

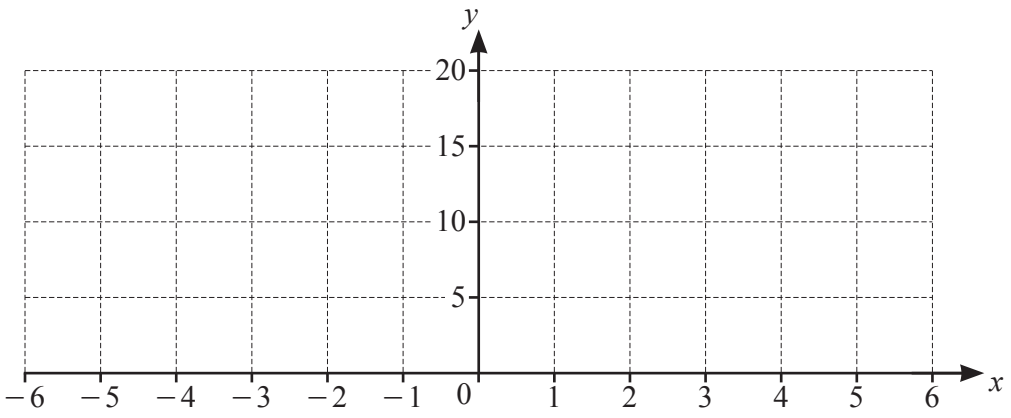
*Formulae for $\Delta ABC$*

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

**1**




**(a)** On the axes, draw the graphs of $y = 5 + |3x - 2|$ and $y = 11 - x$. [4]

**(b)** Using the graphs, or otherwise, solve the inequality $11 - x < 5 + |3x - 2|$. [2]

**2** **(a)** Expand $(2-3x)^4$, evaluating all of the coefficients. [4]

**(b)** The sum of the first three terms in ascending powers of $x$ in the expansion of $(2-3x)^4\left(1+\frac{a}{x}\right)$ is $\frac{32}{x}+b+cx$, where $a$, $b$ and $c$ are integers. Find the values of each of $a$, $b$ and $c$. [4]

**3** **(a)** Show that $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 2\cot x \operatorname{cosec} x$. [4]

**(b)** Hence solve the equation $\frac{1}{\sec x - 1} + \frac{1}{\sec x + 1} = 3\sec x$ for $0° < x < 360°$. [4]

**4** **(a)** Find the $x$-coordinates of the stationary points on the curve $y = 3\ln x + x^2 - 7x$, where $x > 0$. [5]

**(b)** Determine the nature of each of these stationary points. [3]

**5** **(a)** Solve the following simultaneous equations.

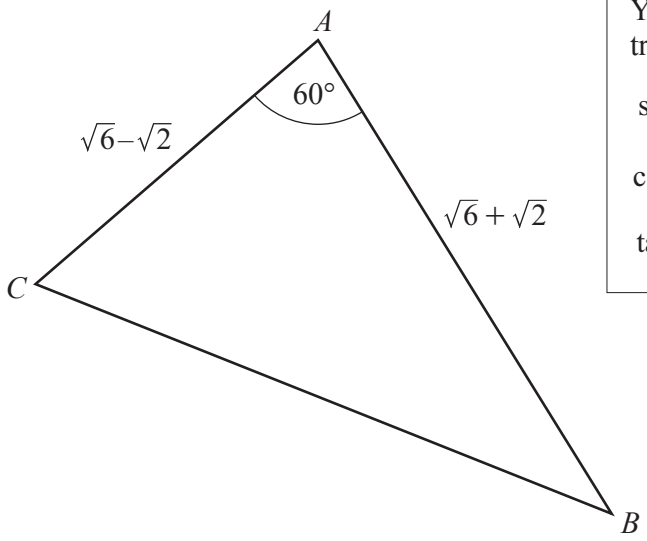
$$e^x + e^y = 5$$
$$2e^x - 3e^y = 8$$

[5]

**(b)** Solve the equation $e^{(2t-1)} = 5e^{(5t-3)}$. [4]

**6 DO NOT USE A CALCULATOR IN THIS QUESTION.**

All lengths in this question are in centimetres.




You may use the following trigonometrical ratios.

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

$\tan 60° = \sqrt{3}$

The diagram shows triangle $ABC$ with $AC = \sqrt{6} - \sqrt{2}$, $AB = \sqrt{6} + \sqrt{2}$ and angle $CAB = 60°$.

**(a)** Find the exact length of $BC$. [3]

**(b)** Show that $\sin ACB = \frac{\sqrt{6} + \sqrt{2}}{4}$. [2]

**(c)** Show that the perpendicular distance from $A$ to the line $BC$ is 1. [2]

**7** It is given that $\frac{d^2y}{dx^2} = e^{2x} + \frac{1}{(x+1)^2}$ for $x > -1$.

**(a)** Find an expression for $\frac{dy}{dx}$ given that $\frac{dy}{dx} = 2$ when $x = 0$. [3]

**(b)** Find an expression for $y$ given that $y = 4$ when $x = 0$. [3]

**8** Variables $x$ and $y$ are such that when $\sqrt{y}$ is plotted against $\log_2(x+1)$, where $x > -1$, a straight line is obtained which passes through $(2, 10.4)$ and $(4, 15.4)$.

**(a)** Find $\sqrt{y}$ in terms of $\log_2(x+1)$. [4]

**(b)** Find the value of $y$ when $x = 15$. [1]

**(c)** Find the value of $x$ when $y = 25$. [3]

**9** **(a)** Find the equation of the normal to the curve $y = x^3 + x^2 - 4x + 6$ at the point $(1, 4)$. [5]

**(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.**

Find the exact $x$-coordinate of each of the two points where the normal cuts the curve again. [5]

**10** **(a)** The first three terms of an arithmetic progression are $x$, $5x-4$ and $8x+2$. Find $x$ and the common difference. [4]

**(b)** The first three terms of a geometric progression are $y$, $5y-4$ and $8y+2$.

**(i)** Find the two possible values of $y$. [4]

**(ii)** For each of these values of $y$, find the corresponding value of the common ratio. [2]

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