

Cambridge O Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

ADDITIONAL MATHEMATICS

4037/23

Paper 2 October/November 2020

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has 16 pages. Blank pages are indicated.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where *n* is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\left\{2a + (n-1)d\right\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \ (r \neq 1)$$

$$S_{\infty} = \frac{a}{1-r} \ (|r| < 1)$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$
$$\sec^2 A = 1 + \tan^2 A$$
$$\csc^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$\Delta = \frac{1}{2}bc \sin A$$

1 Solve
$$|3x-2| = 4+x$$
. [3]

2 Solve the simultaneous equations.

$$x^2 + 3xy = 4$$
$$2x + 5y = 4$$
 [5]

3 Find the values of k for which the equation $x^2 + (k+9)x + 9 = 0$ has two distinct real roots. [4]

4 It is given that $y = \ln(1 + \sin x)$ for $0 < x < \pi$.

(a) Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
. [2]

(b) Find the value of $\frac{dy}{dx}$ when $x = \frac{\pi}{6}$, giving your answer in the form $\frac{1}{\sqrt{a}}$, where a is an integer. [2]

(c) Find the values of x for which $\frac{dy}{dx} = \tan x$. [5]

5 Solve the following simultaneous equations.

$$3^x \times 9^{y-1} = 243$$

$$8 \times 2^{y - \frac{1}{2}} = \frac{2^{2x + 1}}{4\sqrt{2}} \tag{5}$$

6	A 4-how	A 4-digit code is to be formed using 4 different numbers selected from 1, 2, 3, 4, 5, 6, 7, 8 and 9. Find how many different codes can be formed if									
	(a)	there are no restrictions,	[1]								
	(b)	only prime numbers are used,	[1]								
	(c)	two even numbers are followed by two odd numbers,	[2]								
	(d)	the code forms an even number.	[2]								

7 A curve has equation $y = x \cos x$.

(a) Find
$$\frac{\mathrm{d}y}{\mathrm{d}x}$$
. [2]

(b) Find the equation of the normal to the curve at the point where $x = \pi$, giving your answer in the form y = mx + c. [4]

(c) Using your answer to part (a), find the exact value of $\int_0^{\frac{\pi}{6}} x \sin x \, dx$. [5]

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

$$\log_2(y+1) = 3 - 2\log_2 x$$
$$\log_2(x+2) = 2 + \log_2 y$$

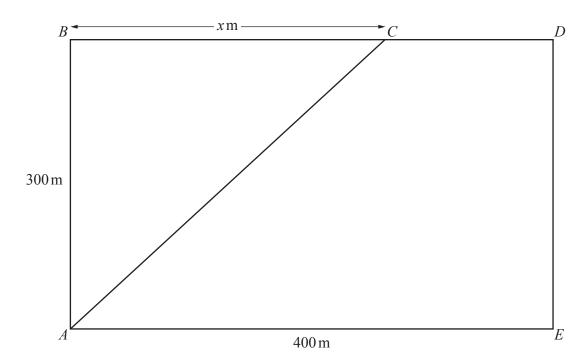
(a) Show that
$$x^3 + 6x^2 - 32 = 0$$
. [4]

[4]

(b) Find the roots of $x^3 + 6x^2 - 32 = 0$.

(c) Give a reason why only one root is a valid solution of the logarithmic equations. Find the value of *y* corresponding to this root. [2]

9



The rectangle ABCDE represents a ploughed field where $AB = 300 \,\mathrm{m}$ and $AE = 400 \,\mathrm{m}$. Joseph needs to walk from A to D in the least possible time. He can walk at $0.9 \,\mathrm{ms}^{-1}$ on the ploughed field and at $1.5 \,\mathrm{ms}^{-1}$ on any part of the path BCD along the edge of the field. He walks from A to C and then from C to D. The distance $BC = x \,\mathrm{m}$.

(a) Find, in terms of x, the total time, Ts, Joseph takes for the journey. [3]

(b) Given that *x* can vary, find the value of *x* for which *T* is a minimum and hence find the minimum value of *T*. [6]

10 (a) The sum of the first 4 terms of an arithmetic progression is 38 and the sum of the next 4 terms is 86. Find the first term and the common difference. [5]

(b) The third term of a geometric progression is 12 and the sixth term is −96. Find the sum of the first 10 terms of this progression. [6]

11 DO NOT USE A CALCULATOR IN THIS QUESTION.

Solve the quadratic equation $(\sqrt{7}-2)x^2-4x+(\sqrt{7}+2)=0$, giving each of your answers in the form $a+b\sqrt{7}$, where a and b are constants.

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