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ADDITIONAL MATHEMATICS

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Paper 2

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MARK SCHEME

Maximum Mark: 80

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MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.


Types of mark

- M Method marks, awarded for a valid method applied to the problem.
- A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.
- B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation ‘**dep**’ is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

| | |
|------|----------------------------|
| awrt | answers which round to |
| cao | correct answer only |
| dep | dependent |
| FT | follow through after error |
| isw | ignore subsequent working |
| nfww | not from wrong working |
| oe | or equivalent |
| rot | rounded or truncated |
| SC | Special Case |
| soi | seen or implied |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|---|
| 1(a) |  | B2 | B1 for each |
| 1(b) | $n(P') = 18$ | B1 | |
| | $n((Q \cup R) \cap P) = 11$ | B1 | |
| | $n(Q' \cup P) = 29$ | B1 | |
| 2 | $3x - 1 = 5 + x \quad x = 3$ | B1 | |
| | $3x - 1 = -5 - x$ oe | M1 | M1 not earned if incorrect equation(s) present |
| | $x = -1$ | A1 | |
| 3 | $\frac{p(\sqrt{3}+1) + (\sqrt{3}-1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = q + 3\sqrt{3}$ | M1 | on LHS take common denominator or rationalise each term or multiply throughout |
| | $p(\sqrt{3}+1) + (\sqrt{3}-1) = 2q + 6\sqrt{3}$ oe | A1 | correct eqn with no surds in denominators of LHS |
| | equate surd/non surd parts | M1 | equate and solve for p or q ($\neq 0$) |
| | $p = 5$ and $q = 2$ | A1 | |
| 4 | $\log_3 3 = 1$ or $\log_3 9 = 2$ | B1 | implied by one correct equation |
| | $x + 1 = 3y$ | B1 | |
| | $x - y = 9$ | B1 | |
| | solve correct equations for x or y | M1 | |
| | $x = 14$ and $y = 5$ | A1 | |
| 5(i) | $\overrightarrow{OX} = \lambda(1.5\mathbf{b} + 3\mathbf{a})$ | B1 | |
| 5(ii) | $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ or $\overrightarrow{BA} = \mathbf{a} - \mathbf{b}$ | B1 | |
| | $\overrightarrow{OX} = \mathbf{a} + \mu(\mathbf{b} - \mathbf{a})$ | B1 | |
| 5(iii) | $1.5\lambda = \mu$ or $3\lambda = 1 - \mu$ | M1 | $\overrightarrow{OX} = \overrightarrow{OX}$ and equate for \mathbf{a} or \mathbf{b} |
| | $\mu = \frac{1}{3} \quad \lambda = \frac{2}{9}$ | A2 | A1 for each |

| Question | Answer | Marks | Guidance |
|----------|---|-------------|---|
| 5(iv) | $\frac{AX}{XB} = \frac{1}{2}$ | B1 | Accept 1 : 2 but not $\frac{1}{2} : 1$ |
| 5(v) | $\frac{OX}{XD} = \frac{2}{7}$ | B1 | Accept 2 : 7 but not $\frac{2}{7} : 1$ |
| 6(i) | $f^2 = f(f)$ used algebraic $([(x+2)^2 + 1] + 2)^2 + 1$ | M1 | numerical or algebraic |
| | 17 | A1 | |
| 6(ii) | $x = \frac{y-2}{2y-1}$ | M1 | change x and y |
| | $2xy - x = y - 2 \rightarrow y(2x-1) = x-2$ | M1 | M1dep multiply, collect y terms, factorise |
| | $y = \frac{x-2}{2x-1} \quad [=g(x)]$ | A1 | correct completion |
| 6(iii) | $gf(x) = \frac{[(x+2)^2 + 1] - 2}{2[(x+2)^2 + 1] - 1}$ oe | B1 | |
| | $\frac{(x+2)^2 - 1}{2(x+2)^2 + 1} = \frac{8}{19}$ $3(x+2)^2 = 27$ oe $3x^2 + 12x - 15 = 0$ | M1 | <i>their</i> $gf = \frac{8}{19}$ and simplify to quadratic equation |
| | solve quadratic | M1 | M1dep Must be of equivalent form |
| | $x=1 \quad x=-5$ | A1 | |
| 7(i) | $v=0 \rightarrow \cos 2t = \frac{1}{3}$ | M1 | set $v=0$ and solve for $\cos 2t$ |
| | $\rightarrow t = 0.615$ or 0.616 | A1 | |
| 7(ii) | $s = \frac{3}{2} \sin 2t - t \quad (+c)$ | M1A1 | M1 for $\sin 2t$ and $\pm t$ |
| | $t = \frac{\pi}{4} \rightarrow s = 1.5 - \frac{\pi}{4} \quad (= 0.715)$ | A1 | |
| 7(iii) | $a = -6 \sin 2t$ | M1A1 | M1 for $-\sin 2t$ |
| | $t = 0.615 \rightarrow a = -5.66$ or -5.65 or $-2\sqrt{8}$ | A1 | condone substitution of degrees |

| Question | Answer | Marks | Guidance |
|----------|--|-----------|---|
| 8(i) | $\cos \alpha = \frac{1}{3}$ oe | M1 | |
| | $\alpha = 70.5^\circ$ | A1 | |
| 8(ii) | speed = $\sqrt{3^2 - 1^2}$ | M1 | Pythagoras/trig ratio/cosine rule |
| | $\sqrt{8}$ or $2\sqrt{2}$ or 2.83 m s^{-1} | A1 | |
| 8(iii) | time = $\frac{50}{\text{their} \sqrt{8}}$ | M1 | |
| | $\frac{25\sqrt{2}}{2}$ or 17.7s | A1 | |
| 8(iv) | <i>their</i> 8(iii) seen | B1 | |
| | $BC = 10\sqrt{2}$ or 14.1 m or 14.2 m | B1 | |
| 9(i) | $\frac{d}{dx}(\ln x) = \frac{1}{x}$ and $\frac{d}{dx}x^3 = 3x^2$ or $\frac{d}{dx}x^{-3} = -3x^{-4}$ | B1 | seen |
| | Substitution of <i>their</i> derivatives into quotient rule | M1 | |
| | $\frac{d}{dx}\left(\frac{\ln x}{x^3}\right) = \frac{x^3 \times \frac{1}{x} - 3x^2 \ln x}{x^6}$ oe | A1 | correct completion |
| 9(ii) | $\frac{dy}{dx} = 0 \rightarrow 1 - 3\ln x = 0$ $\ln x = \frac{1}{3}$ | M1 | equate given $\frac{dy}{dx}$ to zero and solve for $\ln x$ or x |
| | $x = e^{\frac{1}{3}}$ | A1 | seen |
| | $y = \frac{1}{3e}$ | A1 | seen |
| 9(iii) | $\frac{\ln x}{x^3} = \int \frac{1 - 3\ln x}{x^4} dx$ oe | M1 | use given statement in (i) |
| | $\int \frac{1}{x^4} dx = \frac{-1}{3x^3}$ | B1 | seen anywhere |
| | $\int \frac{\ln x}{x^4} dx = -\frac{1}{9x^3} - \frac{\ln x}{3x^3}$ (+C) oe | A2 | A1 for each term |

| Question | Answer | Marks | Guidance |
|-----------|--|-----------|--|
| 10(a) | $\text{LHS} = \frac{\sin^2 x + (1 + \cos x)^2}{\sin x(1 + \cos x)}$ | B1 | correct addition of fractions |
| | $= \frac{1 + 2\cos x + 1}{\sin x(1 + \cos x)}$ | B1 | expansion and use of identity |
| | $= \frac{2(1 + \cos x)}{\sin x(1 + \cos x)} = 2\text{cosec} x$ | B1 | factorisation and completion |
| 10(b)(i) | $\text{cosec}^2 y - 1 + \text{cosec} y - 5 = 0$ $\text{cosec}^2 y + \text{cosec} y - 6 = 0$ | M1 | use of identity for $\cot^2 y$ to obtain quadratic in cosec y |
| | $(\text{cosec} y - 2)(\text{cosec} y + 3) = 0$ | M1 | solve 3 term quadratic for cosec y |
| | $\sin y = \frac{1}{2}, \sin y = -\frac{1}{3}$ | M1 | obtain values for sin y |
| | $y = 30^\circ, 150^\circ, 199.5^\circ, 340.5^\circ$ | A2 | A1 for 2 values |
| 10(b)(ii) | $2z + \frac{\pi}{4} = \frac{5\pi}{6} \text{ or } \frac{7\pi}{6} \quad (2.6\dots, 3.6\dots)$ | M2 | M1 equate to $\frac{5\pi}{6}$ M1 equate to $\frac{7\pi}{6}$ |
| | $z = \frac{7\pi}{24} \text{ or } \frac{11\pi}{24} \quad (0.916, 1.44)$ | A2 | A1 for 1 value |
| 11(i) | Other root = 4 | B1 | |
| | $f(x) = (x-3)(x-3)(x-4)$ $= x^3 - 10x^2 + 33x - 36$ | M1 | multiply out $(x-3)(x-3)(x \pm p)$ |
| | $a = -10 \quad b = 33$ | A2 | A1 for each Can be implied by correct cubic |
| 11(ii) | $x = 6, x = 6, x = 1$ $x = 2, x = 2, x = 9$ $x = 1, x = 1, x = 36$ | B4 | B1 for each of first two sets B2 for third set |