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4037/11

May/June 2022

2 hours

You must answer on the question paper.

No additional materials are needed.

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

- 1 Find constants a , b and c such that $\frac{\sqrt{p}q^{\frac{2}{3}}r^{-3}}{(pq^{-1})^2r^{-1}} = p^a q^b r^c$. [3]

- 2 A particle moves in a straight line such that its displacement, s metres, from a fixed point, at time t seconds, $t \geq 0$, is given by $s = (1 + 3t)^{-\frac{1}{2}}$.

(a) Find the exact speed of the particle when $t = 1$. [3]

(b) Show that the acceleration of the particle will never be zero. [2]

3 A function f is such that $f(x) = \ln(2x+1)$, for $x > -\frac{1}{2}$.

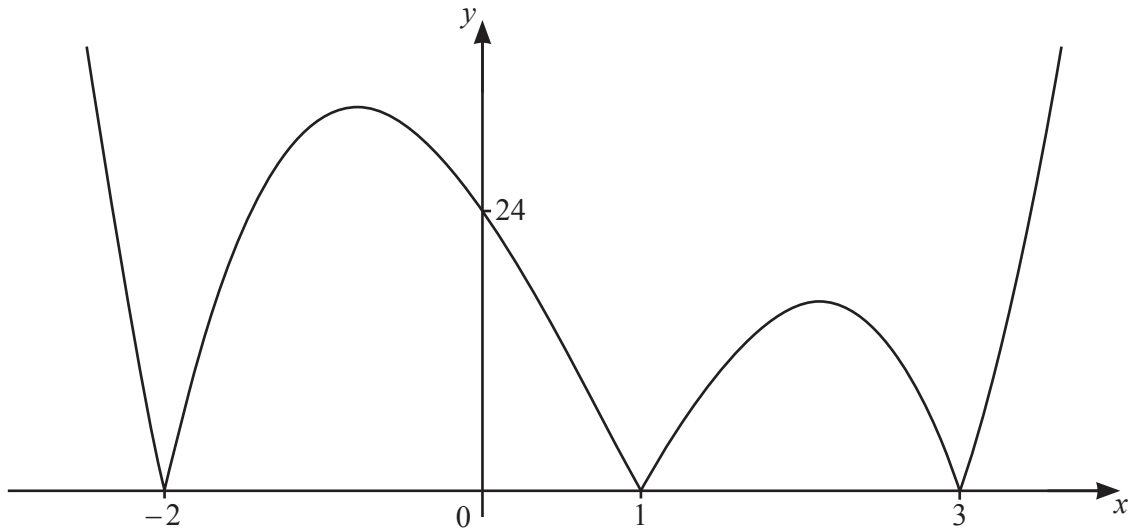
(a) Write down the range of f . [1]

A function g is such that $g(x) = 5x-7$, for $x \in \mathbb{R}$.

(b) Find the exact solution of the equation $gf(x) = 13$. [3]

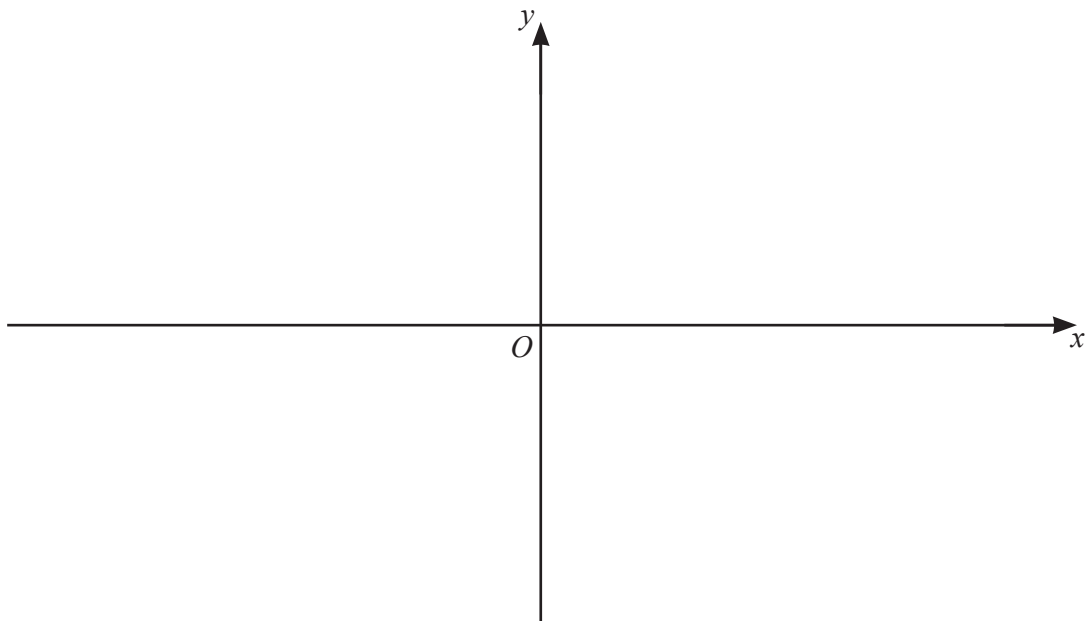
(c) Find the solution of the equation $f'(x) = g^{-1}(x)$. [6]

4 (a)



The diagram shows the graph of $y = |f(x)|$, where $f(x)$ is a cubic. Find the possible expressions for $f(x)$. [3]

- (b) (i) On the axes below, sketch the graph of $y = |2x + 1|$ and the graph of $y = |4(x - 1)|$, stating the coordinates of the points where the graphs meet the coordinate axes. [3]



- (ii) Find the exact solutions of the equation $|2x + 1| = |4(x - 1)|$. [4]

- 5 (a) Find the vector which is in the opposite direction to $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ and has a magnitude of 8.5. [2]

- (b) Find the values of a and b such that $5\begin{pmatrix} 3a \\ b \end{pmatrix} + \begin{pmatrix} 2a+1 \\ 2 \end{pmatrix} = 6\begin{pmatrix} b+a \\ 2 \end{pmatrix}$. [3]

- 6 (a) Write down the values of k for which the line $y = k$ is a tangent to the curve $y = 4 \sin\left(x + \frac{\pi}{4}\right) + 10$. [2]

(b) (i) Show that $\frac{1 + \tan \theta}{1 - \cos \theta} + \frac{1 - \tan \theta}{1 + \cos \theta} = \frac{2(1 + \sin \theta)}{\sin^2 \theta}$. [4]

(ii) Hence solve the equation $\frac{1 + \tan \theta}{1 - \cos \theta} + \frac{1 - \tan \theta}{1 + \cos \theta} = 3$, for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 7 (a) The first three terms of an arithmetic progression are $\lg 3$, $3\lg 3$, $5\lg 3$. Given that the sum to n terms of this progression can be written as $256 \lg 81$, find the value of n . [5]

(b) DO NOT USE A CALCULATOR IN THIS PART OF THE QUESTION.

The first three terms of a geometric progression are $\ln 256$, $\ln 16$, $\ln 4$. Find the sum to infinity of this progression, giving your answer in the form $p \ln 2$. [4]

8 DO NOT USE A CALCULATOR IN THIS QUESTION.

- (a) Find the exact coordinates of the points of intersection of the curve $y = x^2 + 2\sqrt{5}x - 20$ and the line $y = 3\sqrt{5}x + 10$. [4]

- (b) It is given that $\tan \theta = \frac{\sqrt{3}-1}{2+\sqrt{3}}$, for $0 < \theta < \frac{\pi}{2}$. Find $\operatorname{cosec}^2 \theta$ in the form $a + b\sqrt{3}$, where a and b are constants. [5]

- 9 A circle, centre O and radius r cm, has a sector OAB of fixed area 10 cm^2 . Angle AOB is θ radians and the perimeter of the sector is P cm.

(a) Find an expression for P in terms of r . [3]

(b) Find the value of r for which P has a stationary value. [3]

(c) Determine the nature of this stationary value. [2]

(d) Find the value of θ at this stationary value. [1]

- 10** The normal to the curve $y = \tan\left(3x + \frac{\pi}{2}\right)$ at the point P with coordinates $(p, -1)$, where $0 < p \leq \frac{\pi}{6}$, meets the x -axis at the point A and the y -axis at the point B . Find the exact coordinates of the mid-point of AB . [10]

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