



# Cambridge O Level

CANDIDATE  
NAME

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**ADDITIONAL MATHEMATICS**

**4037/22**

Paper 2

**May/June 2021**

**2 hours**

You must answer on the question paper.

No additional materials are needed.

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages.

**Mathematical Formulae****1. ALGEBRA***Quadratic Equation*

For the equation  $ax^2 + bx + c = 0$ ,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

*Binomial Theorem*

$$(a + b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where  $n$  is a positive integer and  $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

*Arithmetic series*      $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

*Geometric series*      $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

**2. TRIGONOMETRY***Identities*

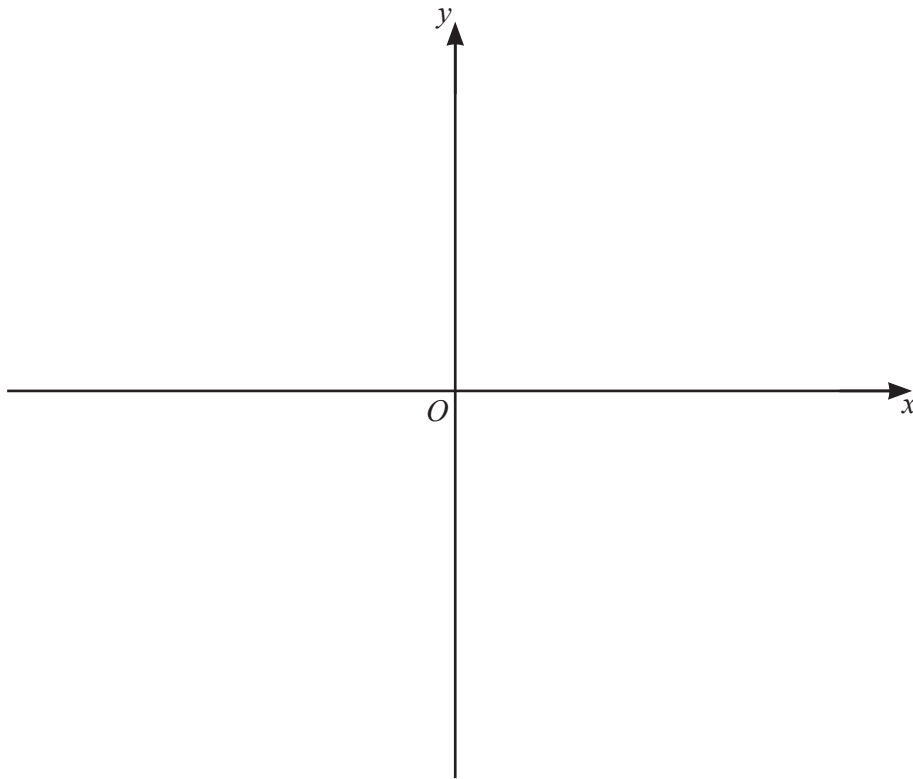
$$\begin{aligned} \sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A \end{aligned}$$

*Formulae for  $\triangle ABC$* 

$$\begin{aligned} \frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A \end{aligned}$$

1 Using the binomial theorem, expand  $(1 + e^{2x})^4$ , simplifying each term. [2]

2 On the axes, sketch the graph of  $y = 3(x - 3)(x - 1)(x + 2)$  stating the intercepts with the coordinate axes. [3]



- 3 Find the values of the constant  $k$  for which  $(2k-1)x^2 + 6x + k + 1 = 0$  has real roots. [5]

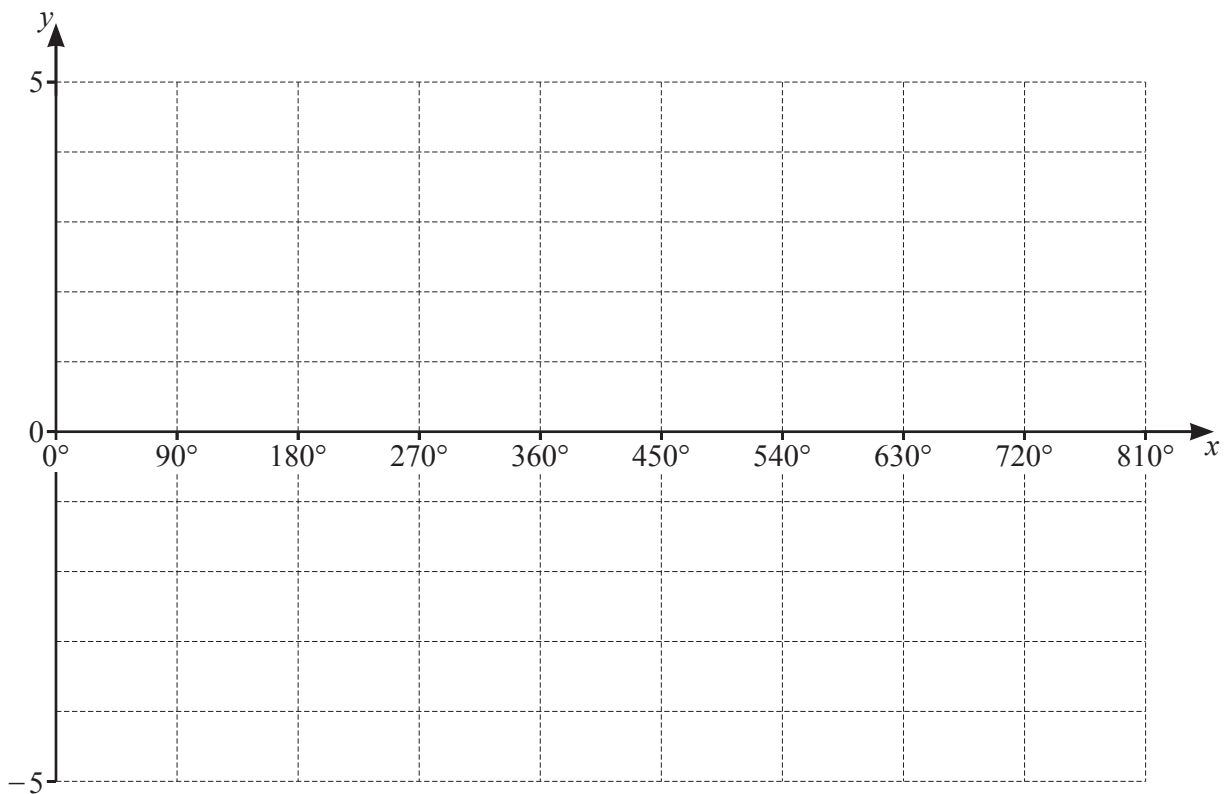
- 4 The polynomial  $p(x) = mx^3 - 29x^2 + 39x + n$ , where  $m$  and  $n$  are constants, has a factor  $3x - 1$ , and remainder 6 when divided by  $x - 1$ . Show that  $x - 2$  is a factor of  $p(x)$ . [6]

5 The function  $f$  is defined, for  $0^\circ \leq x \leq 810^\circ$ , by  $f(x) = -2 + \cos \frac{2x}{3}$ .

(a) Write down the amplitude of  $f$ . [1]

(b) Find the period of  $f$ . [2]

(c) On the axes, sketch the graph of  $y = f(x)$ . [2]



6 The points  $A(5, -4)$  and  $C(11, 6)$  are such that  $AC$  is the diagonal of a square,  $ABCD$ .

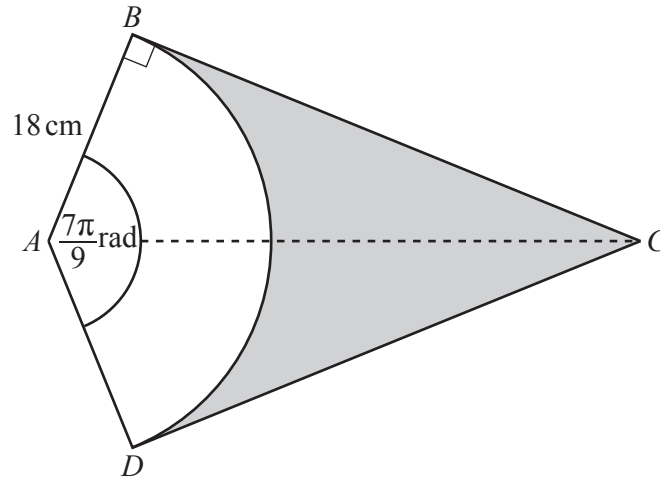
(a) Find the length of the line  $AC$ . [2]

(b) (i) The coordinates of the centre,  $E$ , of the square are  $(8, y)$ . Find the value of  $y$ . [1]

(ii) Find the equation of the diagonal  $BD$ . [3]

(iii) Given that the  $x$ -coordinate of  $B$  is less than the  $x$ -coordinate of  $D$ , write  $\overrightarrow{EB}$  and  $\overrightarrow{ED}$  as column vectors. [2]

7



$DAB$  is a sector of a circle, centre  $A$ , radius  $18\text{ cm}$ . The lines  $CB$  and  $CD$  are tangents to the circle. Angle  $DAB$  is  $\frac{7\pi}{9}$  radians.

(a) Find the perimeter of the shaded region.

[3]

(b) Find the area of the shaded region.

[3]



8 A particle moves in a straight line so that,  $t$  seconds after passing through a fixed point  $O$ , its velocity,  $v \text{ ms}^{-1}$ , is given by  $v = 3t^2 - 30t + 72$ .

(a) Find the distance between the particle's two positions of instantaneous rest. [6]

(b) Find the acceleration of the particle when  $t = 2$ . [2]

9 Solve the following simultaneous equations.

$$4x^2 + 3xy + y^2 = 8$$

$$xy + 4 = 0$$

[6]

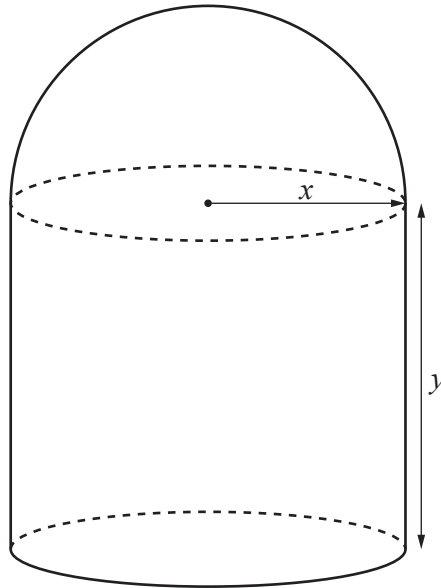
10 (a) Find  $\int (e^{x+1})^3 dx$ . [2]

(b) (i) Differentiate, with respect to  $x$ ,  $y = x \sin 4x$ . [2]

(ii) Hence show that  $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x \cos 4x dx = \frac{1}{8} - \frac{\pi\sqrt{3}}{6}$ . [4]

11 In this question all lengths are in centimetres.

The volume and surface area of a sphere of radius  $r$  are  $\frac{4}{3}\pi r^3$  and  $4\pi r^2$  respectively.



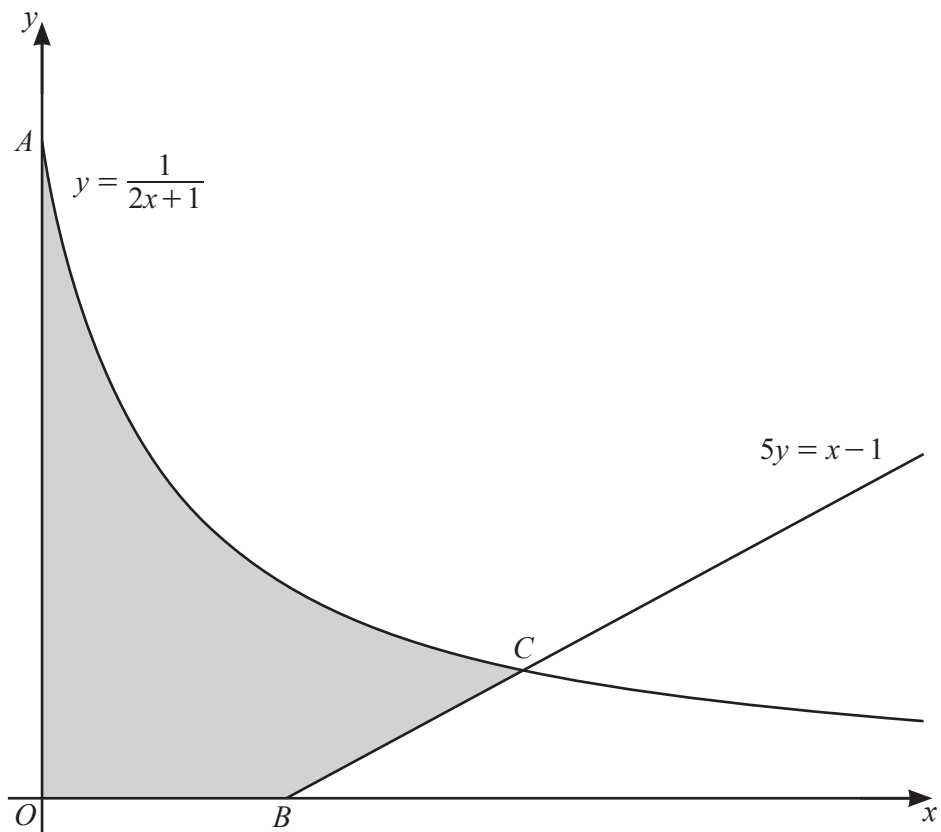
The diagram shows a solid object made from a hemisphere of radius  $x$  and a cylinder of radius  $x$  and height  $y$ . The volume of the object is  $500\text{ cm}^3$ .

(a) Find an expression for  $y$  in terms of  $x$  and show that the surface area,  $S$ , of the object is given by

$$S = \frac{5}{3}\pi x^2 + \frac{1000}{x}. \quad [4]$$

- (b) Given that  $x$  can vary and that  $S$  has a minimum value, find the value of  $x$  for which  $S$  is a minimum. [4]

## 12 DO NOT USE A CALCULATOR IN THIS QUESTION.



The diagram shows part of the curve  $y = \frac{1}{2x+1}$  and part of the line  $5y = x - 1$ .

The curve meets the  $y$ -axis at point  $A$ . The line meets the  $x$ -axis at point  $B$ . The line and curve intersect at point  $C$ .

(a) (i) Find the coordinates of  $A$  and  $B$ . [1]

(ii) Verify that the  $x$ -coordinate of  $C$  is 2. [2]

(b) Find the exact area of the shaded region.

[5]

**Question 13 is printed on the next page.**

13 The functions  $f$  and  $g$  are defined, for  $x > 0$ , by

$$f(x) = \frac{2x^2 - 1}{3x},$$

$$g(x) = \frac{1}{x}.$$

(a) Find and simplify an expression for  $fg(x)$ . [2]

(b) (i) Given that  $f^{-1}$  exists, write down the range of  $f^{-1}$ . [1]

(ii) Show that  $f^{-1}(x) = \frac{px + \sqrt{qx^2 + r}}{4}$ , where  $p$ ,  $q$  and  $r$  are integers. [4]

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