Cambridge O Level

Cambridge Assessment International Education
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/22**

Paper 2 **May/June 2019**

MARK SCHEME

Maximum Mark: 80

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

MARK SCHEME NOTES

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method marks, awarded for a valid method applied to the problem.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. For accuracy marks to be given, the associated Method mark must be earned or implied.

B Mark for a correct result or statement independent of Method marks.

When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. The notation 'dep' is used to indicate that a particular M or B mark is dependent on an earlier mark in the scheme.

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
nfww	not from wrong working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied

Question	Answer	Marks	Partial Marks
1	**Either:** Quotient rule: $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}(\ln x^2) = \frac{2}{x}$ oe soi	**B1**	
	$\frac{(\ln x^2)(\textit{their} \cos x) - (\sin x)\left(\textit{their}\ \frac{2}{x}\right)}{(\ln x^2)^2}$ oe	**M1**	
	$\frac{(\ln x^2)\cos x - (\sin x)\left(\frac{2}{x}\right)}{(\ln x^2)^2}$ oe isw	**A1**	
	Or: Product rule on $y = (\sin x)(\ln x^2)^{-1}$ $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ soi	**B1**	
	$\frac{\mathrm{d}}{\mathrm{d}x}\left((\ln x^2)^{-1}\right) = -(\ln x^2)^{-2} \times \frac{2}{x}$	**B1**	
	$(\sin x) \times \textit{their}\left(-(\ln x^2)^{-2} \times \frac{2}{x}\right)$ $+ (\textit{their} \cos x)(\ln x^2)^{-1}$ oe	**M1**	
	$(\sin x) \times \left(-(\ln x^2)^{-2} \times \frac{2}{x}\right)$ $+ (\cos x)(\ln x^2)^{-1}$ oe isw	**A1**	
2	$k^2 - 4(k-1)(-k)$ oe	**B1**	
	$k(5k-4)$	**M1**	
	Correct critical values 0, 0.8 oe	**A1**	
	$k < 0, k > 0.8$ oe	**A1**	**FT** *their* critical values provided *their* $ak^2 + bk + c > 0$ has positive a and there are 2 values; mark final answer If **B1 M0** allow **SC1** for a final answer of $k > 0.8$ oe

Question	Answer	Marks	Partial Marks
3(i)	Uses x = 2 as a root: $a(2^3)-12(2^2)+5(2)+6=0$	**M1**	or $\begin{array}{c\|cccc} 2 & a & -12 & 5 & 6 \\ & \downarrow & 2a & -24+4a & -38+8a \\ \hline & a & -12+2a & -19+4a & -32+8a=0 \end{array}$
	Solves $8a-48+10+6=0$ to find a = 4	**A1**	or solves $-32+8a=0$ to find $a=4$ If **M0** then **SC1** for $4(2^3)-12(2^2)+5(2)+6=0$ or showing that the synthetic division with a = 4 results in a remainder of 0
3(ii)	$(x-2)(4x^2-4x-3)$	**B2**	**B1** for any two terms correct in quadratic factor
	Product of three correct linear factors: $(x-2)(2x-3)(2x+1)$	**B1**	
	$x=2,\ x=1.5,\ x=-0.5$ oe	**B1**	**dep** on all previous marks having been earned If **B2 B0** then award **SC1** for correct factorisation of correct quadratic factor leading to 3 correct roots
4	**Either:** $A=\dfrac{1}{4}\pi x^2$ oe, soi	**B1**	
	$\dfrac{dA}{dx}=\dfrac{2}{4}\pi x$ oe, soi	**B1**	
	$\dfrac{dx}{dt}=0.01$ soi	**B1**	
	$\dfrac{2}{4}\pi(6)\times 0.01$	**M1**	**FT** *their* $\dfrac{dA}{dx}$ when x = 6
	0.03π or exact equivalent	**A1**	mark final answer
	Or: $A=\pi r^2$ and $r=\dfrac{x}{2}$ soi	**B1**	
	$\dfrac{dA}{dr}=2\pi r$ oe soi	**B1**	
	$\dfrac{dr}{dt}=0.005$ oe soi	**B1**	
	$2\pi(3)\times 0.005$	**M1**	
	0.03π or exact equivalent	**A1**	

Question	Answer	Marks	Partial Marks
5(i)	$5(x-1.5)^2-10.25$ isw	**B3**	**B1** for each of p, q, r correct in correctly formatted expression; allow correct equivalent values If **B0** then **SC2** for $5(x-1.5)-10.25$ or **SC1** for correct values but other incorrect format
5(ii)	$\dfrac{\textit{their}\ -10.25}{5}$ is least value when $x=\textit{their}\,1.5$	**B2**	**STRICT FT** *their* part (i); **B1 STRICT FT** for each
6(a)	2×4 or 2 by 4	**B1**	
6(b)(i)	$(\mathbf{A}^{-1}=)\ \dfrac{1}{2}\begin{pmatrix}3 & 4\\ 1 & 2\end{pmatrix}$ isw	**B2**	**B1** for $\dfrac{1}{2}\times \textit{their}\begin{pmatrix} & \\ & \end{pmatrix}$ or for $k\times\begin{pmatrix}3 & 4\\ 1 & 2\end{pmatrix}$, where k is not 0 or 1
6(b)(ii)	$\mathbf{B}=\dfrac{1}{4}\begin{pmatrix}13 & 20\\ 5 & 8\end{pmatrix}$ oe isw	**B3**	**FT** *their* $\mathbf{A}^{-1}$ provided **B1** earned in **(b)(i)** **B1** for the strategy of using $\mathbf{A}^{-1}$: $\mathbf{B}=\mathbf{A}^{-1}\mathbf{A}^{-1}$ soi or $\mathbf{AB}=\mathbf{A}^{-1}$ soi or $\mathbf{BA}=\mathbf{A}^{-1}$ soi or $\mathbf{B}=(\mathbf{A}^{-1})^2$ and **B1** for two or three elements of **B** correct or correct FT
7(a)	$\lg(x^2-3)=\lg 1$ soi	**M1**	
	-2 and 2	**A1**	Implies **M1**
7(b)(i)	**Two separate terms in numerator:** $(\sin(2x+5))\ln a$ or $\log_a a^{\sin(2x+5)}=(\sin(2x+5))\log_a a$	**B1**	**Combines terms in numerator:** or $\ln\left(\dfrac{a^{\sin(2x+5)}}{a}\right)$
	$-\ln a$ or $\log_a a^{-1}=-\log_a a$	**B1**	or $(\sin(2x+5)-1)\ln a$ or $\dfrac{\ln\left(\dfrac{a^{\sin(2x+5)}}{a}\right)}{\ln a}=\log_a\left(a^{\sin(2x+5)-1}\right)$
	$\sin(2x+5)-1$	**B1**	**dep** all previous marks awarded;
7(b)(ii)	$-\dfrac{1}{2}\cos(2x+5)+(\ \textit{their}\ -1)\ x\ \ (+\ c)$	**B3**	**FT** *their* numerical k **B2** for $-\dfrac{1}{2}\cos(2x+5)$ seen or **B1** for $a\cos(2x+5)$, $a<0$ or for $\dfrac{1}{2}\cos(2x+5)$ or for $-\dfrac{1}{2}\cos 2x+5$ seen

Question	Answer	Marks	Partial Marks
8(a)	$-\frac{20}{8}a^3\left[x^3\right]$ and $-\frac{6}{32}a\left[x^5\right]$ oe soi or $\frac{20}{8}a^3\left[x^3\right]$ and $\frac{6}{32}a\left[x^5\right]$ oe soi	**B2**	**B1** for either $-\frac{20}{8}a^3\left[x^3\right]$ or $-\frac{6}{32}a\left[x^5\right]$ oe or for $-\frac{{}^6C_3}{8}a^3\left[x^3\right]$ **and** $-\frac{{}^6C_5}{32}a\left[x^5\right]$ oe, or for $\frac{{}^6C_3}{8}a^3\left[x^3\right]$ **and** $\frac{{}^6C_5}{32}a\left[x^5\right]$ oe or for $\frac{20}{8}a^3\left[x^3\right]$ **and** $ka\left[x^5\right]$ oe where $k > 0$ or for $ka^3\left[x^3\right]$ **and** $\frac{6}{32}a\left[x^5\right]$ oe where $k > 0$
	their $\frac{20}{8}a^3 = 120 \times$ *their* $\frac{6}{32}a$ oe soi	**M1**	
	$[a =] \pm 3$	**A1**	
8(b)(i)	$1 + 40x + 760x^2 + 9120x^3$	**B2**	**B1** for three out of the four terms correct If **B0** then **SC1** for $1, 40x, 760x^2, 9120x^3$ seen but not summed
8(b)(ii)	$1 + 40(-0.01) + 760(-0.01)^2 + 9120(-0.01)^3$ or 1 – 0.4 + 0.076 – 0.00912 oe leading to 0.66688 cao	**B2**	or **M1** for use of $x = -0.01$ oe in *their* expansion seen or implied by e.g. 0.66688 without working or 1 – 0.4 + 0.076 – 0.00912
9(a)	$6(1-\cos^2 x) - 13\cos x = 1$ oe	**B1**	
	Solves or factorises *their* 3-term quadratic	**M1**	
	70.5 and 289.5	**A2**	with no extras in range **A1** for either, ignoring extras in range
9(b)(i)	Numerator: Substitution of $\tan y = \frac{\sin y}{\cos y}$	**M1**	
	Denominator: Substitution of $1 + \tan^2 y = \sec^2 y$ or substitution of $1 + \tan^2 y = 1 + \frac{\sin^2 y}{\cos^2 y}$ **and** correct rearrangement to $\frac{1}{\cos^2 y}$ oe	**M1**	
	Correct completion to $4\sin y$ cao	**A1**	

Question	Answer	Marks	Partial Marks
9(b)(ii)	–0.848[06…] rot to 3 or more figures	**B1**	with no extras in range
10(a)	$\sqrt{5^2+(-15)^2}$ seen	**M1**	
	$\frac{1}{5\sqrt{10}}(5\mathbf{i}-15\mathbf{j})$ oe, isw	**A1**	
10(b)(i)	$\begin{pmatrix}9\\12\end{pmatrix}$ oe, soi	**B1**	
	$\begin{pmatrix}3\\-5\end{pmatrix}+\frac{2}{3}\left(their\begin{pmatrix}9\\12\end{pmatrix}\right)$ oe, soi or $\begin{pmatrix}12\\7\end{pmatrix}-\frac{1}{3}\left(their\begin{pmatrix}9\\12\end{pmatrix}\right)$ oe, soi	**M1**	
	$\begin{pmatrix}9\\3\end{pmatrix}$	**A1**	If B1 M0, award SC1 for a final answer of $\begin{pmatrix}-9\\-3\end{pmatrix}$ oe
10(b)(ii)	Forms a valid vector relationship using $\overrightarrow{DC}$ and e.g. $\overrightarrow{OD}$ or $\overrightarrow{DB}$ e.g. $their\begin{pmatrix}9\\3\end{pmatrix}=\overrightarrow{OD}+\begin{pmatrix}6\\1.25\end{pmatrix}$ oe or $\overrightarrow{DB}=\begin{pmatrix}6\\1.25\end{pmatrix}+\frac{1}{3}\times their\begin{pmatrix}9\\12\end{pmatrix}$ oe	**M1**	or $\overrightarrow{DC}=\overrightarrow{OC}-\overrightarrow{OD}$ $\begin{pmatrix}6\\1.25\end{pmatrix}=their\begin{pmatrix}9\\3\end{pmatrix}-\frac{1}{\lambda}\begin{pmatrix}12\\7\end{pmatrix}$ soi or $\overrightarrow{DC}=\overrightarrow{DB}-\overrightarrow{CB}$ $\begin{pmatrix}6\\1.25\end{pmatrix}=\frac{\lambda-1}{\lambda}\begin{pmatrix}12\\7\end{pmatrix}-\frac{1}{3}\times their\begin{pmatrix}9\\12\end{pmatrix}$ soi
	Finds a correct proportion e.g. $\overrightarrow{OB}=4\overrightarrow{OD}$ oe soi or $3\overrightarrow{OB}=4\overrightarrow{DB}$ oe soi	**A1**	or solves a correct equation in λ e.g. $6=9-\frac{1}{\lambda}\times 12$
	$\lambda=4$	**A1**	from a fully correct method
11(i)	$v\neq 0$ or $v>0$ oe	**B1**	
11(ii)	Differentiates : $4\times -3(t+1)^{-4}$ oe, isw	**B2**	**B1** for $k(t+1)^{-4}$ with $k\neq -12$
	$-\frac{1}{108}$ oe or –0.00926	**B1**	
11(iii)	Integrates: $[s=]-2(t+1)^{-2}+2$ oe, isw	**B3**	**B2** for $-2(t+1)^{-2}$ $[+c]$ or **B1** for $k(t+1)^{-2}$ $[+c]$ with $k\neq -2$

Question	Answer	Marks	Partial Marks
11(iv)	Finds *their* s from **(iii)** when $t = 4$ or when $t = 3$ or finds $\left[\textit{their}\left(-2(t+1)^{-2}\ [+2]\ \right)\right]_3^4$ $= \textit{their}\left(-2(4+1)^{-2} - \left(-2(3+1)^{-2}\right)\right)$	**M1**	
	$\frac{9}{200}$ or 0.045	**A1**	
12(a)(i)	g > –9	**B1**	
12(a)(ii)	$x > 1$	**B1**	
12(a)(iii)	$[\mathrm{gf}(x) =]\quad 4(5x-2)^2 - 9$	**B1**	
	$100x^2 - 80x - 38 = 0$ or $(5x-2)^2 = \frac{45+9}{4}$	**M1**	
	$[x =]\quad \frac{-(-80) \pm \sqrt{(-80)^2 - 4(100)(-38)}}{2(100)}$ leading to $\frac{4+3\sqrt{6}}{10}$ oe only or $\frac{1}{5}\left(2+\sqrt{\frac{54}{4}}\right)$ or better only	**A1**	
12(b)(i)	(They are) **reflection**s (of each other) in (the line) $\boldsymbol{y = x}$ oe	**B1**	
12(b)(ii)	$x^2 = y^2 + 1$ or $y^2 = x^2 + 1$	**M1**	
	$x = [\pm]\sqrt{y^2+1}$ or $y = [\pm]\sqrt{x^2+1}$	**A1**	
	$-\sqrt{x^2+1}$ nfww	**A1**	