Cambridge International Examinations
Cambridge Ordinary Level

ADDITIONAL MATHEMATICS **4037/11**

Paper 1 **May/June 2016**

MARK SCHEME

Maximum Mark: 80

Published

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

Question	Answer	Marks	Guidance
1 (i)	-27	**B1**	
(ii)	$9-8k=0$	**M1**	for use of discriminant with a complete method to get to $k=$
	$k=\frac{9}{8}$	**A1**	
	Or $\frac{\mathrm{d}y}{\mathrm{d}x}=4x-3$ when $\frac{\mathrm{d}y}{\mathrm{d}x}=0$, $x=\frac{3}{4}$	**M1**	for a complete method to get to $k=$
	so $k=\frac{9}{8}$	**A1**	
	Or completing the square $y=2\left(x-\frac{3}{4}\right)^2+k-\frac{9}{8}$	**M1**	for a complete method to get to $k=$
	$k=\frac{9}{8}$	**A1**	
2 (a)	$2^{4(3x-1)}=2^{3(x+2)}$ or $4^{2(3x-1)}=4^{\frac{3}{2}(x+2)}$ or $8^{\frac{4}{3}(3x-1)}=8^{x+2}$ or $16^{3x-1}=16^{\frac{3}{4}(x+2)}$	**B1**	**B1** for a correct statement
		M1	for equating indices
	leading to $x=\frac{10}{9}$ cao	**A1**	
(b)	$p=\frac{5}{3}$	**B1**	
	$q=-2$	**B1**	

Question	Answer	Marks	Guidance
3	On x-axis, $2x^2 - 7 = 1$ $x = 2$	M1 A1	for equating to 1
	$\frac{dy}{dx} = \frac{4x}{2x^2 - 7}$	B1	
	When $x = 2$, $\frac{dy}{dx} = 8$		
	Gradient of normal = $-\frac{1}{8}$		
	Equation of normal $y = -\frac{1}{8}(x - 2)$	M1	for attempt at perpendicular through *their* $(2, 0)$, must be using $y = 0$
	Required form $x + 8y - 2 = 0$	A1	must be equated to zero with integer coefficients
4 (a)	$\mathbf{A}^2 = \begin{pmatrix} 7 & -2 \\ -3 & 6 \end{pmatrix}$	B1	
	$\mathbf{A}^2 - 2\mathbf{B} = \begin{pmatrix} 1 & -2 \\ -5 & 2 \end{pmatrix}$	M1 A1	for their $\mathbf{A}^2 - 2\mathbf{B}$
(b)	$\begin{pmatrix} 4 & 1 \\ 10 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$		
	so $\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 3 & -1 \\ -10 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$	M1	for pre-multiplication by *their* inverse matrix
	leading to $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$	DM1	**DM1** for attempt at matrix multiplication
	$x = 1$	A1	Allow in matrix form
	$y = -3$	A1	
5 (i)	$\frac{d}{dx}\left(\frac{e^{4x}}{4} - xe^{4x}\right) = e^{4x} - \left(\left(x \times 4e^{4x}\right) + e^{4x}\right)$	B1	for $\frac{d}{dx}\left(\frac{e^{4x}}{4}\right) = e^{4x}$
		M1 A1	for attempt to differentiate a product for a correct product
	$= -4xe^{4x}$	A1	for correct final answer
(ii)	$\int_0^{\ln 2} xe^{4x}\, dx = -\frac{1}{4}\left[\frac{e^{4x}}{4} - xe^{4x}\right]_0^{\ln 2}$	B1FT	**FT** for use of *their* $\frac{1}{p} \times \left(\frac{e^{4x}}{4} - xe^{4x}\right)$, must be numerical *p*, but $\neq 0$
	$= -\frac{1}{4}\left(\left(\frac{16}{4} - 16\ln 2\right) - \frac{1}{4}\right)$	B1 M1	for $e^{4\ln 2} = 16$ for correct use of limits, must be an integral of the correct form
	$= 4\ln 2 - \frac{15}{16}$	A1	

Question	Answer	Marks	Guidance
6 (i)	$2-\sqrt{5} < \mathrm{f}(x) \leqslant 2$	B2	**B1** for $\leqslant 2$ **B1** for $2-\sqrt{5}<$ or awrt –0.24 Must be using f, f(x) or y, $2-\sqrt{5}<$, if not then **B1** max
(ii)	 $\mathrm{f}^{-1}(x)=(2-x)^2-5$ Domain $2-\sqrt{5}<x\leqslant 2$ Range y or $-5\leqslant \mathrm{f}^{-1}(x)<0$	M1 A1 B1 B1	for a correct method to find the inverse Must be using the correct variables for the B marks
(iii)	$\mathrm{fg}(x)=\mathrm{f}\left(\frac{4}{x}\right)$ $=2-\sqrt{\frac{4}{x}+5}$ leading to $x=-4$	M1 DM1 A1	for correct order of functions for solution of equation
7 (i)	Finding an angle of 68.2° or 21.8° $\frac{4.5}{\sin 68.2}=\frac{2.4}{\sin\alpha}$ leading to $\alpha=29.7^\circ$ (allow ± 0.1) Direction is 82.1° to the bank, upstream(allow $\pm 0.1^\circ$)	B1 B1 B1 B1	 for the sine rule
(ii)	$\frac{4.5}{\sin 68.2}=\frac{2.4}{\sin 29.7}=\frac{v_r}{\sin 82.1}$ leading to $v_r=4.8$ time taken = $\frac{80.78}{4.8}=16.8$	B1 B1 M1 A1	for the sine rule for resultant velocity for attempt to find AB and hence the time taken
	Alternative method: Finding an angle of 68.2° or 21.8° $4.5^2=2.4^2+v_r^2-(2\times 2.4\times v_r\cos 68.2)$ leading to $v_r=4.8$	 B1 B1 B1	 for correct use of the cosine rule for resultant velocity
	Use of sine rule to obtain angle and direction to obtain direction is 82.1° to the bank, upstream	B1 B1 B1	for use of the sine rule for $\alpha=29.7^\circ$ for 82.1°
	 Use of time taken = $\frac{80.78}{4.8}=16.8$	M1 A1	for attempt to find AB and hence the time taken

Question	Answer	Marks	Guidance
8 (i)	$y-6=-\frac{4}{12}(x+8)$ $(\ 3y+x=10\ )$	M1 A1	for a correct method allow unsimplified
(ii)	$y-7=3(x+1)$ $(y=3x+10)$	DM1 A1	for attempt at a perpendicular line using $(-1, 7)$ allow unsimplified
(iii)	point of intersection $(-2, 4)$ which is the midpoint of AB	M1 M1 A1	for attempt to find the point of intersection using simultaneous equations for attempt to find midpoint for all correct
	Alternative method: Midpoint $(-2, 4)$ Verification that this point lies on CP.	M1 M1 A1	for attempt to find midpoint for full verification for all correct
(iv)	$CP=\sqrt{10}$ or 3.16	B1	
(v)	Area = $\frac{1}{2}\times\sqrt{10}\times4\sqrt{10}$ $=20$	M1 A1	for correct method **using *CP*** for 19.9 – 20.1

Question	Answer	Marks	Guidance
9 (i)	$2\cos x\cot x = \cot x + 2\cos x$		
	$2\cos x\dfrac{\cos x}{\sin x}+1=\dfrac{\cos x}{\sin x}+2\cos x$	M1	for use of $\cot x=\dfrac{\cos x}{\sin x}$ for both terms
	$2\cos^2 x+\sin x=\cos x+2\cos x\sin x$	DM1	for multiplication throughout by $\sin x$
	$2\cos^2 x-2\cos x\sin x=\cos x-\sin x$		
	$2\cos x(\cos x-\sin x)=\cos x-\sin x$	DM1	for attempt to factorise
	$(2\cos x-1)(\cos x-\sin x)=0$	A1	for completely correct solution www
	Alternative method:		
	$a\cos^2 x-a\cos x\sin x-b\cos x$ $+b\sin x=0$	M1	for expansion of RHS
	$a\cos x\cot x-a\cos x-b\cot x+b=0$	DM1 DM1	for division by $\sin x$ for comparing like terms to obtain both a and b
	$a=2,\quad b=1$	A1	for both correct www
(ii)	$(2\cos x-1)(\cos x-\sin x)=0$		
	$\cos x=\dfrac{1}{2}$, $\tan x=1$	M1	for either
	$x=\dfrac{\pi}{3}$, $x=\dfrac{\pi}{4}$	A1,A1	**A1** for each, penalise extra solutions within the range by withholding the last A mark
	Alternative method: $(2\cos x-1)(\cot x-1)=0$		
	Leading to $\cos x=\dfrac{1}{2}$, $\tan x=1$	M1	for attempt to factorise the original equation and attempt to solve
	$x=\dfrac{\pi}{3}$, $x=\dfrac{\pi}{4}$	A1,A1	**A1** for each, penalise extra solutions within the range by withholding the last A mark
10 (i)	$f(-2)=-32-2k+p=0$	M1	for attempt at $f(-2)$
	$f'\left(\dfrac{1}{2}\right)=\dfrac{12}{4}+k=0$	M1	for attempt at $f'\left(\dfrac{1}{2}\right)$
	leading to $k=-3$ and $p=26$	A1,A1	**A1** for each
(ii)		B1FT	**FT** for *their* $\dfrac{p}{2}$
	$(x+2)(4x^2-8x+13)$	B1	all correct
(iii)	Showing that $4x^2-8x+13=0$ has no real roots	M1,	M1 for a valid attempt at solution of equation leading to no solution or consideration of the discriminant
	so $x=-2$ only www	A1	

Question	Answer	Marks	Guidance
11 (i)	$AB = 2r\sin\theta$ or $\sqrt{r^2 + r^2 - 2r^2\cos 2\theta}$ or $\dfrac{r\sin 2\theta}{\sin\left(\dfrac{\pi}{2} - \theta\right)}$ or $\dfrac{r\sin 2\theta}{\cos\theta}$	B1	
(ii)	$2r\sin\theta + 2r\theta = 20$ $r = \dfrac{10}{\theta + \sin\theta}$	M1 A1	for use of (i) + arc length = 20, oe must be convinced
(iii)	$\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -\dfrac{10(1+\cos\theta)}{(\theta + \sin\theta)^2}$ When $\theta = \dfrac{\pi}{6}$, $\dfrac{\mathrm{d}r}{\mathrm{d}\theta} = -17.8$	M1 A2,1,0 A1	for a correct attempt to differentiate –1 each error allow awrt -17.8
(iv)	$\dfrac{\mathrm{d}r}{\mathrm{d}t} = 15$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = \dfrac{\mathrm{d}r}{\mathrm{d}t} \div \dfrac{\mathrm{d}r}{\mathrm{d}\theta}$ $\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = -0.842$	B1 M1 A1	may be implied for use of $\dfrac{15}{\textit{their}\text{ (iii)}}$ allow -0.84 or -0.843