

**CAMBRIDGE INTERNATIONAL EXAMINATIONS**  
GCE Ordinary Level

**MARK SCHEME for the May/June 2014 series**

**4037 ADDITIONAL MATHEMATICS**

**4037/12**

Paper 1, maximum raw mark 80

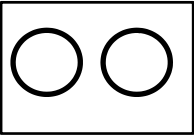
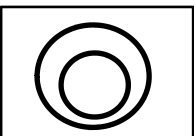
This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

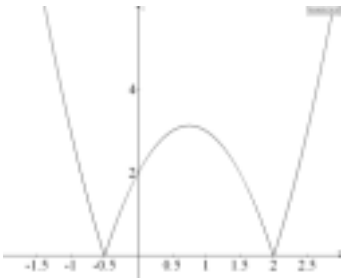
Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2014 series for most IGCSE, GCE Advanced Level and Advanced Subsidiary Level components and some Ordinary Level components.

Page 2	Mark Scheme	Syllabus	Paper
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1	$\frac{\cos^2 A + (1 + \sin A)^2}{(1 + \sin A)\cos A}$ $\frac{\cos^2 A + 1 + 2 \sin A + \sin^2 A}{(1 + \sin A)\cos A}$ $= \frac{2(1 + \sin A)}{(1 + \sin A)\cos A}$ $= \frac{2}{\cos A} = 2 \sec A$ <p><b>Alternative:</b></p> $\frac{\cos A(1 - \sin A)}{(1 + \sin A)(1 - \sin A)} + \frac{1 + \sin A}{\cos A}$ $= \frac{\cos A(1 - \sin A)}{1 - \sin^2 A} + \frac{1 + \sin A}{\cos A}$ $= \frac{\cos A(1 - \sin A)}{\cos^2 A} + \frac{1 + \sin A}{\cos A}$ $= \frac{1 - \sin A}{\cos A} + \frac{1 + \sin A}{\cos A}$ $= \frac{2}{\cos A} = 2 \sec A$	<p><b>M1</b></p> <p><b>M1</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>M1</b> for obtaining a single fraction, correctly</p> <p><b>M1</b> for expansion of <math>(1 + \sin A)^2</math> and use of identity</p> <p><b>DM1</b> for factorisation and cancelling of <math>(1 + \sin A)</math> factor</p> <p><b>A1</b> for use of <math>\frac{1}{\cos A} = \sec A</math> and final answer</p> <p><b>M1</b> for multiplying first term by <math>\frac{1 - \sin A}{1 - \sin A}</math></p> <p><b>M1</b> for expansion of <math>(1 - \sin A)(1 + \sin A)</math> and use of identity</p> <p><b>M1</b> for simplification of the 2 terms</p> <p><b>A1</b> for use of <math>\frac{1}{\cos A} = \sec A</math> and final answer</p>
2 (a) (i)	  (i)   (b) (i) 6 (ii) 5 (iii) 9	<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p>	

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<p>3 (i)</p>  <p>(ii)</p> <p>Maximum point occurs when <math>y = \frac{25}{8}</math></p> <p>so <math>k &gt; \frac{25}{8}</math></p>		<p><b>B1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>B1</b> for shape</p> <p><b>B1</b> for <math>y = 2</math> (must have a graph)</p> <p><b>B1</b> for <math>x = -0.5</math> and <math>2</math> (must have a graph)</p> <p><b>M1</b> for obtaining the value of <math>y</math> at the maximum point, by either completing the square, differentiation, use of discriminant or symmetry.</p> <p><b>A1</b> Must have the correct sign for <b>A1</b> Ignore any upper limits</p>
<p>4</p>	$\int_0^a \sin 3x \, dx = \frac{1}{3} \quad dx = \frac{1}{3}$ $\left[ -\frac{2}{3} \cos 3x \right]_0^a = \frac{1}{3}$ $\left( -\frac{2}{3} \cos 3a \right) - \left( -\frac{2}{3} \right) = \frac{1}{3}$ $\cos 3a = 0.5$ $3a = \frac{\pi}{3}, \quad a = \frac{\pi}{9}$	<p><b>B1, B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>B1</b> for <math>k \cos 3x</math> only, <b>B1</b> for <math>-\frac{2}{3} \cos 3x</math> only</p> <p><b>M1</b> for correct substitution of the correct limits into their result</p> <p><b>A1</b> for correct equation</p> <p><b>M1</b> for correct method of solution of equation of the form <math>\cos ma = k</math></p> <p><b>A1</b> allow 0.349, must be a radian answer</p>
<p>5 (i)</p> <p>(ii)</p>	<p><math>2^{5x} \times 2^{2y} = 2^{-3}</math> leads to <math>5x + 2y = -3</math></p> <p><math>7^x \times 49^{2y} = 1</math> can be written as <math>x + 4y = 0</math></p> <p>Solving <math>5x + 2y = -3</math> and <math>x + 4y = 0</math> leads to</p> $x = -\frac{2}{3}, \quad y = \frac{1}{6}$	<p><b>B1, B1</b></p> <p><b>DB1</b></p> <p><b>B1</b></p> <p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>B1</b> for <math>2^{2y}</math>, <b>B1</b> for <math>2^{-3}</math>, <b>B1</b> for dealing with indices correctly to obtain given answer</p> <p><b>B1</b> for either <math>7^{4y}</math> or <math>7^0</math> seen</p> <p><b>B1</b> for <math>x + 4y = 0</math></p> <p><b>M1</b> for solution of their simultaneous equations, must both be linear</p> <p><b>A1</b> for both, allow equivalent fractions only</p>

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6	(a) YX and ZY	<b>B1,B1</b>	<b>B1</b> for each, must be in correct order,
	(b) $\mathbf{B} = \mathbf{A}^{-1} \begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix},$	<b>M1</b>	<b>M1</b> for pre-multiplication by $\mathbf{A}^{-1}$
	$= -\frac{1}{3} \begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$	<b>B1,B1</b>	<b>B1</b> for $-\frac{1}{3}$ , <b>B1</b> for $\begin{pmatrix} 1 & 2 \\ 4 & 5 \end{pmatrix}$
	$= -\frac{1}{3} \begin{pmatrix} -9 & 3 \\ -18 & 21 \end{pmatrix}$ or $\begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$	<b>DM1</b>	<b>DM1</b> for attempt at matrix multiplication
	<b>Alternative method:</b>	<b>A1</b>	<b>A1</b> allow in either form
	$\begin{pmatrix} 5 & -2 \\ -4 & 1 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 3 & 9 \\ -6 & -3 \end{pmatrix}$	<b>M1</b>	<b>M1</b> for a complete method to obtain 4 equations
	Leads to $5a - 2c = 3$ , $5b - 2d = 9$ $-4a + c = -6$ , $-4b + d = -3$	<b>A2,1,0</b>	-1 for each incorrect equation
	Solutions give matrix	<b>M1</b>	<b>M1</b> for solution to find 4 unknowns
	$-\frac{1}{3} \begin{pmatrix} -9 & 3 \\ -18 & 21 \end{pmatrix}$ or $\begin{pmatrix} 3 & -1 \\ 6 & -7 \end{pmatrix}$	<b>A1</b>	<b>A1</b> for a correct, final matrix

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<p>7 (i)</p>	<p><math>\sin \frac{\theta}{2} = \frac{6}{8}, \frac{\theta}{2} = 0.8481</math> or better</p> <p>or <math>12^2 = 8^2 + 8^2 - 128 \cos \theta</math></p> <p><math>\theta = 1.6961</math> or better</p> <p>or using areas</p> <p><math>\frac{1}{2} \times 12 \times 2\sqrt{7} = \frac{1}{2} 8^2 \sin \theta</math> oe</p> <p><math>\sin \theta = 0.9922, \theta = 1.4455</math> or 1.6961</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>M1</b> for a complete method to find either <math>\theta</math> or <math>\frac{\theta}{2}</math></p> <p>Answer given.</p> <p><b>M1</b> for using the area of the triangle in 2 different forms</p> <p><b>A1</b> for choosing the correct angle.</p>
<p>(ii)</p>	<p>Arc length = <math>(2\pi - 1.696) \times 8</math></p> <p>(36.697 or 36.7)</p> <p>Perimeter = <math>12 + (2\pi - 1.696) \times 8</math></p> <p>= 48.7</p>	<p><b>M1</b></p> <p><b>A1</b></p> <p><b>A1</b></p>	<p><b>M1</b> for correct attempt at a minor or major arc length</p> <p><b>A1</b> for correct major arc length, allow unsimplified</p> <p><b>A1</b> for 48.7 or better</p>
<p>(iii)</p>	<p>Area = <math>\frac{8^2}{2} (2\pi - 1.696) + \frac{8^2}{2} \sin 1.696</math></p> <p>= 178.5, 178.6, awrt 179</p> <p><b>Alternative:</b></p> <p>Area = <math>\pi 8^2 - \left( \frac{1}{2} 8^2 (1.696) - \frac{8^2}{2} \sin 1.696 \right)</math></p>	<p><b>M1, M1</b></p> <p><b>A1</b></p>	<p><b>M1</b> for correct attempt to find area of major sector</p> <p><b>M1</b> for correct attempt to find area of triangle, using any method</p> <p><b>M1</b> for attempt at area of circle – area of minor sector</p> <p><b>M1</b> for area of triangle</p>

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8	(a) (i)	720	B1	
	(ii)	240	B1	
	(iii)	Starts with either a 2 or a 4: 48 ways	B1	allow unevaluated
		Does not start with either a 2 or a 4: 96 ways (i.e. starts with 1 or 5)	B1	allow unevaluated
		Total = 144	B1	must be evaluated
	<b>Alternative 1:</b>			
	Ends with a 2, starts with a 1,4 or 5 : 72 ways Ends with a 4, starts with a 1,2 or 5 : 72 ways Total =144		B1 B1 B1	
	<b>Alternative 2:</b>			
	$240 - (2 \times 2 \times {}^4P_3)$ or $(4 \times {}^4P_3 \times 2) - (2 \times {}^4P_3)$ =144		B2 B1	B2 for correct expression seen, allow $P$ notation
	<b>Alternative 3:</b>			
${}^3P_1 \times {}^4P_3 \times {}^2P_1$ or $3 \times 4 \times 2$ =144		B2 B1	Allow $P$ notation here, for B2	
(b)	With twins : ${}^{16}C_4$ (=1820)	B1		
	Without twins: ${}^{16}C_6$ (=8008)	B1		
	Total: 9828	B1		
	<b>Alternative:</b>			
${}^{18}C_6 - (2 \times {}^{16}C_5)$ = 9828		B1,B1 B1	B1 for ${}^{18}C_6 - \dots$ , B1 for $2 \times {}^{16}C_5$	

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<p>9 (i)</p>	$h = \frac{4000}{\pi r^2} \text{ or } \pi r^2 h = 4000$ $A = 2\pi r h + 2\pi r^2$ $A = 2\pi r \frac{4000}{\pi r^2} + 2\pi r^2$	<p><b>B1</b></p> <p><b>M1</b></p> <p><b>A1</b></p>	<p><b>M1</b> for substitution of <math>h</math> or <math>\pi r h</math> into <i>their</i> equation for <math>A</math></p> <p><b>A1</b> Answer given</p>
<p>(ii)</p>	$\frac{dA}{dr} = -\frac{8000}{r^2} + 4\pi r$ <p>When <math>\frac{dA}{dr} = 0</math>, <math>r^3 = \frac{8000}{4\pi}</math></p> <p>leading to <math>A = 1395, 1390</math></p> $\frac{d^2 A}{dr^2} = \frac{16000}{r^3} + 4\pi,$ <p>which, is positive so a minimum.</p>	<p><b>B1, B1</b></p> <p><b>M1</b></p> <p><b>M1</b></p> <p><b>A1</b></p> <p><math>\sqrt{\mathbf{B1}}</math></p>	<p><b>B1</b> for each term correct</p> <p><b>M1</b> for equating to zero and attempt to find <math>r^3</math></p> <p><b>M1</b> for substitution of their <math>r</math> to obtain <math>A</math>.</p> <p><b>A1</b> for 1390 or awrt 1395</p> <p><math>\sqrt{\mathbf{B1}}</math> for a complete correct method and conclusion.</p>

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10 (i)	Velocity = $26 \times \frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$ = $10\mathbf{i} + 24\mathbf{j}$	M1	M1 for $\frac{1}{13}(5\mathbf{i} + 12\mathbf{j})$
		A1	
	<b>Alternative 1:</b> $ 10\mathbf{i} + 24\mathbf{j}  = \sqrt{10^2 + 24^2}$ = 26	M1	M1 for working from given answer to obtain the given speed
	Showing that one vector is a multiple of the other, hence same direction	A1	A1 for a completely correct method
	<b>Alternative 2:</b> $\sqrt{5^2 + 12^2} = 13$ , $13k = 26$ , so $k = 2$ Velocity = $2(5\mathbf{i} + 12\mathbf{j})$ ,	M1	M1 for attempt to obtain the 'multiple' and apply to the direction vector
	Velocity = $10\mathbf{i} + 24\mathbf{j}$	A1	A1 for a completely correct method
	<b>Alternative 3:</b> Use of trig: $\tan \alpha = \frac{12}{5}$ , $\alpha = 67.4^\circ$ Velocity $26 \cos 67.4^\circ \mathbf{i} + 26 \sin 67.4^\circ \mathbf{j}$	M1	M1 for reaching this stage
	Velocity = $10\mathbf{i} + 24\mathbf{j}$	A1	A1 for a completely correct method
	(ii) Position vector = $4(10\mathbf{i} + 24\mathbf{j})$ or $40\mathbf{i} + 96\mathbf{j}$	B1	Allow either form for B1
	(iii) $(40\mathbf{i} + 96\mathbf{j}) + (10\mathbf{i} + 24\mathbf{j})t$ oe	M1	M1 for <i>their</i> (ii) + $(10\mathbf{i} + 24\mathbf{j})t$ or $(10\mathbf{i} + 24\mathbf{j}) \times (t + 4)$
	A1	A1 correct answer only	
(iv) $(120\mathbf{i} + 81\mathbf{j}) + (-22\mathbf{i} + 30\mathbf{j})t$ oe	B1		
(v) $40 + 10t = 120 - 22t$ or $96 + 24t = 81 + 30t$  $t = 2.5$ or 18:30  Position vector = $65\mathbf{i} + 156\mathbf{j}$	M1	M1 for equating like vectors	
	A1	A1 Allow for $t = 2.5$	
	DM1	DM1 for use of $t$ to obtain position vector	
	A1	A1 cao	



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<b>11 (a)</b>	$\tan x(\tan x + 5) = 0$ $\tan x = 0, \quad x = 0^\circ, 180^\circ$ $\tan x = -5, \quad x = 101.3^\circ$	<b>B1,B1</b> <b>B1</b>	<b>B1</b> for each, must be from correct work
<b>(b)</b>	$2(1 - \sin^2 y) - \sin y - 1 = 0$ $2 \sin^2 y + \sin y - 1 = 0$ $(2 \sin y - 1)(\sin y + 1) = 0$ $\sin y = \frac{1}{2}, y = 30^\circ, 150^\circ$  $\sin y = -1, y = 270^\circ$	<b>M1</b>  <b>A1,A1</b>  <b>A1</b>	<b>M1</b> for use of correct identity and attempt to solve resulting 3 term quadratic equation.
<b>(c)</b>	$\cos\left(2z - \frac{\pi}{6}\right) = \frac{1}{2}$  $\left(2z - \frac{\pi}{6}\right) = \frac{\pi}{3}$ $z = \frac{\pi}{4}$ or 0.785 or better  $\left(2z - \frac{\pi}{6}\right) = \frac{5\pi}{3}$  $z = \frac{11\pi}{12}$ or 2.88 or better	<b>M1</b>  <b>A1</b>  <b>M1</b>  <b>A1</b>	<b>M1</b> for dealing with sec correctly and obtaining $\frac{\pi}{3}$ or 1.05  <b>M1</b> for obtaining a second equation $\left(2z - \frac{\pi}{6}\right) = 2\pi - \text{their } \frac{\pi}{3}$ oe