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CANDIDATE NAME
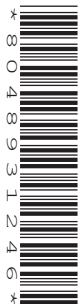
CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS   **0580/42**

Paper 4 (Extended)   **October/November 2021**

                                                                                                              2 hours 30 minutes

You must answer on the question paper.

You will need: Geometrical instruments

INSTRUCTIONS
●   Answer **all** questions.
●   Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
●   Write your name, centre number and candidate number in the boxes at the top of the page.
●   Write your answer to each question in the space provided.
●   Do **not** use an erasable pen or correction fluid.
●   Do **not** write on any bar codes.
●   You should use a calculator where appropriate.
●   You may use tracing paper.
●   You must show all necessary working clearly.
●   Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
●   For π, use either your calculator value or 3.142.

INFORMATION
●   The total mark for this paper is 130.
●   The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

                                                                                                              [Turn over

1 **(a)** Malena has 450 fruit trees.
The fruit trees are in the ratio   apple : pear : plum = 8 : 7 : 3.

    (i) Show that Malena has 200 apple trees.

[2]

    (ii) Find the number of plum trees.

.... [1]

    (iii) Malena wants to increase the number of pear trees by 32%.

Calculate the number of extra pear trees she needs.

.... [2]

    (iv) Each apple tree produces 48.5 kg of apples.
The apples have an average mass of 165 g each.

Calculate the total number of apples produced by the 200 trees.
Give your answer correct to the nearest 1000 apples.

.... [3]

(b) Malena's land is valued at three million and seventy-five thousand dollars.

    (i) Write this number in figures.

                .... [1]

    (ii) Write your answer to **part (b)(i)** in standard form.

                .... [1]

(c) In 2020, each plum tree produced 37.7 kg of plums.
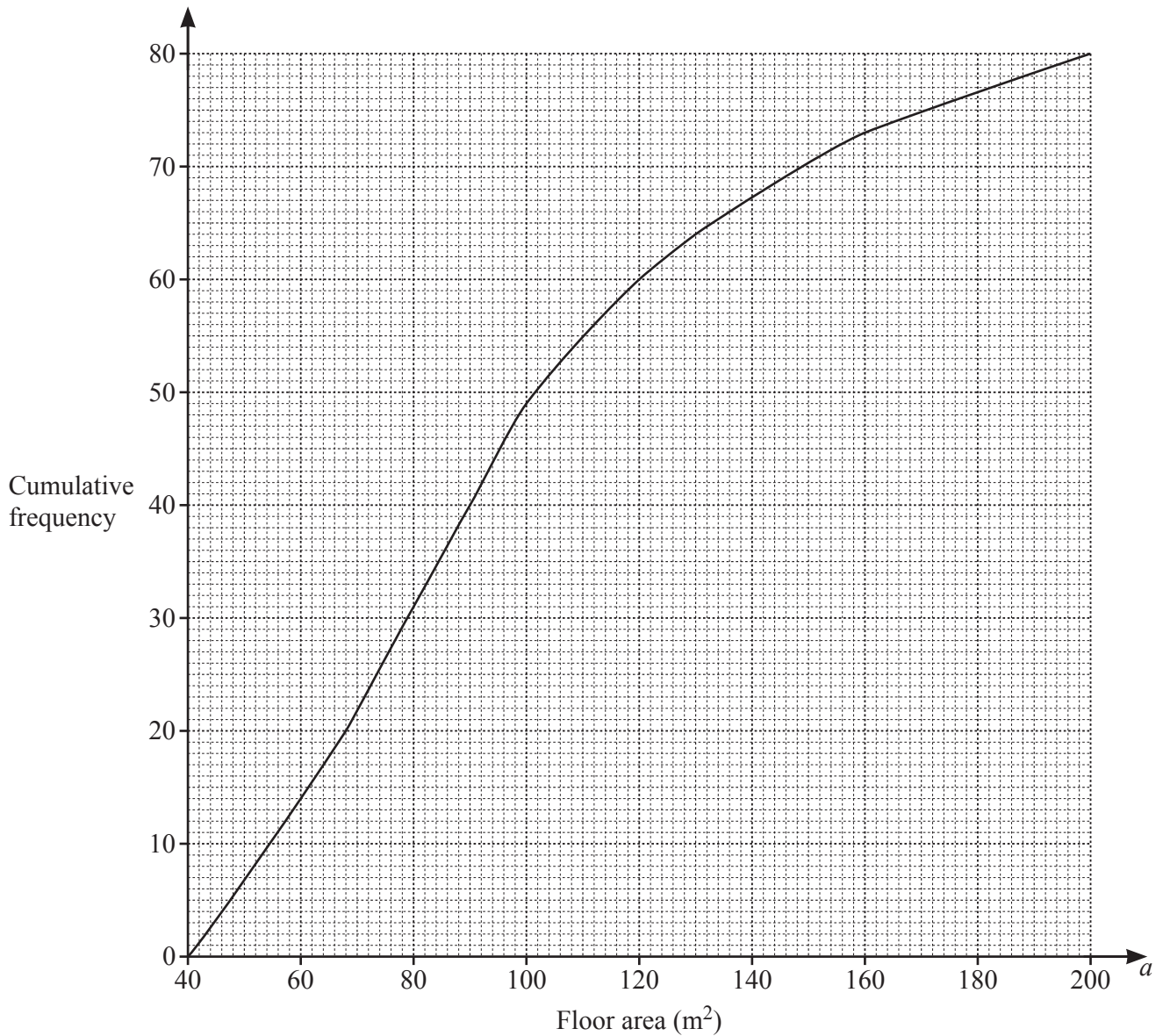This was 16% more than in 2019.

Calculate the mass of plums produced by each plum tree in 2019.

.... kg [2]

(d) Malena invests \$1800 at a rate of 2.1% per year compound interest.

Calculate the value of her investment at the end of 15 years.

\$ .... [2]

2 **(a)** The cumulative frequency diagram shows information about the floor area, $a \, m^2$, of each of 80 houses.

Use the diagram to find an estimate of

(i) the median,

.... m^2 [1]

(ii) the lower quartile,

.... m^2 [1]

(iii) the interquartile range,

.... m^2 [1]

(iv) the number of houses with a floor area greater than $120 \, m^2$.

.... [2]

(b) The information about the 80 floor areas is shown in this frequency table.

Floor area (a m^2)	$40 < a \leqslant 60$	$60 < a \leqslant 80$	$80 < a \leqslant 100$	$100 < a \leqslant 130$	$130 < a \leqslant 160$	$160 < a \leqslant 200$
Frequency	14	17	18	15	9	7

(i) Calculate an estimate of the mean floor area.

.... m^2 [4]

(ii) Complete the histogram to show the information in the frequency table.
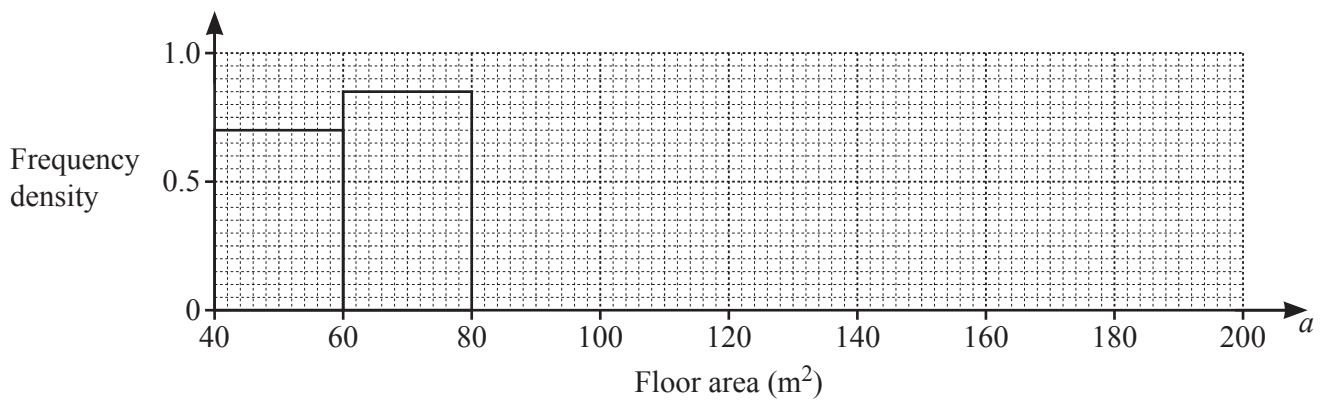
Floor area (m^2)

[4]

(iii) Two of the houses are picked at random.

Find the probability that one of the houses has a floor area greater than $130\,\text{m}^2$ and the other has a floor area $60\,\text{m}^2$ or less.

.... [3]

3 **(a)**
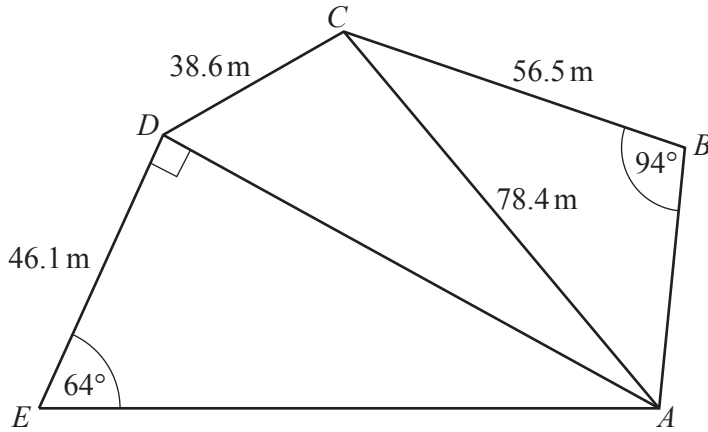
ABCDE is a pentagon.

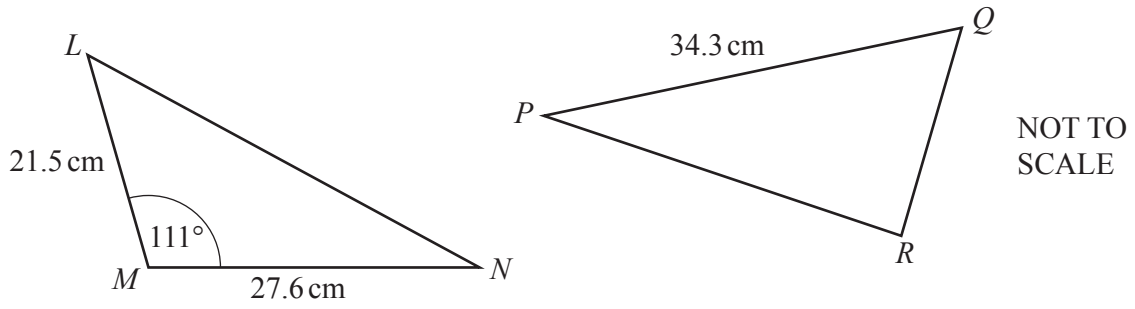**(i)** Calculate *AD* and show that it rounds to 94.5 m, correct to 1 decimal place.

[2]

(ii) Calculate angle *BAC*.

Angle *BAC* = ... [3]

(iii) Calculate the largest angle in triangle *CAD*.

.... [4]

(b)

Triangle *PQR* has the same area as triangle *LMN*.

Calculate the shortest distance from *R* to the line *PQ*.

.... cm [3]

4
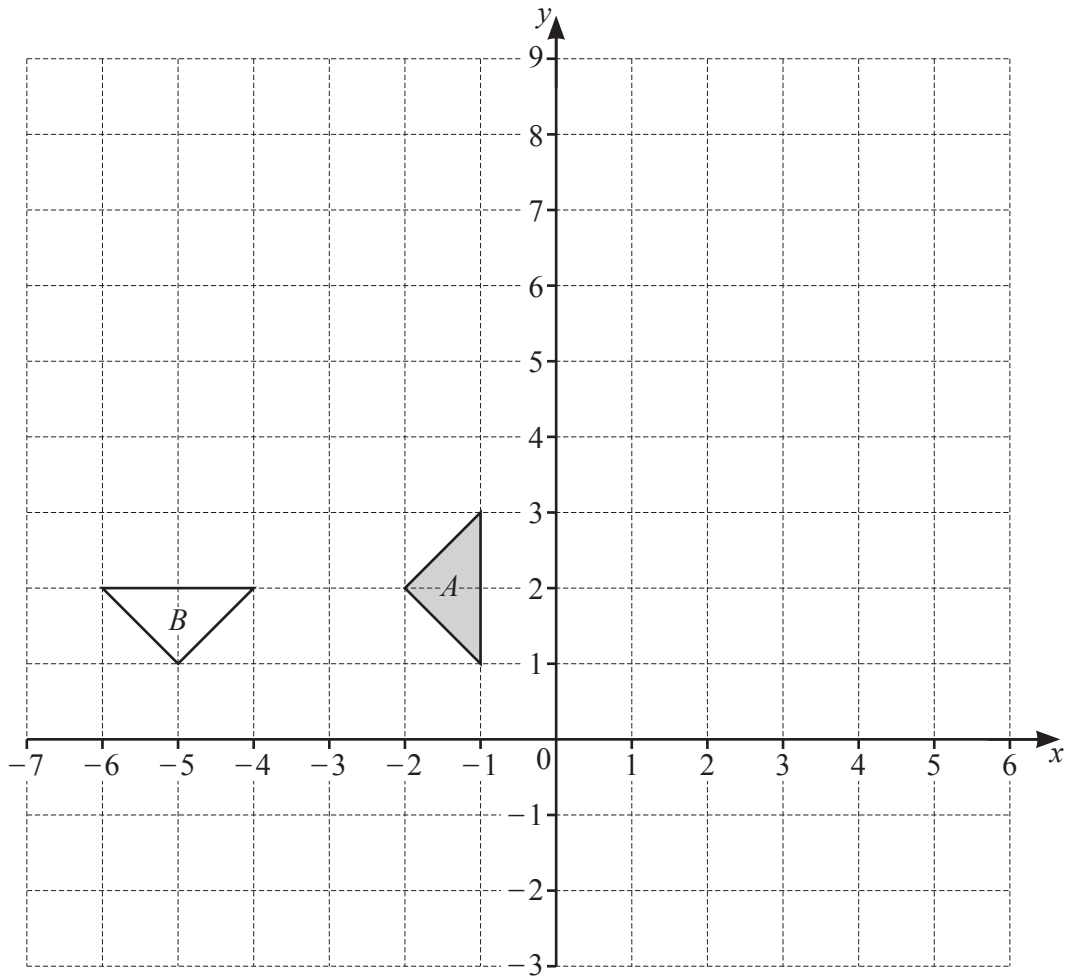
(a) On the grid, draw the image of triangle A after

  (i) a translation by the vector $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$, [2]

  (ii) a reflection in the line $x = 1$, [2]

  (iii) an enlargement, scale factor 2 and centre $(-5, -2)$. [2]

(b) Describe fully the **single** transformation that maps triangle A onto triangle B.
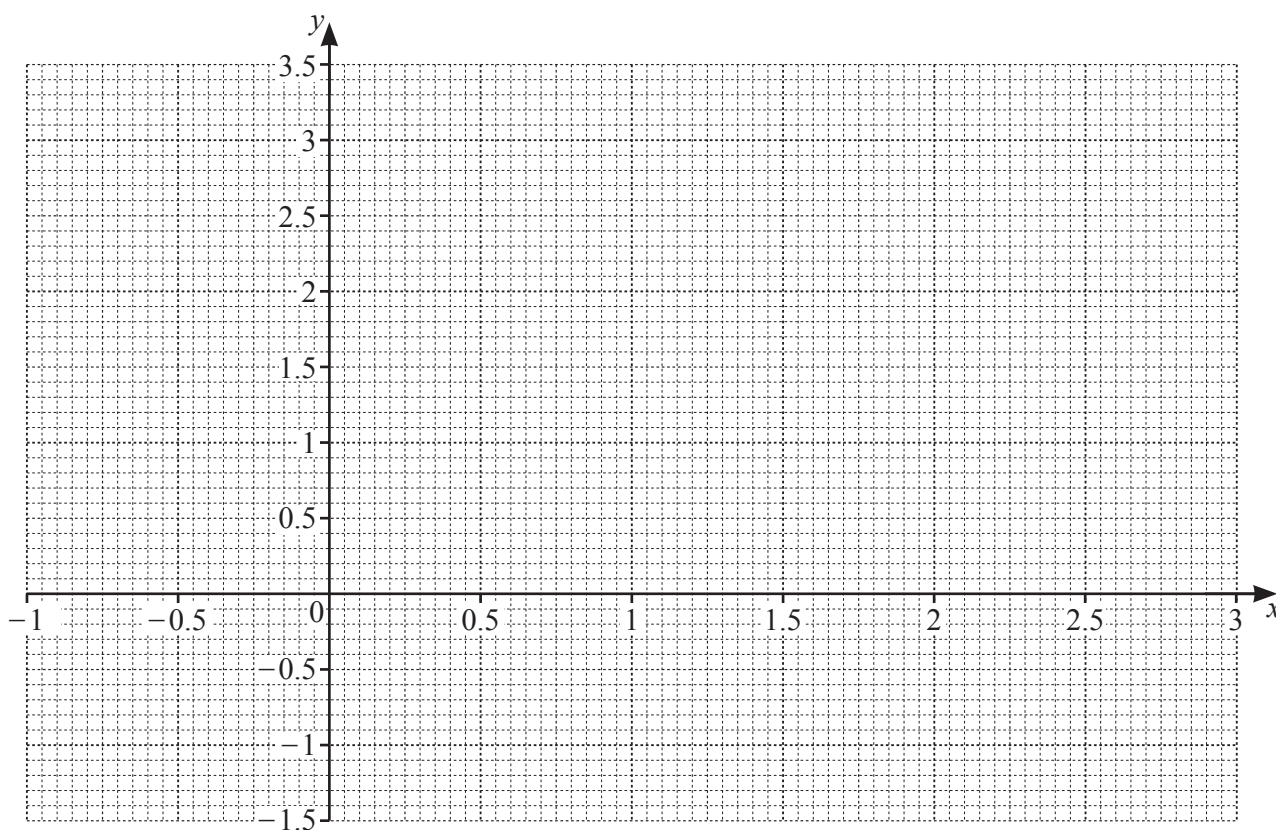
  ....

  .... [3]

5 The table shows some values for $y = x^3 - 3x^2 + 3$.

x	-1	-0.5	0	0.5	1	1.5	2	2.5	3
y		2.125	3	2.375	1		-1	-0.125	

(a) Complete the table. [3]

(b) On the grid, draw the graph of $y = x^3 - 3x^2 + 3$ for $-1 \leqslant x \leqslant 3$.

[4]

(c) By drawing a suitable straight line on the grid, solve the equation $x^3 - 3x^2 + x + 1 = 0$.

$x =$ .... or $x =$ .... or $x =$ .... [4]

6 **(a)** Solve.

(i) $4(2x - 3) = 24$

$x =$ .... [3]

(ii) $6x + 14 > 6$

.... [2]

(b) Rearrange the formula $V = 2x^3 - 3y^3$ to make y the subject.

$y =$ .... [3]

(c) Show that $(2n - 5)^2 - 13$ is a multiple of 4 for all integer values of n.

[3]

(d) The expression $5 + 12x - 2x^2$ can be written in the form $q - 2(x + p)^2$.

    (i) Find the value of p and the value of q.

$$p = \text{................} , q = \text{................} \quad [3]$$

    (ii) Write down the coordinates of the maximum point of the curve $y = 5 + 12x - 2x^2$.

$$(\text{................} , \text{................}) \quad [1]$$

(e) The energy of a moving object is directly proportional to the square of its speed.
The speed of the object is increased by 30%.

Calculate the percentage increase in the energy of the object.

$$\text{..} \, \% \quad [2]$$

7 **(a)** The diagram shows a container for storing grain.

The container is made from a hemisphere, a cylinder and a cone, each with radius 2 m. The height of the cylinder is 5.2 m and the height of the cone is h m.

NOT TO SCALE
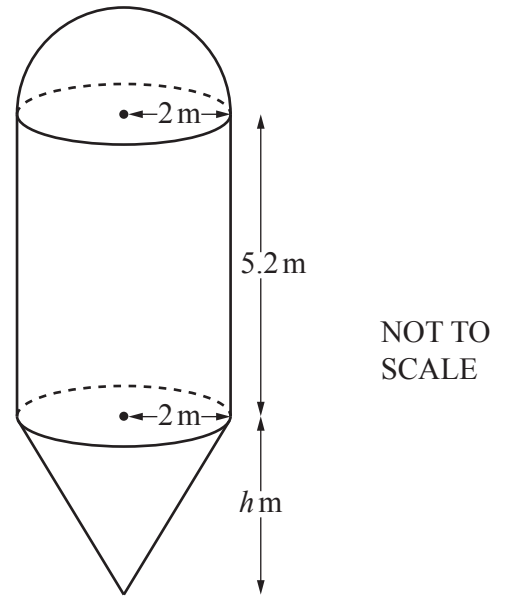
(i) Calculate the volume of the hemisphere. Give your answer as a multiple of π.

[The volume, V, of a sphere with radius r is $V = \dfrac{4}{3}\pi r^3$.]

.... m^3 [2]

(ii) The total volume of the container is $\dfrac{88\pi}{3}$ m^3.

Calculate the value of h.

[The volume, V, of a cone with radius r and height h is $V = \dfrac{1}{3}\pi r^2 h$.]

$h = $ .... [4]

(iii) The container is full of grain.
Grain is removed from the container at a rate of 35 000 kg per hour.
1m^3 of grain has a mass of 620 kg.

Calculate the time taken to empty the container.
Give your answer in hours and minutes.

.... h .... min [3]

(b)
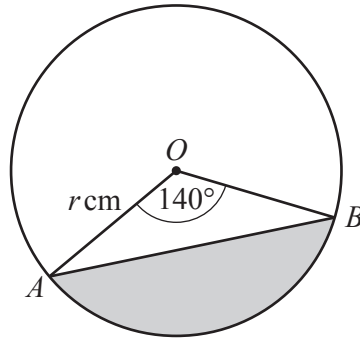
A and *B* are points on a circle, centre *O*, radius *r* cm.
The area of the shaded segment is 65cm^2.

Calculate the value of *r*.

$r =$ .... [4]

8 **(a)** Kaito runs along a 12 km path at an average speed of x km/h.

(i) Write down an expression, in terms of x, for the number of hours he takes.

.... hours [1]

(ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito.
She walks at an average speed of $(x-4)$ km/h.

Write down an equation, in terms of x, and show that it simplifies to $x^2 - 4x - 32 = 0$.

[4]

(iii) Solve by factorisation.
$$x^2 - 4x - 32 = 0$$

$x =$ .... or $x =$ .... [3]

(iv) Find the number of hours it takes Yuki to walk along the 12 km path.

.... hours [2]

(b) A bus travels 440 km, correct to the nearest 10 km.
The time taken to complete the journey is 6 hours, correct to the nearest half hour.

Calculate the lower bound of the speed of the bus.

.... km/h [3]

9 **(a)** F is the point $(5, -2)$ and $\overrightarrow{FG} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$.

Find

(i) the coordinates of point G,

(................... , ...) [1]

(ii) $5\overrightarrow{FG}$,

$\begin{pmatrix} \\ \end{pmatrix}$ [1]

(iii) $\left|\overrightarrow{FG}\right|$.

.... [2]

(b)
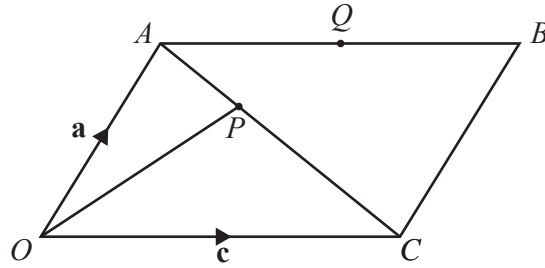
OABC is a parallelogram.
P is a point on *AC* and *Q* is the midpoint of *AB*.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$.

(i) Find, in terms of **a** and/or **c**

    (a) $\overrightarrow{AQ}$,

$\overrightarrow{AQ}$ = ... [1]

    (b) $\overrightarrow{OQ}$.

$\overrightarrow{OQ}$ = ... [1]

(ii) $\overrightarrow{OP} = \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{c}$

    (a) Show that *O*, *P* and *Q* lie on a straight line.

[2]

    (b) Write down the ratio *OP* : *OQ*.
          Give your answer in the form 1 : *n*.

1 : .... [1]

10 **(a)** Find the coordinates of the turning points of the graph of $y = x^3 - 12x + 6$.
You must show all your working.

(........... , ...) and (........... , ...) [5]

(b) Determine whether each turning point is a maximum or a minimum.
Show how you decide.

[3]

11 (a)
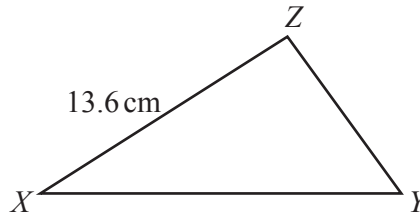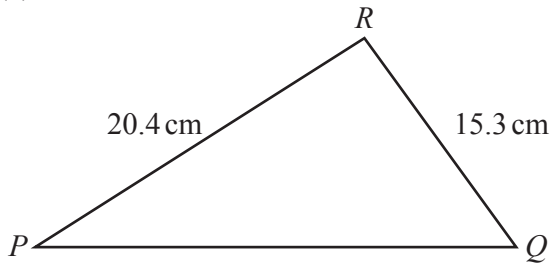
Triangle *PQR* is mathematically similar to triangle *XYZ*.

(i) Find *YZ*.

YZ = ... cm [2]

(ii) The area of triangle *XYZ* is $63.6 \, \text{cm}^2$.

Calculate the area of triangle *PQR*.

.... cm^2 [3]

(b) Two containers are mathematically similar.
The larger container has a capacity of 64.8 litres and a surface area of $0.792 \, \text{m}^2$.
The smaller container has a capacity of 37.5 litres.

Calculate the surface area of the smaller container.

.... m^2 [3]

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