

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2015 series

9709 MATHEMATICS

9709/63

Paper 6, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously ‘correct’ answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

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| AEF | Any Equivalent Form (of answer is equally acceptable) |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| BOD | Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear) |
| CAO | Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed) |
| CWO | Correct Working Only – often written by a 'fortuitous' answer |
| ISW | Ignore Subsequent Working |
| MR | Misread |
| PA | Premature Approximation (resulting in basically correct work that is insufficiently accurate) |
| SOS | See Other Solution (the candidate makes a better attempt at the same question) |
| SR | Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |

Penalties

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| MR –1 | A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting. |
| PA –1 | This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting. |

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| 1 | coded mean = 0.3 oe $sd = \sqrt{\frac{96.1}{250} - (0.3)^2}$ $= 0.543$ Alt: $\Sigma(t-2.5)^2$ expanded $\Sigma t^2 = 2033.6$ $sd = \sqrt{\frac{2033.6}{250} - 2.8^2}$ $= 0.543$ | B1 | $\Sigma(t - 2.5) = 75$ B0 until $\div 250$ |
| | | M1 | Subst in variance formula both terms coded |
| | | A1 3 | Correct answer |
| | | Or B1 | |
| | | M1 | Substituting their Σt^2 from expanded 3-term expression, 250 and 2.8 in variance formula |
| | | A1 3 | |
| 2 | (i) $P(X) = \frac{20}{28} \left(\frac{5}{7} \right) (0.714), 71.4\%$ | B1 1 | oe |
| | (ii) $P(F) = \frac{20}{28} \times \frac{1}{4} \times \frac{8}{28} \times \frac{6}{10} = \frac{7}{20}$ | M1 | Summing two 2-factor probs created by One of $\frac{1}{4}$ or $\frac{3}{4}$ multiplied by $\frac{20}{28}$ or $\frac{8}{28}$ |
| | | A1 2 | Added to $\frac{4}{10}$ or $\frac{6}{10} \times$ altn population prob Correct answer |
| | (iii) $P(X F) = \frac{5/28}{7/20} = \frac{25}{49} (0.510)$ | M1 | Their unsimplified country X probability ($\frac{5}{28}$) as num or denom of a fraction |
| | | A1 2 | Or (their fair hair population) \div (total fair hair pop) Correct answer |
| 3 | (i) $P(S) = \frac{3}{16}$ $P(T) = \frac{4}{16}$ $P(S \cap T) = \frac{2}{16}$ $P(S) \times P(T) = \frac{3}{64} \neq \frac{2}{16}$ Not independent | M1 | Sensible attempt at $P(S)$ |
| | | M1 | Sensible attempt at $P(T)$ |
| | | B1 | Correct $P(S \cap T)$ |
| | | M1 | comp $P(S) \times P(T)$ with $P(S \cap T)$ (their values), evaluated |
| | | A1 5 | Correct conclusion following all correct working |
| | (ii) not exclusive since $P(S \cap T) \neq 0$ Or counter example e.g. 1 and 3 Or $P(S \cup T) \neq P(S) + P(T)$ with values | B1 1 | FT their $P(S \cap T)$, not obtained from $P(S) \times P(T)$, with value and statement. |
| 4 | (i) $z = 1.127$ $1.127 = \frac{136 - 125}{\sigma}$ $\sigma = 9.76$ | B1 | ± 1.127 seen accept rounding to ± 1.13 |
| | | M1 | Standardising no cc no sq rt, with attempt at z |
| | | A1 3 | (not $\pm 0.8078, \pm 0.5517, \pm 0.13, \pm 0.87$) Correct ans |

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|--------------|--|--|--|------|--|---|---|---|-------------------------------------|---|---|---|-------------------------------------|---|---|---|-------------------------------------|---|---|---|-------------------------------------|---|---|---|---|---|---|---|-------------------------------------|---|--|
| (ii) | $P(131 < x < 141) = P\left(\frac{131-125}{9.76} < z < \frac{141-125}{9.76}\right)$ $= \Phi(1.639) - \Phi(0.6147)$ $= 0.9493 - 0.7307$ $= 0.2186$ <p>Number = $0.2186 \times 170 = 37$ or 38 or awrt 37.2</p> | M1 M1 M1 A1 | Standardising once with their sd, no $\sqrt{2}$, allow cc Correct area $\Phi 2 - \Phi 1$ Mult by 170, $P < 1$ Correct answer, nfw | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 5 (a) | e.g. ** (AAOOOI) ***** $\frac{8!}{2!2!} \times \frac{6!}{2!3!} = 604800$ | B1 M1 A1 | 8! ($8 \times 7!$) or 6! seen anywhere, either alone or in numerator) Dividing by at least 3 of 2!2!2!3! (may be fractions added) Correct answer | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| (b) | <table border="0"> <tr> <td>C(7)</td> <td>E(6)</td> <td>A(4)</td> <td></td> </tr> <tr> <td>1</td> <td>1</td> <td>2</td> <td>$= 7 \times 6 \times {}^4C_2 = 252$</td> </tr> <tr> <td>1</td> <td>2</td> <td>1</td> <td>$= 7 \times {}^6C_2 \times 4 = 420$</td> </tr> <tr> <td>1</td> <td>3</td> <td>0</td> <td>$= 7 \times {}^6C_3 \times 1 = 140$</td> </tr> <tr> <td>2</td> <td>1</td> <td>1</td> <td>$= {}^7C_2 \times 6 \times 4 = 504$</td> </tr> <tr> <td>2</td> <td>2</td> <td>0</td> <td>$= {}^7C_2 \times {}^6C_2 \times 1 = 315$</td> </tr> <tr> <td>3</td> <td>1</td> <td>0</td> <td>$= {}^7C_3 \times 6 \times 1 = 210$</td> </tr> </table> <p>Total = 1841</p> | C(7) | E(6) | A(4) | | 1 | 1 | 2 | $= 7 \times 6 \times {}^4C_2 = 252$ | 1 | 2 | 1 | $= 7 \times {}^6C_2 \times 4 = 420$ | 1 | 3 | 0 | $= 7 \times {}^6C_3 \times 1 = 140$ | 2 | 1 | 1 | $= {}^7C_2 \times 6 \times 4 = 504$ | 2 | 2 | 0 | $= {}^7C_2 \times {}^6C_2 \times 1 = 315$ | 3 | 1 | 0 | $= {}^7C_3 \times 6 \times 1 = 210$ | M1 A1 M1* DM1 A1 | Mult 3 appropriate combinations together assume $6 = {}^6C_1, 1 = {}^4C_0$ etc., $\sum r=4$, C&E both present At least 3 correct unsimplified products Listing at least 4 different correct options Summing at least 4 outcomes, involving 3 combs or perms, $\sum r=4$ Correct answer SC if CE removed, M1 available for listing at least 4 different correct options for remaining 2. DM1 for ${}^7C_1 \times {}^6C_1 \times (\text{sum of at least 4 outcomes})$ |
| C(7) | E(6) | A(4) | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 1 | 2 | $= 7 \times 6 \times {}^4C_2 = 252$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 2 | 1 | $= 7 \times {}^6C_2 \times 4 = 420$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 1 | 3 | 0 | $= 7 \times {}^6C_3 \times 1 = 140$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 1 | 1 | $= {}^7C_2 \times 6 \times 4 = 504$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 2 | 2 | 0 | $= {}^7C_2 \times {}^6C_2 \times 1 = 315$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 3 | 1 | 0 | $= {}^7C_3 \times 6 \times 1 = 210$ | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| 6 (i) | fd 0.9, 3, 4.2, 5.2, 1.4 | M1 A1 B1 B1 | Attempt at scaled freq [$f/(attempt\ at\ cw)$] Correct heights seen on diagram Scale no less than 1cm to 1 unit Correct bar widths visually no gaps Labels (ht/metres and fd or freq per 20m etc.) and end points at 20.5 etc. condone 2 end point errors, scale no less than 1cm to 5m for 20,30... unless clearly accurate, linear scale between 20.5 and 80 | | | | | | | | | | | | | | | | | | | | | | | | | | | | |

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| | (ii) $(30.5 \times 18 + 43 \times 15 + 48 \times 21 + 55.5 \times 52 + 70.5 \times 28)/134$ $= \frac{7062}{134} = 52.701$ $\text{Var} = (30.5^2 \times 18 + 43^2 \times 15 + 48^2 \times 21 + 55.5^2 \times 52 + 70.5^2 \times 28)/134 - 52.701^2$ $= 392203.5/134 - 52.701^2 = 149.496$ $\text{sd} = 12.2$ | M1 | Attempt at unsimplified, mid points (at least 4 within 0.5) |
| | | M1 A1 | Attempt at Σfx their mid points $\div 134$ Correct mean rounding to 53 |
| | | M1 | Attempts at Σfx^2 their mid points \div their $\Sigma f - \text{mean}^2$ |
| | | A1 5 | Correct answer, nfw |
| 7 | (i) $P(0, 1, 2) = (0.92)^{19} + {}^{19}C_1(0.08)(0.92)^{18} + {}^{19}C_2(0.08)^2(0.92)^{17}$ $= 0.809$ | M1 M1 | Binomial term ${}^{19}C_x p^x (1-p)^{19-x}$ seen $0 < p < 1$ Correct unsimplified expression |
| | | A1 3 | Correct answer (no working SC B2) |
| | (ii) $P(\text{at least } 1) = 1 - P(0)$ $= 1 - P(0.92)^n > 0.90$ $0.1 > (0.92)^n$ $n > 27.6$ Ans 28 | M1 M1 | Eqn with their 0.92^n , 0.9 or 0.1, 1 not nec Solving attempt by logs or trial and error, power eqn with one unknown power |
| | (iii) $np = 1800 \times 0.08 = 144$ $npq = 132.48$ $P(\text{at least } 152) = P\left(z > \left(\frac{151.5 - 144}{\sqrt{132.48}}\right)\right)$ $= P(z > 0.6516)$ $= 1 - 0.7429$ $= 0.257$ | B1 M1 M1 M1 A1 | correct unsimplified np and npq seen accept 132.5, 132, 11.5, awrt 11.51 standardising, with $\sqrt{\quad}$ cont correction 151.5 or 152.5 seen correct area $1 - \Phi$ (probability) correct answer |
| | (iv) Use because 1800×0.08 (and 1800×0.92 are both) > 5 | B1 1 | $1800 \times 0.08 > 5$ is sufficient $np > 5$ is sufficient if clearly evaluated in (iii) If $npq > 5$ stated then award B0 |