

GCE Examinations  
Advanced Subsidiary

## Core Mathematics C3

Paper L

Time: 1 hour 30 minutes

### *Instructions and Information*

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Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration.

Full marks may be obtained for answers to ALL questions.

Mathematical formulae and statistical tables are available.

This paper has eight questions.

### *Advice to Candidates*

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You must show sufficient working to make your methods clear to an examiner.  
Answers without working may gain no credit.



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1.  $f(x) \equiv \frac{2x-3}{x-2}, x \in \mathbb{R}, x > 2.$

(a) Find the range of  $f$ . (2)

(b) Show that  $ff(x) = x$  for all  $x > 2$ . (3)

(c) Hence, write down an expression for  $f^{-1}(x)$ . (1)

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2. Solve each equation, giving your answers in exact form.

(a)  $e^{4x-3} = 2$  (3)

(b)  $\ln(2y-1) = 1 + \ln(3-y)$  (4)

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3. The curve  $C$  has the equation  $y = 2e^x - 6 \ln x$  and passes through the point  $P$  with  $x$ -coordinate 1.

(a) Find an equation for the tangent to  $C$  at  $P$ . (4)

The tangent to  $C$  at  $P$  meets the coordinate axes at the points  $Q$  and  $R$ .

(b) Show that the area of triangle  $OQR$ , where  $O$  is the origin, is  $\frac{9}{3-e}$ . (4)

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4. (a) Express

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)}$$

as a single fraction in its simplest form. (5)

(b) Hence, show that the equation

$$\frac{x-10}{(x-3)(x+4)} - \frac{x-8}{(x-3)(2x-1)} = 1$$

has no real roots. (4)

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5. Find the values of  $x$  in the interval  $-180 < x < 180$  for which

$$\tan (x + 45)^\circ - \tan x^\circ = 4,$$

giving your answers to 1 decimal place. (9)

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6. (a) Sketch on the same diagram the graphs of  $y = |x| - a$  and  $y = |3x + 5a|$ , where  $a$  is a positive constant.

Show on your diagram the coordinates of any points where each graph meets the coordinate axes. (6)

- (b) Solve the equation

$$|x| - a = |3x + 5a|. \quad (4)$$


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7. (a) Use the identity

$$\cos (A + B) \equiv \cos A \cos B - \sin A \sin B$$

to prove that

$$\cos x \equiv 1 - 2 \sin^2 \frac{x}{2}. \quad (3)$$

- (b) Prove that, for  $\sin x \neq 0$ ,

$$\frac{1 - \cos x}{\sin x} \equiv \tan \frac{x}{2}. \quad (3)$$

- (c) Find the values of  $x$  in the interval  $0 \leq x \leq 360^\circ$  for which

$$\frac{1 - \cos x}{\sin x} = 2 \sec^2 \frac{x}{2} - 5,$$

giving your answers to 1 decimal place where appropriate. (6)

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**Turn over**

8. A curve has the equation  $y = (2x + 3)e^{-x}$ .

(a) Find the exact coordinates of the stationary point of the curve. (4)

The curve crosses the  $y$ -axis at the point  $P$ .

(b) Find an equation for the normal to the curve at  $P$ . (2)

The normal to the curve at  $P$  meets the curve again at  $Q$ .

(c) Show that the  $x$ -coordinate of  $Q$  lies in the interval  $[-2, -1]$ . (3)

(d) Use the iterative formula

$$x_{n+1} = \frac{3 - 3e^{x_n}}{e^{x_n} - 2},$$

with  $x_0 = -1$ , to find  $x_1, x_2, x_3$  and  $x_4$ . Give the value of  $x_4$  to 2 decimal places. (3)

(e) Show that your value for  $x_4$  is the  $x$ -coordinate of  $Q$  correct to 2 decimal places. (2)

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**END**